No. 1605 [EN]

IMI Working Paper

The Effects of Monetary Policy on Input Inventories

Dai Tiantian and Liu Xiangbo

INTERNATIONAL MONETARY INSTITUTE

For further information, please visit
http://www.imi.org.cn/

Weibo

WeChat

IMI Working Paper No. 1605 [EN]

The effects of monetary policy on input inventories

By **DAI TIANTIAN and LIU XIANGBO*[*]**

March 2016

Abstract

This paper explores the long-run and short-run effects of monetary policy on input inventories in a search model with monetary prorogation and two-stage production. Inventories arise endogenously due to search frictions. In the long run, monetary policy has hump-shaped real effects on steady-state input inventory investment, inventory-to-sales ratio as well as sales. We show that the effect of an increase in the money growth rate is driven by both the extensive and intensive margin in the finished goods market. The model is then calibrated to the U.S. data to study the short-run effect of monetary policy. We find that our model can reproduce the stylized facts of input inventories well and input inventories can amplify aggregate fluctuations.

JEL Classification: E32, E52, G31 D83

Keywords: monetary shock; input inventories; business cycles; monetary search

---

[*] Dai Tiantian: Renmin University of China, 59 zhongguancun Street, Beijing, China 100872 (e-mail: dait@ruc.edu.cn); Liu Xiangbo (corresponding author): Renmin University of China, 59 zhongguancun Street, Beijing, China 100872 (e-mail: xiangbo.liu@ruc.edu.cn). We are grateful to Allen Head and Thorsten Koeppl for their enormous guidance and encouragement. We also would like to thank Jonathan Chiu, Oleksiy Kryvtsov, Lealand Morin, Shouyong Shi, Hongfei Sun, and seminar participants at the Canadian Economics Association - Annual Meeting 2011, 2013, and the 2013 China Meeting of Econometric Society for comments and discussions.

1. Introduction

The importance of studying inventory behavior in understanding the propagation mechanism of the business cycle has been widely acknowledged in the literature. The importance of inventories can be implied by a set of stylized facts. Using quarterly U.S. private nonfarm inventory data from 1953:1-2002:1, Khan and Thomas (2007a) establish: (1) the volatility of inventory investment is large, accounting for, on average, 29.5% of the volatility of GDP; (2) net inventory investment is procyclical; (3) net inventory investment is positively correlated with final sales, and its respective correlation coefficients with GDP and final sales are 0.67 and 0.41, which implies that output is more volatile than sales; (4) the inventory-to-sales ratio is countercyclical. The third empirical finding is often viewed as evidence that inventories can play a destabilizing role over business cycles. In other words, inventories may amplify aggregate fluctuations over the business cycle.

In fact, such amplification effects are mainly attributed to input instead of output inventories. Blinder and Maccini (1991) first point out the importance of input inventories. By decomposing the monthly U.S. manufacturing data by sector from 1959-1986, they found that, investment in input inventories is larger and more volatile than investment in out-put inventories and concluded that “most researchers seem to have barked up the wrong tree." This finding is robust to the narrowest definition of input inventories. Humphreys, Maccini and Schuh (2001), who examined longer time series from 1959-1994, also confirmed this finding. They found that manufacturing firms hold on average more than twice the amount of input inventories than output inventories, and input inventories are three times more volatile than output inventories. Recent evidence also shows different cyclical behavior between input and output inventories. As presented by Iacoviello, Schiantarelli and Schuh (2011), the input inventory to sales ratio is countercyclical, and the output inventory-sales ratio is “mildly procyclical".

Existing models,[1] (see, among others, Blinder and Maccini (1991), Khan and Thomas (2007a,b) and Fisher and Hornstein (2000)) in the inventory literature can match the stylized facts of inventories with technology shocks in many dimensions. However, some of those models fail to match the behavior of inventory investment, inventory-to-sales ratio and more cyclical production relative to sales. Compared to technology shocks, the importance of demand shocks for explaining inventory behaviors are far from conclusive. To name but a few, Kahn (1987) shows that the excess velocity of production relative to sales can be reproduced only by shocks to demand. Maccini, Moore and Schaller (2004) reinforce the puzzle that observed by Blinder and Maccini (1991): interest rate has no significant effect on inventories; and suggests that “. . . the long-run relationship between the inventories and the real interest rate may be fruitful". Jung and Yun (2006) show that output inventory behavior can be reproduced by a sticky price model with shocks to federal funds rate. Nevertheless, to the best of our knowledge, no paper in the literature has studied the effect of monetary shocks on

[1] Blinder and Maccini (1991) use production smoothing model, whereas Khan and Thomas (2007b) and Fisher and Hornstein (2000) use (S, s) model to rationalize inventories. Khan and Thomas (2007a) reviewed and evaluated stockout avoidance model and the (S, s) model. Also see Ramey and West (1999) for a comprehensive review on production smoothing model.

input inventory behavior. To fill the gap, we employ Shi (1998), a search model with monetary prorogation, and extend the model from single-stage production to two-stage production by endogenizing the choice of material inputs, to study both the long-run and short-run effects of monetary policy on input inventories. Inventories arise naturally in our model because of a search friction in the finished goods market. To highlight the unique feature of input inventories, we exclude output inventories. Thus, unmatched finished goods producers hold unused intermediate goods at the end of each period.

We find that monetary shocks have long run real effects on input inventories. Comparative statics show that monetary policy has hump-shaped real effects on steady state output per match, final sales, input net inventory investment (NII) and inventory-to-sales ratio. In particular, NII first increases with the growth rate of money, before falling for large growth rates. Intuitively, NII depends on the quantity of intermediate goods held by each output producer and the number of unmatched producers. When the money growth rate is low, a positive monetary shock reduces the real money balance, and buyers search more intensively, which generates a positive effect on the total number of matches (extensive margin effect), and implies a negative effect on the number of unmatched producers. On the other hand, monetary shock has a positive effect on the output per match (intensive margin effect), which implies a positive effect on the intermediate goods held. For a sufficiently low level of money growth, the positive effect on NII dominates the negative effect. NII increases with the money growth rate. When the money growth rate becomes sufficiently high, the negative effect would dominate, and NII decreases with the money growth rate.

This positive intensive margin effect is novel and is due to the two-stage production structure. The intuition is as follows. Output per match depends on the real money balances, labor and intermediate goods. If the money growth rate is low, a higher money growth reduces the real money balances, and hence has a negative effect on firms' profits. On the other hand, reduced real money balances lower firms' costs of inputs, which have a positive effect on firms' profits. Therefore, the positive effect dominates the negative effect for low levels of money growth; firm's profitability increases, and households hire more labor and produce more intermediate goods. With more inputs, output producers can produce more once matched.

Finally, the positive intensive and extensive margin effects also imply that final sales increase with the money growth rate when it is low. Since input inventories move with final sales in the same direction, our results suggest that input inventories amplify the effects of monetary shocks on GDP in the long run.

When we calibrate the model to match the quarterly U.S. data from 1967:Q1 to 2010:Q4, we find that our model can reproduce the stylized facts of input inventories quite well. Specifically, our model predicts procyclical inventory investment, a countercyclical inventory-to-sales ratio, more volatile output relative to final sales and a positive correlation between NII and final sales. We also revisit the debate about the role of inventories over business cycles, and show that input inventories amplify aggregate fluctuations.

To model input inventories, researchers also consider a multi-stage production, although they use different types of frictions to rationalize inventories. Nevertheless, most papers rely on real shocks to capture inventory regularities. If there were demand shocks, there would be tradeoffs between inventory investment and final sales. Example include, but not limited to, Blinder and Maccini (1991), Khan and Thomas (2007a), Ramey and West (1999), and Wang and Shi (2006). As a result, these models, for example the production smoothing model and stockout avoidance model (see, among others, Bils (2004), Bils and Kahn (2000), Coen-Pirani (2004), and Wen (2011)), usually predict counterfactual results, in particular, countercyclical inventory investment and a negative correlation between final sales and inventory investment. Khan and Thomas (2007a) show that by introducing idiosyncratic shocks, the generalized stockout avoidance model without capital is able to reproduce procyclical inventory investment and a positive relationship between inventory investment and final sales under preference shocks. But such improvements have to severely sacrifice the ability to match the long-run average inventory-to-sales ratio.

The model developed in the current study is different from that in Shi (1998) and thus delivers contrasting implications in matching real data. In particular, Shi (1998) employs a single-stage production model with output inventories, whereas we construct a multi-stage production model with input inventories. Under his model, Shi found that output inventory investment decreases monotonically with the money growth rate. This is because the quantity of goods per match decreases with the money growth rate monotonically in his model. In this way, output inventories serve as buffer stocks and respond negatively to a positive monetary shock as shown by Shi (1998) and Menner (2006), and hence, output inventories decrease whenever sales increase. Thus, the single-stage production model with output inventories is like the production smoothing model, in which inventories smooth production. It is difficult to generate more volatile output relative to final sales and a positive correlation between NII and final sales. However, in our multi-stage production model, output per match increases with moderate inflation. Such positive responses are strong enough to cause input inventories to move with final sales in the same direction during the transition, which is essential for reproducing the positive correlation between NII and final sales and more volatile output. Thus, the multi-stage production model with input inventories is like the stockout avoidance model, in which inventories amplify aggregate fluctuations.

The remainder of this paper is organized as follows. In section I, we describe a benchmark search model with input inventories. In section II, we define equilibrium and show how that monetary policy has long-run effects on inventories. In section III, we study the short-run dynamics of input inventories. Results of sensitivity analysis are also provided. Finally, section IV concludes this paper.

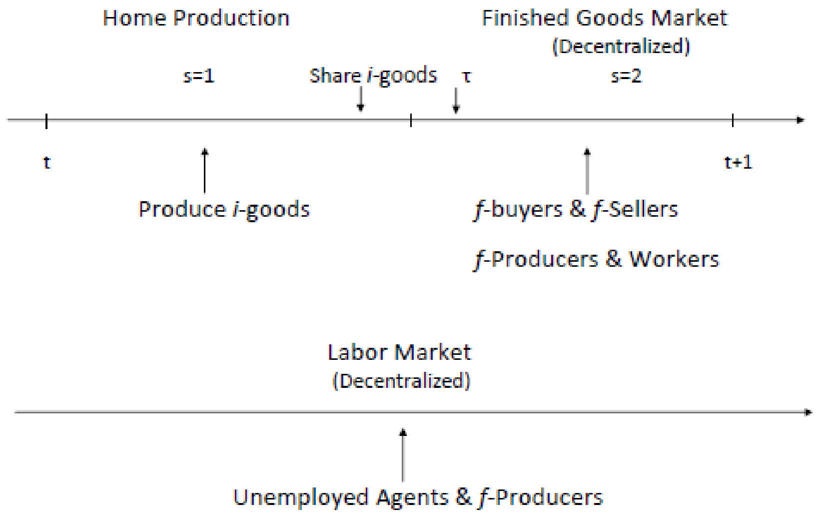
2. A Search Model with Multi-stage Production

2.1 The Environment

This is a large household model. Households are different in types, which set is denoted by H. The number of households in each type is large and normalized to one. There are also many types of finished goods, which set is denoted by H^f with same measures as H. Each household can produce both intermediate goods and household specific finished goods, which means that the finished goods $h^f \in H^f$ produced by household *h* is desired only by some other types of households.

Each household consists of six groups of agents (with the accompanying measure in parentheses): intermediate goods producers (a^i_p), finished goods buyers (a^f_b) and entrepreneurs (a^f_p), leisure seekers (n_0), workers ($a^f_p n_t$), and unemployed agents (u). Each entrepreneur consists of a finished goods producer and a finished goods seller. The number of agents ($a^i_p; a^f_b; a^f_p; u$) remain constant, while the number of effective buyers is endogenous, because I allow households to choose search intensities every period. Meanwhile, the number of leisure seekers and workers (n_0, n_t) varies over time which captures employment fluctuations.

Time is discrete. Figure 1 depicts the timing of $_t$ the model. In the first subperiod, intermediate goods producers produce intermediate goods (q_t^i) for their own finished goods production with disutility $\varphi(q_t^i)$. The function φ satisfies $\varphi' > 0$, $\varphi'' > 0$ for $q^i > 0$, and $\varphi'(0) = \varphi(0) = 0$. At the end of the first subperiod, newly produced intermediate goods, together with input inventories, are evenly shared among entrepreneurs. Households do not consume intermediate goods. Intermediate goods are storable across periods.



In the second subperiod, finished goods market opens. Assume there are search frictions in the finished goods market. Once a buyer and a seller have been matched, the seller places the order for her customer. The corresponding finished goods producer then produces the product. As discussed in the introduction, this paper focuses on input instead of output inventories by assuming that finished goods producers produce if and only if they are matched. An intrinsically useless object, called fait money, can facilitate trades in this market. Furthermore, assume there is no double coincidence of wants, so barter trades are excluded. At the beginning of each period, household divides the nominal money balance evenly among its finished goods buyers and chooses their search intensities (s^f_t). The terms of trade include the quantity of goods and the quantity of money $(\hat{q}_t^f, \hat{m}_t^f)$ are determined by Nash bargaining. Notice that the price level in the finished goods market is $P_t = m^f_t / q^f_t$. During the second subperiod, each household receives a lump-sum transfer (τ_t) which will be added to next period's nominal money balance. At the end of the period, finished goods buyers bring trade receipts, entrepreneurs bring profits and unused intermediate goods, and workers bring wage income back to the household. At the end, the household share consumption with its agents. Since agents regard the household's utility as a common objective and share consumption and inventories with each other, the idiosyncratic risk generated by search friction is smoothed within each household. The household carries the new nominal money balance (M_{t+1}) and input inventories which depreciate at a rate of δ_i over each period. Workers hired in the last period separate from current jobs at an exogenous rate r_n.

In the finished goods market, the total number of matches is determined by the following Cobb-Douglas matching function. Variables with a hat refer to an arbitrary household.

$$g(\hat{s}^f) = z_1^f (a_b^f \hat{s}^f)^{\xi} a_p^{f\,1-\xi}, \quad \xi \in (0,1), \tag{1}$$

where $z^f_1 > 0$ is a constant. Denote the ratio of buyers to sellers as $B^f = a^f_b / a_p^f$ and $z^f = z_1{}^f (B^f)^{\varepsilon-1}$. Then the matching rate for each unit of a buyer's search intensity is $g^f(\hat{s}^f)$ and the matching rate for each seller is $g^f(\hat{s}_s^f)$, where,

$$g_b^f \equiv z^f (\hat{s}^f)^{\xi-1}, \tag{2}$$

$$g_s^f \equiv z^f B^f (\hat{s}^f)^{\xi}. \tag{3}$$

Thus buyers and sellers get desirable matches at rates $s^f g^f_b$ and $g_s{}^f$ respectively.

In the labor market, each finished goods producer posts vacancies v_t. Unemployed agents search for jobs. Matched workers start to work in the next period and supply one unit of labor inelastically. Wages (W_t) are negotiated according to Nash bargaining and are paid in nominal terms, regardless of whether or not their employees formed a match. As in the standard labor search model (e.g., Blanchard and Diamond (1989)), the matching technology is: $\bar{\mu}(a_p^f \hat{v})^{\phi} u^{1-\phi}$, where $\varphi \in (0,1)$ and $\bar{\mu}$ is a constant; and the hat on variables refers to an arbitrary producer. The total number of matches for each firm is $\mu(\hat{v})v$, where $\mu(\hat{v}) = \bar{\mu}(a_p^f v / u)^{\phi-1}$ is the number of matches per vacancy. Hence, the number of matches per unemployed agent is $\mu(\hat{v}) a_p^f \hat{v} / u$.

2.2 The Household's Decision Problem

At the beginning of each period, the household divides the nominal money balance evenly among its finished goods buyers and chooses buyers' search intensities (s^f_t), consumption level (c_t), the number of vacancies for each firm (v_t), the next period's employment level (n_{t+1}), nominal money balance (M_{t+1}) and input inventory level (i_{t+1}) that carried over period, taking the terms of trade as given. Assume search intensities of both sellers and unemployed workers are inelastic with no cost to households.

The household's utility function, U(c), is strictly increasing and concave, and satisfies $\lim_{c\to 0} cU'(c) = \infty$ and $\lim_{c\to\infty} cU'(c) = 0$. $\varphi(q_t^i)$ is the disutility of producing intermediate goods. φ^f is the disutility of working in the finished goods market. $\phi^f(s_t^f)$ is the disutility of searching in the finished goods market. The function ϕ^f satisfies $\phi^f(s_t^f)$ and $\phi^{f''} > 0$ for s > 0 and $\phi^f(0) = \phi^{f'}(0) = 0$. Finally, $K(v_t)$ is the disutility of posting vacancies, which has the same properties as ϕ^f. Let F_{bt}^* (with measure $s_t^f g_{bt}^f a_b^f$) be the set of matched finished goods buyers in the period t. Similarly, F_{pt} (with measure $g_{st}^f a_p^f$) is the set of matched finished goods sellers in the current period.

Since we focus on input inventories, we assume the production function of the finished good is a Leontief production function:

$$q_t^f = \min\{a_t, n_t\}, \tag{4}$$

where a_t is the quantity of material inputs and n_t is labor inputs hired in the last period. A Leontief production function implies that intermediate goods and labor are not substitutable[2]. Moreover, this assumption gives us analytical results.

Then, the representative household's decision problem can be summarized as follows. The representative household taking the sequence $\{\hat{q}_t^f, \hat{m}_t^f, \hat{W}_t\}_{t\geq 0}$ and initial conditions $\{M_0, i_0, n_0\}$ as given, chooses $\{C_t, q_t^i, s_t^f, v_t, M_{t+1}, i_{t+1}, | n_{t+1}\}_{t\geq 0}$ to maximize its expected lifetime utility:

$$\max \sum_{t=0}^{\infty} \beta^t \mathbb{E}_{-1}[U(c_t) - \varphi(q_t^i) - a_p \hat{n}_t \varphi^f - a_b^f \Phi^f(s_t^f) - a_p^f K(v_t)] \tag{5}$$

subject to the following constraints for all $t \geq 0$

$$
\begin{aligned}
c_t &\leq s_t^f g_{bt}^f a_b^f \hat{q}_t^f, && (6)\\
\frac{M_t}{a_b^f} &\geq \hat{m}_t^f, \quad \forall F_{bt}{}^* && (7)\\
q_t^f &= \min\{a_t, n_t\}, && (8)\\
a_t &\leq i_t + q_t^i / a_p^f, \quad \forall F_{pt}{}^* && (9)\\
q_t^f &\geq \hat{q}_t^f, \quad \forall F_{pt}{}^* && (10)\\
M_{t+1} &\leq M_t + \tau_t + a_p^f \hat{n}_t \hat{P}_t^f \hat{W}_t - s_t^f g_{bt}^f a_b^f \hat{m}_t^f + g_{st}^f a_p^f \hat{m}_t^f - \hat{P}_t^f a_p^f \hat{W}_t n_t, && (11)\\
0 &\leq a_p^f[(1-\delta_n)n_t + v_t \mu_t - n_{t+1}], && (12)\\
a_p^f i_{t+1} &\leq (1-\delta_i)[a_p^f i_t + a_p^f q_t^i - g_{st}^f a_p^f a_t]. && (13)
\end{aligned}
$$

Constraint (6) and (7) are standard in a large household model. Constraint (6) is a budget constraint, which requires that the household's consumption does not exceed the total amount of finished goods obtained by its buyers. Constraint (7) states that in order to successfully trade with a matched seller, the buyer must have enough money. Constraint (8) implies that the usage of intermediate goods and labor are equal.

The intuition behind constraint (9) and constraint (10) is similar to the money constraint (7). Constraint (9) states that finished goods producer cannot use more intermediate goods than their holdings. Similarly, constraint (10) requires that matched finished goods

[2] A simple example would be to consider automobile manufacturers that cannot substitute labor for auto parts. But for the sake of comparison, we use a Cobb-Douglas production function for my quantitative analysis.

producers should have enough workers and intermediate goods to produce.

Constraint (11) is the law of motion of money, which states that the nominal money balance at the beginning of next period will be no larger than the nominal money balance carried from last period plus changes in the nominal money balance. The changes in the nominal money balance come from the lump-sum transfer received in the second subperiod, the money spent by finished goods buyers, profits from entrepreneurs and wages earned by workers. Entrepreneurs obtain money if, and only if, their sellers can find desired matches, while wages have to be paid to workers at the end of the period, regardless of whether they matched or not.

Constraint (12) is the law of motion of employment, which states that at the beginning of next period, the number of workers in each firm is no larger than the number of workers who still stay with the current job, plus newly hired workers. The last constraint is the law of motion of inventories, which implies that the household's next period inventory level is no larger than unused intermediate goods depreciated at a rate of $\delta_i \in (0,1)$.

Denote the multipliers of money constraint (7) by Λ_t^f. Let Ω_{at} be the shadow price of (9) at the beginning of period t + 1. We are interested in equilibria with a positive inventory level, which requires that inventories have positive values in each period(ex. $\Omega_{at} > 0$). The multiplier of (10) is denoted by Ω_f. Since entrepreneurs get positive surplus from trading finished goods, it is optimal for them to hire enough workers and have enough intermediate goods in hand. Let the shadow prices of (11), (12), and (13) at the beginning of period $t+1$ be Ω_{mt}, Ω_{nt} and Ω_{it} respectively, which are measured in terms of the household's period t utility.

Constraint (7) and (9) are restricted to be binding in equilibrium. By plugging c_t into the household's utility function, substituting a_t and q_t^f by n_t, and holding conditions (10), (11), (12) and (13) with equality, we can derive the first-order conditions with respect to $(M_{t+1}, i_{t+1}, n_{t+1}, s_t^f, v_t, q_t^i)$:

$$\Omega_{Mt} = \beta \mathbb{E}[\Omega_{Mt+1} + s_{t+1}^f g_{bt+1}^f \Lambda_{t+1}^f], \tag{14}$$

$$\Omega_{it} = \beta \mathbb{E}[(1-\delta_i)\Omega_{it+1} + g_{st+1}^f \Omega_{at+1}], \tag{15}$$

$$\begin{aligned} \Omega_{nt} &= \beta \mathbb{E}[(1-\delta_n)\Omega_{nt+1} + g_{st+1}^f [\Omega_{ft+1} - \Omega_{at+1} \\ &\quad - (1-\delta_i)\Omega_{it+1}] - \hat{P}_{t+1}^f \hat{W}_{t+1} \Omega_{Mt+1}], \end{aligned} \tag{16}$$

$$\Phi^{f\prime}(s_t^f) = g_{bt}^f [U'(C_t) - \omega_t^f] q_t^f, \tag{17}$$

$$\Omega_{nt} = K'(v_t)/\mu(\hat{v}_t), \tag{18}$$

$$\varphi'(q_t^i) = g_{st}^f \Omega_{at} + (1-\delta_i)\Omega_{it}. \tag{19}$$

Condition (14) equates the opportunity cost of obtaining one more unit of money and the expected benefits of carrying the money over to the next period. Such benefits include the shadow price of money and the shadow value of relaxing the money constraint in the finished goods market. Similarly, condition (15) equates the opportunity cost of obtaining an additional unit of input inventory and the expected benefits of carrying it over to the next period. Such benefits include the shadow value of inventories and the shadow value of relaxing the intermediate goods usage constraint.

Condition (16) equates the opportunity cost of hiring an additional worker and the expected benefits generated by this worker in the next period. This opportunity cost does not only include the shadow value of labor and the wage paid in terms of period t + 1 utilities $(\beta P_{t+1} W_{t+1} \Omega_{Mt+1})$, but also includes the expected cost of tightening the period t + 1 intermediate goods usage constraint and the expected shadow value of inventories discounted at the proper rate.

Condition (17) states that the opportunity cost of increasing the search intensity, which involves search costs and the real money balance, equals the marginal utility of consumption. Condition (18) equates the marginal cost of posting a vacancy and the expected benefits. The last condition equates the marginal cost of producing one more unit of intermediate goods and the marginal benefits which include the shadow value of inventories and the cost of relaxing the second subperiod intermediate goods usage constraint.

2.3 Terms of Trade

Let us specify the terms of trade for the finished goods market and the labor market. Be-cause there are large households in this model, each agent in the representative household is negligible and can be viewed as an identity of a small measure (ε). Since each agent's contribution to the household is also negligible, we compute the terms of trade brought by each agent first, then take the limit $\varepsilon \to 0$. Variables with a bar refer to the buyer in the other household and are taken as a given by the representative household.

The terms of trade in the finished goods market are denoted by $(q_t^f \varepsilon, \overline{m}_t^f \varepsilon)$, where $q_t^f \varepsilon$ is the quantity of finished goods and $\overline{m}_t^f \varepsilon$ is the quantity of money. Thus, the trading surpluses of these two agents to their households are:

$$\text{seller's trade surplus:} \quad \Omega_{Mt} \bar{m}_t^f \varepsilon - \Omega_{ft} q_t^f \varepsilon, \tag{20}$$

$$\text{buyer's trade surplus:} \quad U(\bar{c}_t + q_t^f \varepsilon) - U(\bar{c}_t) - (\bar{\Lambda}_t^f + \bar{\Omega}_{Mt}) \bar{m}_t^f \varepsilon. \tag{21}$$

Normalizing surpluses by ε, the terms of trade are determined by Nash bargaining between buyer and seller with equal weights:

$$\max_{\bar{m}_t^f, q_t^f} \left[\Omega_{Mt} \bar{m}_t^f - \Omega_{ft} q_t^f\right]^{1/2} \times \left[\frac{U(\bar{c}_t + q_t^f \varepsilon) - U(\bar{c}_t)}{\varepsilon} - (\bar{\Lambda}_t^f + \bar{\Omega}_{Mt}) \bar{m}_t^f\right]^{1/2}. \tag{22}$$

By substituting $\overline{m}_t^f = P_t^f q_t^f$ solving for the first-order conditions and taking the limit,we can get the following equations:

$$(\bar{\Omega}_{Mt} + \bar{\Lambda}_t^f) P_t^f = U'(c_t), \tag{23}$$

$$P_t^f \Omega_{Mt} = \Omega_{ft}. \tag{24}$$

The first condition equates the marginal utility of consumption with the opportunity cost of spending money. The second condition states that the shadow value of real money balances equals the opportunity cost of obtaining money. Denote the shadow value of real money balance in the finished goods market by $\omega_t^f = P_t^f \Omega_{Mt}$

The wage ($W_{t+1}\varepsilon$) is determined by Nash bargaining between the producer and the unemployed worker in the labor market. Assuming the producer's bargaining weight is σ, where $\sigma \in (0,1)$. Since the producer's surplus of hiring $\sigma\varepsilon$ more workers is $\{\Omega_{nt} - \beta E[(1-\delta_n)\Omega_{nt+1}\}\varepsilon$, by rearranging condition (16), we can reinterpret the producer's surplus in terms of real money balances:

$$\beta \mathbb{E}[\Omega_{ft+1} g_{st+1}^f - \omega_{t+1}^f W_{t+1} - a_p^f g_{st+1}^f \Omega_{at+1} - g_{st+1}^f \Omega_{it+1}(1-\delta_i)]\varepsilon. \quad (25)$$

Meanwhile, the unemployed agent contributes to his household's utility by $\beta(\overline{\omega}_{t+1}^f W_{t+1} - \varphi^f)\varepsilon$, where W_{t+1} is the expected wage income in terms of the real money balance. Normalizing surpluses $\beta\varepsilon$, the wage rate maximizes the weighted Nash product of these two agent's surpluses: $\max_{W_{t+1}}[[\Omega_{ft+1} g_{st+1}^f - \omega_{t+1}^f W_{t+1} - a_p^f g_{st+1}^f \Omega_{at+1} - g_{st+1}^f \Omega_{it+1}(1-\delta_i)]^\sigma * [\overline{\omega}_{t+1}^f W_{t+1} - \varphi^f]^{1-\sigma}]$.

The wage rate can be obtained after taking the limit " ! 0 on the first-order condition:

$$\mathbb{E}\left[\omega_{t+1}^f W_{t+1}\right] = \mathbb{E}\left[(1-\sigma) g_{st+1}^f [\Omega_{ft+1} - \Omega_{at+1} - \Omega_{it+1}(1-\delta_i)] + \sigma\varphi^f\right]. \quad (26)$$

The wage rate equals the weighted sum of the expected future bene_t of hiring " more workers and the opportunity cost of working.

3. Equilibrium

3.1 Characterization

Although households produce and consume different types of goods, they are identical in the sense that they have the same utility function and production technologies. Given this, we define the symmetric search equilibrium as follows:

Definition 1 *A symmetric search equilibrium is a sequence of household's choices* $\{\Gamma_{ht}\}_{t\geq 0}$, $\Gamma_{ht} \equiv (c_t, q_t^i, s_t^f, v_t, M_{t+1}, i_{t+1}, n_{t+1}^f)_h$, *expected quantities in trade* $\{\hat{X}_t\}_{t\geq 0}$, $\hat{X}_t \equiv (\hat{m}_t^f, \hat{q}_t^f, \hat{W}_t)$, *and the terms of trade* $\{X_t\}_{t\geq 0}$, *such that*

1. *all of these variables are identical across households and relevant individuals;*
2. *given* $\{\hat{X}_t\}_{t\geq 0}$ *and the initial conditions* (M_0, i_0, n_0), $\{\Gamma_{ht}\}_{t\geq 0}$ *solves the household's maximization problem, with* $(s^f, v) = (\hat{s}^f, \hat{v})$;
3. X_t *satisfies (25), (26) and (29);*
4. $\hat{X}_t = X_t \ \forall t \geq 0$.

As is standard, in order for money to play the role of a medium of exchange, we have to restrict the equilibrium to $\lambda^f > 0$. Similarly, we assume $\Omega_{at} > 0$, which requires that output producers prefer producing to hoarding intermediate goods in the second subperiod. Denote $k(v_t) = K'(v_t)/\mu(\hat{v}_t)$. Condition (11) and the Leontief production function imply that $i_t = q_t^f - q_t^i$. These three restrictions will be verified in the steady state. Then "hat" and "bar" are suppressed for a symmetric equilibrium. Condition (13) is reduced to $M_t + \tau_t = M_{t+1}$ under symmetry. Define the gross rate of money growth by $\gamma_t = M_{t+1} / M_t = (M_t + \tau_t) / M_t$. By substituting conditions (9), (11), (20), (21), (25), (26), (29) and $P_t^f = M_t^f / q_t^f$ into conditions (14) - (19), we can eliminate $(M, i, n, \lambda^f, \Omega_a, \Omega_f, \Omega_n, m^f, W)$, and the dynamic system is characterized in terms of (s^f, w^f, v, q^i, q^f) by the following conditions:

$$\mathbb{E}[\frac{\gamma_t \omega_t^f q_t^f}{q_{t+1}^f}] = \beta E\{\omega_{t+1}^f + z^f (s_{t+1}^f)^{\xi} [U'(c_{t+1} - \omega_{t+1}^f)]\}, \tag{27}$$

$$\Omega_{it} = \beta \mathbb{E}[\varphi'(q_{t+1}^i)], \tag{28}$$

$$\begin{aligned} k(v_t) &= \beta \mathbb{E}[(1-\delta_n) k(v_{t+1}) + \sigma z^f B^f (s_{t+1}^f)^{\xi} \omega_{t+1}^f \\ &+ (1 - z^f B^f (s_{t+1}^f)^{\xi}) \sigma (1-\delta_i) \Omega_{it+1} - \sigma \varphi'(q_{t+1}^i) - \sigma \varphi^f], \end{aligned} \tag{29}$$

$$\Phi^{f'}(s_t^f) = z^f (s_t^f)^{\xi - 1} [U'(c_t) - w_t^f] q_t^f, \tag{30}$$

$$\mathbb{E}[q_{t+1}^i] = \mathbb{E}\{q_{t+1}^f - (1-\delta_i)[1 - z^f B^f (s_t^f)^{\xi}] q_t^f\}, \tag{31}$$

$$\mathbb{E}[q_{t+1}^f] = (1-\delta_n) q_t^f + v_t \mu(v_t), \tag{32}$$

$$c_t = a_p^f B^f z^f (s_t^f)^{\xi} q_t^f. \tag{33}$$

Condition (32) of the dynamic system is of particular interest. It implies that if a monetary shock were to hit the economy, the corresponding effect would be propagated through the inventory channel. As shown by the right hand side of this equation, the quantity of next period home production (q_{t+1}^f) not only depends on the next period's quantity of finished goods (q_{t+1}^f), but also depends on the current period's quantities (q^f). Since condition (9) is restricted to being binding in equilibrium, the effects of a monetary shock would depend on the inventory level.

By rewriting $(s_t^{f*}, \Omega_i^*, v^*, q^{i*})$ as functions of (w^{f*}, q^{f*}), the steady state system can be reduced to two equations with two unknowns:

$$z^f [s^f (\omega^{f*}, q^{f*})]^{\xi} = \frac{\gamma - \beta}{\beta} \frac{\omega^{f*}}{U'(c(\omega^{f*}, q^{f*})) - w^{f*}}, \tag{34}$$

$$\begin{aligned} (1 - \beta(1-\delta_n)) k(v(q^{f*})) + \beta \sigma \varphi^f &= \beta \{\sigma z^f B^f (s^f (\omega^{f*}, q^{f*}))^{\xi} \omega^{f*} \\ &+ [(1 - z^f B^f (s^f (\omega^{f*}, q^{f*}))^{\xi}) \sigma (1-\delta_i) \beta - \sigma] \varphi'(q^i (\omega^{f*}, q^{f*}))\}. \end{aligned} \tag{35}$$

We can see that the money growth rate has real effects on steady state variables. More-over, an unique steady state, which satisfies $\lambda^f > 0$; $\Omega_a > 0$, can be pinned down by these two equations (see Appendix A for a proof).

3.2 Long Run Effects of Money Growth

As shown in Appendix A, steady state equation (35) is independent of γ, and equation (36) is not monotonic in γ. This implies that the effects of monetary policy has nonmonotonic long run effects on the quantity of finished goods per match, or, the intensive margin effect. Particularly, for low levels of γ, q^f increases with γ, but q^f decreases with γ if γ is high. The possible positive intensive margin effect is novel and is due to the two-stage production structure. The intuition is the following. q^f depends on the real money balances, labor and intermediate goods. If γ is low, a higher γ reduces the real money balances. Reduced real money balances on the one hand have a negative effect on firms' profits. On the other hand, they lower firms' costs of inputs, which have a positive effect on firms' profits. If γ is low, the positive effect dominates the negative effect; firm's profitability increases with γ; and hence households hire more labor and produce more intermediate goods.[3] Because there is a search friction in the finished goods market, households do not observe which output producers would get a match in the first sub-period and have to adjust the input levels for all output producers. In an extreme case of no substitutability between labor and intermediate goods, one more worker requires one more unit of intermediate goods as input; and output producer can produce one more unit once matched.[4]

Moreover, a higher money growth rate also has a positive extensive margin effect. As real money balances decreasing, buyers' surpluses from trade increase, and they search more intensively in order to spend money more quickly. As a result, final sales increases as both intensive and extensive margin effects are positive. In the opposite, when γ is high, firm's profitability decreases; and households hire less labor, produce less intermediate goods, and hence, q^f and final sales decrease with γ. The above analysis can be summarized in the following proposition:

PROPOSITION 1 *For sufficiently low levels of money growth, the steady state employment, quantity of finished goods traded in each match, the production of intermediate goods and final sales increase with the money growth rate, but they decrease with the money growth rate when the level of money growth is sufficiently high (See Appendix B for a proof.).*

The two-stage production model links the input responses in the upstream market to the final sales in the downstream market. To the contrary, in the one sector search model, the quantity of goods per match decreases with the money growth rate monotonically, or the intensive margin effect is always negative. This is because the quantity of goods per match depends only on real money balances. A higher γ reduces real money balances, and hence sellers' surpluses. Then, sellers produce less once matched.

The nonmonotonic response of quantities per match implies that, changes in the money growth rate also have nonmonotonic effects on input inventory investment (NII, as well as input inventories). By plugging the steady state equation of q^i into equation (15), we can see that NII depends on the quantity of intermediate goods held by each

[3] When γ is low, the right-hand side of equation (36) is downward sloping, which equals firm's marginal profitability from hiring $(g_s^f[\Omega_f - a_p^f\Omega_a - \Omega_t(1-\delta_t)] - \omega W)$ and it decreases with ω^f (increases with γ).

[4] This result should hold for other production functions. See the example of Cobb-Douglas production function as shown in the quantitative analysis.

output producer (or output per match) and the number of unmatched producers $(1-g_s^{f*})$:

$$NII^* = a_p^f \delta_i i^* = (1-\delta_i)\delta_i a_p^f [1-g_s^{f*}] q^{f*} \tag{36}$$

where, $q^{f*} = i^* + q^{i*}{}_p / a_p^f$ .Money growth aects the long run input inventory investment along two margins. If γ is low, both intensive margin effect and extensive margin effect are positive for a higher γ. First, positive intensive margin effect implies that each unmatched finished goods producer holds more intermediate goods. Second, positive extensive margin effect increases the total number of matches in the finished goods market, which has a negative effect on the total number of unmatched finished goods producers. If γ is low, the positive effect dominates the negative effect on NII and, as a result, NII increases with γ. But, if γ is high, households acquire less inputs, and NII decreases with γ. These results are summarized in the following proposition (see Appendix B for a proof):

PROPOSITION 2 *For sufficiently low levels of money growth, the steady state net in-put inventory investment (and input inventories) increases with the money growth rate, but decreases with the money growth rate for sufficiently high levels of money growth.*

According to the accounting identity GDP = Final Sales + NII, it is easy to show that GDP has similar nonmonotonic response as final sales and NII. There are hot debates in the literature on whether inventories have a destabilizing or stabilizing role over business cycles. Our model supports the view that inventories amplify the response of output in both long run and short run. This is because search friction forces inventories move with finals sales in the same direction in response to monetary shocks. Our model will also predict positive relationship between final sales and NII during the transitions. Therefore, we provide a microfoundation to the destabilizing role of inventories by introducing search frictions. We summarize the results in the following proposition:

Finally, GDP volatility has substantially decreased since 1984, which leads to two decades of “great moderation".[5] At about the same time, the input inventory-to-sales ratio started to show a significant downward trend.[6] Declining inventory-to-sales ratio is one of the explanations of “great moderation" in the literature, because GDP fluctuations could be amplified by inventories. Iacoviello, Schiantarelli and Schuh (2011) show the empirical evidence that the long run inventory-to-sales ratio decreases as the money growth rate decreases for that two decades. Our model predicts that the calibrated money growth rate is below the critical value. Thus our finds are consistent with theirs and suggest that changes in the money growth rate would be one of the reasons for the decline of the inventory-to-sales ratio and GDP volatility since the mid-1980. In the quantitative analysis, we give numerical examples to illustrate these propositions.

PROPOSITION 3 *For sufficiently low levels of money growth, the steady state input inventory-to-sales ratio increases with the money growth rate. To the contrary, it*

[5] See McConnell and Perez-Quiros (2000) and Ramey and Vine (2004) for identifying the structural break.
[6] Kahn, McConnell and Perez-Quiros (2002) show a similar trend for durable goods

decreases with the money growth rate for sufficiently high levels of money growth. (see Appendix B for a proof)

3.3 Comparisons Between Input and Output Inventories

The model developed in the current study is different from that in Shi (1998) and thus delivers contrasting implications in matching real data. In particular, Shi (1998) employs a single-stage production model with output inventories, whereas we construct a multi-stage production model with input inventories. Under his model, Shi found that output inventory investment decreases monotonically with the money growth rate. This is because the quantity of goods per match decreases with the money growth rate monotonically in his model. The corresponding condition in Shi (1998) is $i^* = q^* - f(n^*)$, where i^* is the output inventories, $f(n^*)$ is the firm's production function and q^* is the quantity of goods per match. Since production can be substituted by output inventories, output inventories serve as buffer stocks and respond negatively to a positive money shock as shown by Shi (1998) and Menner (2006), and hence, output inventories decrease whenever sales increase. Thus, the single-stage production model with output inventories is like the production smoothing model, in which inventories decrease with output and sales. It is hard for his model to generate more volatile output relative to final sales and a positive correlation between NII and final sales.

However, in our multi-stage production model, output per match increases with moderate inflation. By plugging condition (9) into condition (10), the equation $n^* = i^* + q^{i*} / a_p^f = q^{f*}$ holds in equilibrium. It is clear that both n^* and q^{i*} have to be adjusted to match q^{f*}. Thus, given search friction, NII increases with output and sales. Such positive responses are strong enough to cause input inventories to move with final sales in the same direction. Thus the multi-stage production model with input inventories is like the stockout avoidance model, in which inventories amplify aggregate fluctuations.

4. Quantitative Analysis

For the sake of comparing results with related papers, we use a Cobb-Douglas production function instead of the Leontief production function and assume the gross rate of money growth and productivity jointly follow a VAR process as in Wang and Shi (2006). We also include search frictions in the intermediate goods market.

4.1 Full Model

The detailed dierences are the following. First, as the features of the finished goods market, there are search frictions in the intermediate goods market. Accordingly, there are also many types of intermediate goods, which are denoted by H^i. The measures of H, H^i and H^f are the same. Each household can produce the household specific intermediate goods. Trade occurs in the intermediate goods market, because the type $h \in H$ household cannot use its own specific intermediate goods as input to produce its specific finished goods. Let $\Phi^f(s_t^f)$ be a intermediate buyer's disutility of searching in the intermediate goods market, which has the same properties as $\Phi^f(s_t^f)$ in the baseline model.

Second, each household has one more group of agents, namely the intermediate goods buyers (a_b^i). Third, each household has to make two more decisions in each period: (1) How to divide money between markets, with the proportion Δ_{t+1} for intermediate goods buyers. Then, at the beginning of each period, she divides each market's nominal money balance evenly among each type of buyers. (2) Intermediate goods buyers' search intensities (s_t^i).

Once a buyer and seller are matched, the seller produces intermediate goods for his partner on the spot with disutility, $\varphi(\hat{q}_t^i)$, where $\hat{q}_t^i$ denotes the quantities of goods. The input buyer pays the amount of money, $\hat{m}_t^i$, to the intermediate goods seller. Denote the price level in the intermediate goods market by $P_t^i = m_t^i / q_i^t$. The function φ satisfies $\varphi' > 0$ and $\varphi'' > 0$ for $q > 0$, and $\varphi'(0) = \varphi(0) = 0$. Variables with a hat refer to an arbitrary household. At the end of the first sub period, the intermediate goods market closes. Input buyers bring trade receipts back to the household. The household adds traded intermediate goods to the input inventories carried from the last period, and then divides the intermediate goods evenly among entrepreneurs.[7]

Similarly to the benchmark model, the total number of matches in the intermediate goods market is determined by the following matching function: $g(\hat{s}^i) = z_1^i (a_p^f \hat{s}^i)^{\xi} a_p^{i\,1-\xi}, \xi \in (0,1)$. As such, the matching rate for each unit of a buyer's search intensity is $g_b^i = z^i (\hat{s}^i)^{\xi-1}$ and the matching rate per seller is $g_s^i = z^i B^i (\hat{s}^i)^{\xi}$, where $B^i = a_b^i / a_p^i$ and $z^i = z_1^i (B^i)^{\xi-1}$. Finally, a buyer and a seller get desirable matches at rates $s^i g_b^i$ and g_s^i respectively.

The last dierence from the benchmark model is the Cobb-Douglas production function used in this model:

$$q_t^f = A_t a_t^{\alpha} n_t^{1-\alpha}, \tag{37}$$

where a_t is the quantity of material inputs and n_t is labor inputs that are hired in the last period. A_t is total factor productivity. The VAR process has the following form:

$$(\gamma)_{t+1}\, lnA_{t+1} = N_1 + N_2\, (\gamma)_t\, lnA_t + (\epsilon)_{m,t+1}\, \epsilon_{A,t+1} \tag{38}$$

where N_1 is a 2*1 vector and N2 is a 2*2 matrix. ε_m is the shock to the money growth and ε_A is the productivity shock. The household's new decision problem, bargaining solutions for the intermediate goods market and the new dynamic system is described in Appendix C.

4.2 Calibration

The model is log-linearized and calibrated to match the quarterly US data. The sample period is from 1967:Q1 to 2010:Q4. The data is from the Bureau of Economic

[7] Assume intermediate goods sellers would not bring their money holdings back to the household until the end of the period. This assumption simplifies the model in the sense that the equilibrium conditions do not involve the intertemporal price ratio of one good market relative to the other

Analysis, the Bureau of Labor Statistics and the Federal Reserve Bank of St. Louis on inventories for the manufacturing sector, final sales, employment, money stock and the velocity of money to compute my calibration targets. All of the variables are in real terms. The input inventories include inventories of materials and supplies and inventories of work-in-process. Final sales are manufacturing sales. GDP is calculated according to the accounting identity, which equals final sales plus net inventory investment.

My calibration strategy is summarized as follows. The parameters in my model can be grouped into three categories. The first set of parameters $(\gamma^*, \hat{N}1, \hat{N}2, \sigma_m, \sigma_A \sigma_{mA}, \rho, \sigma_g)$ is calculated directly from data. The second set of parameters $(\phi, \sigma, \delta_n, z_1^f, a_p^f, a_b^f, FI, b, K_0, \varphi^i, \varphi_0^i, \varphi^f)$ is pinned down by jointly matching a set of targets. Most of my targets are calculated from my samples, while some of them are taken from other work in the literature. The last set of parameters $(\eta, \varepsilon_i, \varepsilon_f, B^i)$ is hard to determine, and is pinned down by jointly minimizing the difference between the simulated second moments and the observed ones.

The discount factor is set at $\beta = 0{:}995$, which implifies that the annual interest rate is 2%. In order to calibrate the model, we assume the utility function, the disutility functions of searching in the goods markets, the disutility function of producing intermediate goods, and the disutility function of posting vacancies have the following functional forms:

$$U(c_t) = \frac{c_t^{1-\eta} - 1}{1-\eta}; \tag{39}$$

$$\Phi^i(s_t^i) = \varphi^i(\varphi_0^i s_t^i)^{1+1/\epsilon_i}; \tag{40}$$

$$\Phi^f(s_t^f) = \varphi^f(\varphi_0^f s_t^f)^{1+1/\epsilon_f}; \tag{41}$$

$$\varphi(q_t^i) = \frac{b}{2}(q_t^i)^2; \tag{42}$$

$$K(v_t) = K_0 v_t^2; \tag{43}$$

where $\eta, \varphi^i, \varphi_0^i, \varphi^f, \varphi_0^f, \varepsilon_f, b, K_0$ are constants. The gross rate of money growth and productivity jointly follow a VAR(1) process. And, the estimated coefficients and the standard error of both shocks are the following:

$$\hat{N}1 = (0).738$$

$$\vdash 0.958$$

$$, \quad \hat{N}2 = (0).274 - 0.0773$$

$$0.8450.942,$$

$$\sigma_m = 0.0064, \quad \sigma_A = 0.0138, \quad \sigma_{mA} = 0.0045.$$

The time series of total factor productivity are calculated as the Solow residual. Since the production function is in the individual level, it has to be aggregated in order to match the data. The expression for the log of productivity is the following:

$$lnA = ln(c^f) - ln(a_b^f v_c^f B^f) - \alpha ln(inputs/P_i) + \alpha ln(a_b^f v_c^f B^f/(1-FI)) - (1-\alpha) ln(empl/a_p^f), \quad (44)$$

where v_c^f is the velocity of money. The variable inputs are the total cost of material inputs. P_i is the price deflator for material costs. The variable *empl* is the aggregate employment in the manufacturing sector. These series come from NBER databases.

We use M2 as the money stock since it is stable within the sample period.

The steady state money growth rate is calculated as the sample average. We also need to determine the following parameters:

$$(\phi, \eta, \sigma, \delta_n, \xi, u, \delta_i, z_i^f, z_1^f, a_p^i, a_p^f, a_b^f, FI, b, K_0, \varphi^i, \varphi_0^i, \varphi^f, \varphi_0^f, \varepsilon_i, \varepsilon_f, \alpha, B_i, \overline{\mu})$$

Parameters $(\phi, \sigma, \delta_n, u, \delta_i, z_1^f, a_p^f, a_b^f, FI, b, K_0, \varphi^i, \varphi_0^i, \varphi^f, \varphi_0^f, \alpha, \overline{\mu})$ are jointly calibrated to match the following targets: (1) the average labor participation rate is 0.6445 over the same sample period. (2) The average unemployment rate is 0.061. (3) The average velocity of M2 money stock is 1.8236. (4) The average input inventory to final sales ratio is 0.984. (5) The intermediate inputs to final sales ratio is 0.549. (6) The inventory to output ratio is 0.981 and the inventory investment to output ratio is 0.0038. (7) The shopping time of the population is 11.17% of the working time and the working time is 30% of agents' discretionary time. (8) The vacancy posting cost is $3.72 * 10^{-4}$. (10) The average quarterly separation rate from employment to unemployment is 0.102. (11) The average markup is 70%.

The first six targets are calculated from my samples. The seventh target is used to compute the goods markets' search intensities and is taken from Shi (1998).[8] The target value for the vacancy posting cost is taken from Berentsen, Menzio and Wright (2011). The target value for the average quarterly separation rate is taken from Shimer (2005). Since the markup is difficult to determine, we do sensitivity analysis for this value.

The finished good producers' bargaining power (σ), in the labor market, is set to the same value as the elasticity of the labor market matching function (ϕ) to give workers 72% of the rent, as such the Hosios condition holds.[9] The elasticity of the labor market matching function (ϕ) and the associate constant are estimated by the ordinary least squares, regressing the log of the job-finding rate on the log of market tightness. For the sake of comparison, FI is set to 0.269 as outlined in Shi (1998) and Wang and Shi (2006).[10]

[8] Wang and Shi (2006) match the same target.

[9] Also see Hosios (1990); Shi (2006); Shimer (2005) and Rogerson and Shimer (2011)..

[10] Also see Christiano (1988)

Table 1: Parameter Values and Calibration Targets

Parameters	Values	Targets
β	0.995	Annual interest rate: 4%
A^*	1	Normalization
Unemployment u	0.0393	Avg. LP: 0.6445
i-sellers a_p^i	0.2010	Avg. UR: 0.061
f-sellers a_p^f	0.2010	Avg. Velocity of M2: 1.8236
i-buyers a_b^i	0.1005	Avg. ISR: 0.984
f-buyers a_b^f	0.0515	Avg. IPS: 0.549
δ_i	0.0038	Avg. INV/GDP: 0.981
z_1^i	0.0934	Avg. NII/GDP: 0.0038
z_1^f	3.0524	Shopping time/Working time: 11.17%
b	0.6706	Working time/Discretionary time: 30%
K_0	0.0006	Vacancy posting cost: 3.72×10^{-4}
B_f	0.1947	Avg. markup: 70%
α	0.4156	
φ^i	14.2091	
φ_0^i	0.1104	
φ^f	1.6025	
φ_0^f	1.8043	
ϕ	0.28	OLS estimation
$\bar{\mu}$	0.364	OLS estimation
σ	0.28	Give workers 72% of the rent
δ_n	0.105	Avg. monthly separation rate: 0.034
FI	0.269	Shi (1998)

To pin down the parameters (a_p^i, a_b^i, z_1^f) in the intermediate goods market, first, we assume (1) the intermediate goods market tightness equals B_f; (2) the time spent on searching intermediate goods is also 11.17% of the working time; (3) the numbers of sellers in both goods markets are equal; (4) the velocity of the money stock in the intermediate goods market is 0.2. This velocity seems very low, but it fits the data best.

Table 2: Parameter Values and Targets (cont'd)

Parameters	Benchmark Values
ξ	0.8
ϵ_i	0.4
ϵ_f	0.01
η	0.8
B_i	0.2

Finally, we set the benchmark value of $(\eta, \varepsilon_i, \varepsilon_f, \xi, B_i)$ to best fit the data, then do sensitivity analysis on these parameters. We set the elasticity of goods market matching functions ξ to be 0.8, the elasticities of the disutility functions of searching ε_i and ε_f to be 0.4 and 0.01 respectively. Let the relative risk aversion η = 0:8 and the markup to be 70%. We normalize the number of workers hired by each firm to n = 1. The parameter values and corresponding targets are summarized in Table 1. The benchmark values for assumed parameters are summarized in Table 2. The strategy of calibration is described in detail in Appendix D.

4.3 Model Predictions

By using the calibrated parameter values, we give a numerical example in Figure 2 to il-lustrate Proposition 1-3 which shows that, for a low level of money growth, GDP, the net inventory investment, the inventory-to-sales ratio and the quantity of finished goods per match increase with the money growth rate, but decrease with the money growth rate if it reaches a high growth threshold. The critical money growth rate for each variable is slightly different, ranging from 5.68 percent to 7.98 percent.

Now let us look at the quantitative performance of the multi-stage production model. Table 3 reports the model prediction for stylized facts of input inventories. By assuming the economy is hit by a positive shock to the money growth rate, the stylized facts of input inventories can be quantitatively well reproduced, such as procyclical inventory investment, countercyclical inventory-to-sales ratio, negative correlation between final sales and inventory-to-sales ratio and more volatile output relative to final sales.

The most striking result is that the model predicts a positive correlation between final sales and net input inventory investment. As tested in Khan and Thomas (2007b), neither the (S, s) model nor the basic stockout avoidance model can reproduce the positive relationship between final sales and net inventory investment under a preference shock. Even after introducing idiosyncratic shocks, the generalized stockout avoidance model still can only generate a very weak positive correlation, and this slight improvement comes as a result of severely sacrificing the ability to match the long run average inventory-to-sales ratio. Furthermore, unlike the models of Kryvtsov and Midrigan (2010a,b) and Jung and Yun (2006), which require unrealistic high depreciation rate to match the data. Our model can replicate the stylized facts of input inventories with calibrated depreciation rate that is as low as the empirical one.

The positive response of q^f, as shown in the next section is essential for being able to match the stylized facts, because it induces positive responses of employment and production of intermediate goods. Although the total number of matched finished goods producers increases, their unmatched counterparts hold more unused intermediate goods. As a result, the positive effect dominates the negative effect on inventories and input inventories increase with final sales during the transition. While as showed in Menner (2006), output inventories decrease with sales in response to a positive money growth shock, because the number of matched buyers increases, and the quantity of goods per match decreases.

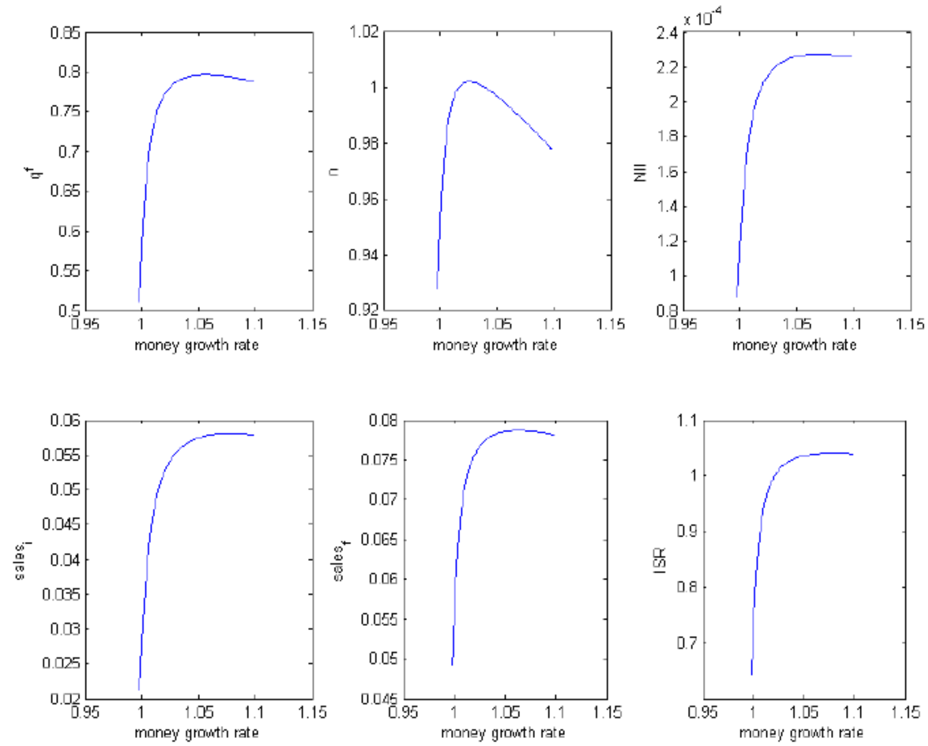


Figure 2: Long Run Effects of Money Growth

The model predicts a higher standard deviation of inventory investment relative to GDP, since there is no adjustment cost in my model. By introducing adjustment cost, the response of inventory investment will be smoother. Of course, it may sacrifice the accurateness of the stylized facts in other dimensions. Nevertheless, the results suggest that monetary policy is important for explaining inventory behaviors and replicating inventory stylized facts.

4.4 Impulse Responses

Figure 3 depicts the impulse response functions to a one positive standard deviation shock to the money growth rate. Particularly, the responses of quantity of finished goods per match, inventories and employment are hump-shaped, and that they stay above the steady state during the entire transition.

The details of the propagation mechanism are as follows. First, when the shock hits the economy, the money growth rate increases and real money balances fall immediately, which stimulates buyers to search more intensively in both goods markets. Since inventories depreciate each period and households are eager to consume, households allocate proportionally more money to the finished goods market, as demonstrated by the fact that Δ_t drops immediately. Given the low money growth rate, households anticipate higher final sales and immediately increase intermediate goods buyers' search intensities in order to obtain more intermediate goods. But households do not know which finished goods producers would get a match in the second sub-period, thus they have to increase the level of intermediate goods for all finished goods producers, as demonstrated by the fact that a_t jumps immediately. With more intermediate goods, finished goods producers produce more if they matched and q_t^f jumps immediately.

Table 3: Model Predictions

	Data	Model	Standard Deviations
corr(GDP, IS)	-0.756	-0.798	(0.011)
corr(GDP, NII)	0.789	0.882	(0.008)
corr(FS, IS)	-0.756	-0.665	(0.018)
corr(FS, GDP)	0.987	0.957	(0.001)
corr(FS, NII)	0.680	0.708	(0.014)
$\sigma(FS)/\sigma(GDP)$	0.837	0.666	(0.008)
$\sigma(NII)/\sigma(GDP)$	0.220	0.411	(0.005)
$\sigma(IS)/\sigma(GDP)$	0.843	0.794	(0.011)

Numbers in parentheses are standard deviations over 1000 simulations. GDP: real GDP;GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: final sales. All data are real series, end of period, seasonally adjusted and chained in 2005 dollars for the period 1967:Q1 to 2010:Q4. Net inventory investment is detrended as a share of GDP. Other series are detrended using a HP filter (λ = 1600). NII are calculated as a share of GDP in order to compare the results with that in Khan and Thomas (2007b).

Second, the money growth shock also induces a positive response of productivity, because we assume that monetary shocks affect productivity contemporarily and past money growth has a positive correlation with current productivity. Since the effects on productivity are very persistent, final sales keep above the steady state for more than twelve quarters. Such effects on final sales transfer back to the intermediate market and keep intermediate sales above the steady state. The positive response of final sales also increases future revenues. Since revenues stay above the steady state from the second period, households post more vacancies immediately, and n_{t+1} stays above the steady state for more than ten periods. As a result, q_t^f continues to rise before slowly going back to the steady state as the effects of technology shock diminishing. The multi-stage production enables q_t^f to synchronize with the responses of employment and material inputs during the transition. Thus the positive response of q^f arises from the interaction between different production stages which is different from the standard search model.

Finally, inventories stay above the steady state since unmatched finished goods producers hold more intermediate goods during the transition, which is shown by the decreased shadow value of inventories.

Figure 4 depicts the short run dynamic responses of the equilibrium to one positive standard deviation shock to productivity. The same qualitative responses were obtained as in the former case, except for the responses related to employment and qf, which decrease monotonically toward the steady state instead of hump-shaped responses. Intuitively speaking, this is because the technology shock decreases monotonically during the transition. The positive technology shock induces a negative response of the money growth, because current money growth is negatively correlated with past productity. Therefore, the responses of both shocks are qualitatively the same as those in Figure 4 from the second period. Quantitatively speaking, the responses are stronger in the current case, since the magnitude of the shock to productivity is bigger.

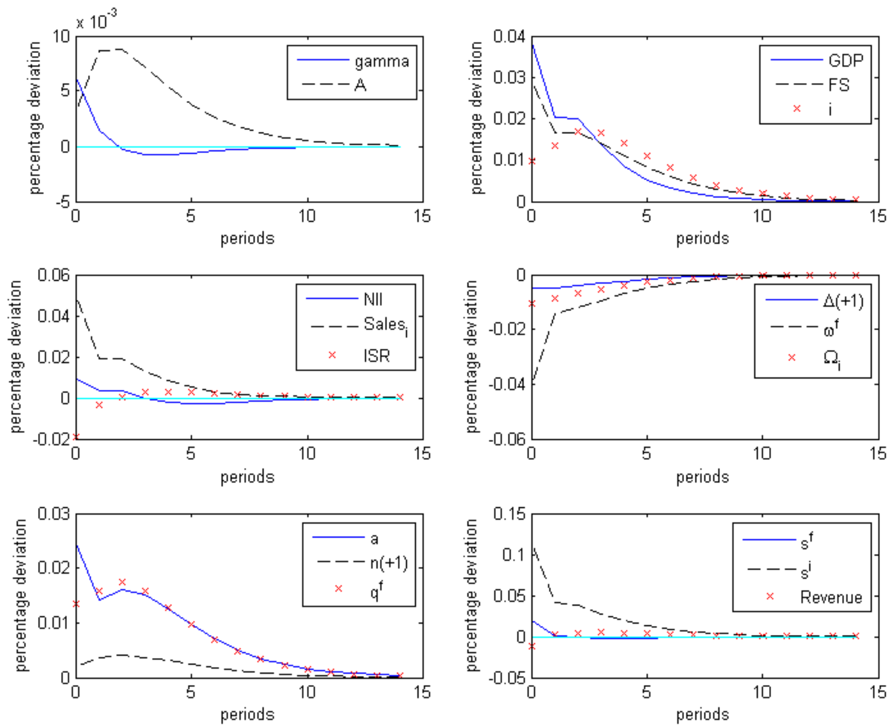


Figure 3: Impulse Response Functions to a Positive Money Growth Shock

The short run responses of input inventories in our model and that of output inventories in Menner (2006) and Shi (1998) are very different. The single-stage production model is similar to the production smoothing model, which always has a tradeo between the inventory and sales (and hence output). Thus, inventories decrease whenever sales increases during the transition, and it is difficult to reproduce the stylized facts of inventories without technology shocks.

Building on Shi's (1998) model, Wang and Shi (2006) also predict a positive response of inventories with the shock to money growth rate.[11] But, in our model, employment responds positively to both shocks. Input inventories are a part of the next period's material inputs in the multi-stage production model, thus higher input inventories have positive effects on final sales and revenues. As a result, households hire more labor if future revenues increase. In contrast, employment responds negatively to both shocks in Wang and Shi (2006), because abundant goods reduce households' profitability to hire labor. And, in their paper, inventories are important because output inventories induce a shortage of future goods supply, which can keep buyers' search intensities above the steady state. Therefore, input and output inventories work differently through the propagation mechanism in these two models.[12]

[11] Our findings are consistent with Chang, Hornstein and Sarte (2006) in which employment (and inventories) increases in response to a permanent and positive shock to productivity in a model with inventories, if the costs of holding inventories are sufficiently low.

[12] All of these papers do not distinguish between input and output inventories when calibrate to data.

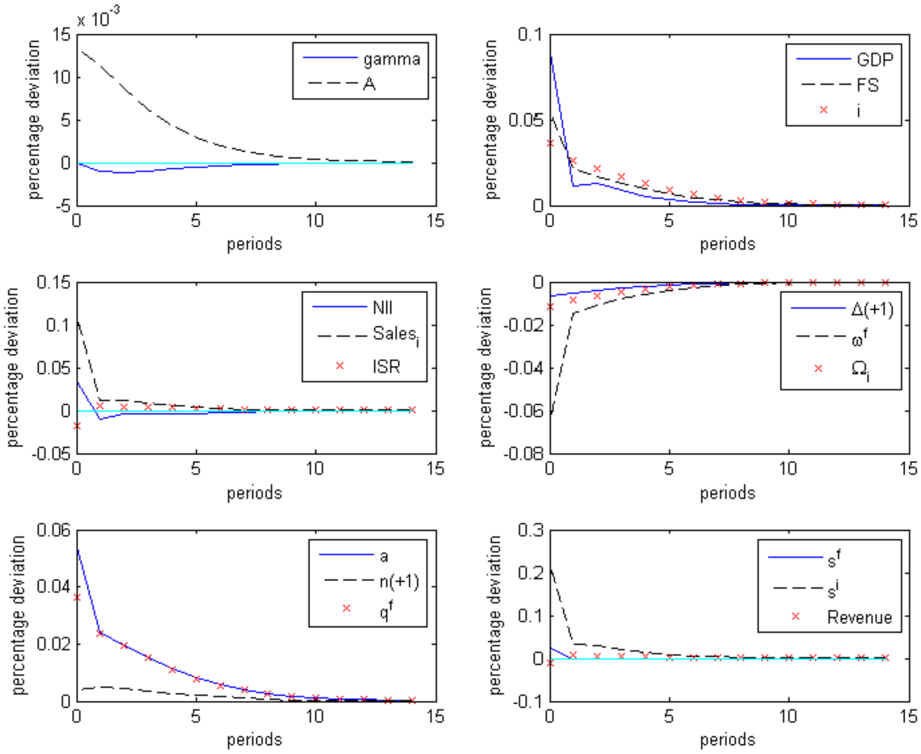


Figure 4: Impulse Response Functions to a Positive Productivity Shock

4.5 The Role of Input Inventories over Business Cycles

There has long been a debate within the inventory literature about the role of inventories over business cycles. Since GDP is more volatile than final sales in the data, most researchers believe that inventories amplify aggregate fluctuations over business cycles. Others argue that inventories smooth business cycles because they smooth productions. In this section, we revisit this debate and show that input inventories play a decentralizing role over business cycles.

We compare two pairs of results by targeting different inventory-to-sales ratios and re-calibrating the model to each ratio. We divide the sample period into two sub periods. The first inventory-to-sales ratio is 1.0402, which is calculated from the first sub period:1967:I-1983:IV, and the second inventory-to-sales ratio is 0.9499, which is calculated from the second sub period: 1984:I-2010:IV. We choose the year 1984 as a break point for my sample because the input inventory-to-sales ratio experiences a declining trend beginning in 1984.[13] Some researchers argue that this declining trend is one of the reasons for "Great Moderation". Our results support this argument.

The calibration results are reported in Table 4. The stylized facts calculated from these two sub samples are similar except for the relative standard deviations of the inventory investment and the inventory-to-sales ratio relative to GDP. The most striking results are that the standard deviations of GDP, the input inventory investment, the inventory-to-sales ratio and final sales are lower in the second sub sample. Thus, our results show that, in the second sub sample, aggregate fluctuations are smoothed by

[13] The output inventory-to-sales ratio exhibits an opposite trend. See Iacoviello, Schiantarelli and Schuh (2011) for details

holding lower input inventory levels.

Table 4: Role of Inventories

	$ISR = 1.0402$		$ISR = 0.9499$		$Data$
corr(GDP, IS)	-0.774	(0.014)	-0.801	(0.011)	-0.756
corr(GDP, NII)	0.896	(0.007)	0.874	(0.008)	0.789
corr(FS, IS)	-0.599	(0.023)	-0.690	(0.017)	-0.756
corr(FS, GDP)	0.940	(0.001)	0.965	(0.001)	0.987
corr(FS, NII)	0.693	(0.014)	0.717	(0.014)	0.680
$\sigma(FS)/\sigma(GDP)$	0.613	(0.008)	0.696	(0.007)	0.837
$\sigma(NII)/\sigma(GDP)$	0.473	(0.005)	0.376	(0.004)	0.220
$\sigma(IS)/\sigma(GDP)$	0.670	(0.009)	0.860	(0.013)	0.843
$\sigma(GDP)$	0.112	(0.003)	0.088	(0.002)	-
$\sigma(NII)$	0.053	(0.001)	0.033	(0.001)	-
$\sigma(INV)$	0.016	(0.001)	0.012	(0.001)	-
$\sigma(FS)$	0.069	(0.002)	0.061	(0.002)	-

Numbers in parentheses are standard deviations over 1000 simulations. GDP: real GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: final sales; n: employment level of each output producer. The model is calibrated to two different ISR targets while keeping all the other parameters unchanged. ISR = 1:0402 is calculated from the first sub-sample: 1967:I- 1983:IV and ISR = 0:9499 is calculated from the second sub-sample: 1984:I - 2010:IV.

Herrera and Pesavento (2005) argue that the better inventory management technology cannot account for most of the decline in volatility of output, because both the volatility of sales and inventories decreased since mid-1980s and most of the decline is due to the decline in that of input inventories.[14] Consistent with the empirical findings in Stock and Watson (2002), our model suggests that besides the better inventory management technology, reduction in volatility of money growth shocks may be another promising reason for explaining the “Great Moderation".

4.6 Sensitivity Analysis

In the preceding sections, we have seen that the search model with shocks to the money growth rate can reproduce stylized facts of inventories. The model also suggests that input inventories amplify aggregate fluctuations over business cycles. Since parameters $\varepsilon_i, \varepsilon_f, \alpha, \eta, B_i$ and markup are hard calibrated from the data, we assumed their values for the best _t to data in the benchmark model. In this section, we will examine the sensitivity of the quantitative results to different values of these parameters. Each parameter will be analyzed separately, and the model is recalibrated to the data for each analysis. The results of sensitivity analysis are reported in Table 5 - 10.

[14] Herrera, Murtazashvili and Pesavento (2008) show that the cross-section correlation among manufacturing inventories and sales increased since the "Great Modaration"

Table 5: Sensitivity analysis: ϵ_i

ϵ_i	0.1	0.4*	1	4	8	Data
corr(GDP, IS)	-0.745	-0.798	-0.413	0.631	0.733	-0.756
	(0.006)	(0.011)	(0.028)	(0.011)	(0.009)	
corr(GDP, NII)	0.507	0.882	0.917	0.937	0.940	0.789
	(0.020)	(0.008)	(0.006)	(0.004)	(0.004)	
corr(FS, IS)	-0.858	-0.665	-0.157	0.821	0.891	-0.756
	(0.006)	(0.018)	(0.038)	(0.010)	(0.007)	
corr(FS, GDP)	0.940	0.957	0.908	0.872	0.866	0.987
	(0.004)	(0.001)	(0.002)	(0.003)	(0.003)	
corr(FS, NII)	0.184	0.708	0.667	0.655	0.654	0.680
	(0.031)	(0.014)	(0.014)	(0.014)	(0.013)	
$\sigma(FS)/\sigma(GDP)$	0.877	0.666	0.532	0.444	0.427	0.837
	(0.007)	(0.008)	(0.009)	(0.008)	(0.007)	
$\sigma(NII)/\sigma(GDP)$	0.346	0.411	0.566	0.663	0.681	0.220
	(0.010)	(0.005)	(0.005)	(0.005)	(0.006)	
$\sigma(IS)/\sigma(GDP)$	2.321	0.794	0.300	0.265	0.294	0.843
	(0.049)	(0.011)	(0.008)	(0.008)	(0.007)	
$\sigma(GDP)$	0.043	0.096	0.161	0.247	0.270	-
	(0.002)	(0.002)	(0.004)	(0.006)	(0.007)	
$\sigma(NII)$	0.015	0.039	0.091	0.164	0.184	-
	(0.000)	(0.001)	(0.002)	(0.005)	(0.005)	
$\sigma(INV)$	0.005	0.013	0.023	0.035	0.039	-
	(0.000)	(0.001)	(0.001)	(0.001)	(0.001)	
$\sigma(FS)$	0.038	0.064	0.086	0.110	0.116	-
	(0.001)	(0.002)	(0.002)	(0.003)	(0.003)	

Numbers in parentheses are standard deviations over 1000 simulations. GDP: real GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: final sales; n: employment level of each output producer. Parameter values with a star are benchmark parameter values.

Table 5 shows that the correlations predicted by this model are sensitive to changes in the value of ε_i, except for the correlation between final sales and GDP. Other correlations match the data more effectively with low value of ε_i. In particular, if ε_i were large, (for example, ten times larger than the benchmark value,) the correlation between inventory-to-sales ratio and GDP (or FS) would turn positive which is not consistent with data.

Table 6: Sensitivity analysis: ϵ_f

ϵ_f	0.01*	0.03	0.1	1	2	Data
corr(GDP, IS)	-0.798	-0.749	-0.576	-0.304	-0.280	-0.756
	(0.011)	(0.014)	(0.020)	(0.021)	(0.020)	
corr(GDP, NII)	0.882	0.889	0.888	0.939	0.943	0.789
	(0.008)	(0.006)	(0.006)	(0.003)	(0.003)	
corr(FS, IS)	-0.666	-0.716	-0.591	-0.352	-0.331	-0.756
	(0.018)	(0.020)	(0.027)	(0.030)	(0.028)	
corr(FS, GDP)	0.957	0.978	0.966	0.950	0.949	0.987
	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)	
corr(FS, NII)	0.709	0.774	0.739	0.784	0.790	0.680
	(0.014)	(0.012)	(0.013)	(0.010)	(0.010)	
$\sigma(FS)/\sigma(GDP)$	0.666	0.722	0.683	0.556	0.542	0.837
	(0.008)	(0.005)	(0.005)	(0.005)	(0.005)	
$\sigma(NII)/\sigma(GDP)$	0.412	0.331	0.383	0.503	0.516	0.220
	(0.005)	(0.004)	(0.004)	(0.005)	(0.005)	
$\sigma(IS)/\sigma(GDP)$	0.794	1.454	1.796	2.128	2.174	0.843
	(0.011)	(0.019)	(0.020)	(0.049)	(0.052)	
$\sigma(GDP)$	0.096	0.096	0.087	0.074	0.073	-
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	
$\sigma(NII)$	0.039	0.032	0.033	0.037	0.038	-
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	
$\sigma(INV)$	0.013	0.013	0.014	0.016	0.016	-
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	
$\sigma(FS)$	0.064	0.069	0.059	0.041	0.040	-
	(0.002)	(0.002)	(0.001)	(0.001)	(0.001)	

Numbers in parentheses are standard deviations over 1000 simulations. GDP: real GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: final sales; n: employment level of each output producer. Parameter values with a star are benchmark parameter values.

The intuitions of these results are as follows. A higher value of ε_i implies that intermediate goods buyers are more responsive to both shocks, which is demonstrated by larger standard deviations of aggregate variables. Intermediate goods buyers search more intensively, while the effect on finished goods buyers' search intensities is limited because ε_f is unchanged. Therefore, the increase in intermediate goods is larger than the increase in final sales, so households accumulate more inventories, which drives the inventory-to-sales ratio up and generates counterfactual results.

Table 6 shows that the model matches the data effectively with low ε_f. A higher value of ε_f implies that finished goods producers are more responsive to both shocks. Since finished goods buyers search more intensively in this case, q^f responds at a lower magnitude to the shocks and GDP and final sales become less volatile. Despite the decreasing volatilities of GDP and final sales, the inventory investment responds more strongly to the shocks. Therefore, the relative standard deviation of the inventory-to-sales ratio (relative to GDP) increases with ε_f, and the correlation between inventory-to-sales ratio and GDP (or FS) are underestimated.

Table 7: Sensitivity analysis: ξ

ξ	0.2	0.4	0.6	0.8*	0.9	Data
corr(GDP, IS)	-0.901	-0.874	-0.833	-0.798	-0.780	-0.756
	(0.005)	(0.004)	(0.008)	(0.011)	(0.013)	
corr(GDP, NII)	0.663	0.809	0.855	0.882	0.892	0.789
	(0.015)	(0.011)	(0.010)	(0.008)	(0.007)	
corr(FS, IS)	-0.912	-0.804	-0.722	-0.665	-0.639	-0.756
	(0.002)	(0.006)	(0.012)	(0.018)	(0.020)	
corr(FS, GDP)	0.981	0.975	0.965	0.957	0.953	0.987
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	
corr(FS, NII)	0.504	0.659	0.690	0.708	0.714	0.680
	(0.018)	(0.016)	(0.015)	(0.014)	(0.013)	
$\sigma(FS)/\sigma(GDP)$	0.867	0.782	0.716	0.666	0.645	0.837
	(0.006)	(0.007)	(0.008)	(0.008)	(0.008)	
$\sigma(NII)/\sigma(GDP)$	0.226	0.294	0.362	0.411	0.432	0.220
	(0.006)	(0.005)	(0.005)	(0.005)	(0.004)	
$\sigma(IS)/\sigma(GDP)$	1.731	1.148	0.927	0.794	0.741	0.843
	(0.020)	(0.018)	(0.013)	(0.011)	(0.010)	
$\sigma(GDP)$	0.035	0.054	0.075	0.096	0.107	-
	(0.001)	(0.002)	(0.002)	(0.002)	(0.003)	
$\sigma(NII)$	0.008	0.016	0.027	0.039	0.046	-
	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	
$\sigma(INV)$	0.004	0.007	0.010	0.013	0.015	-
	(0.000)	(0.000)	(0.000)	(0.001)	(0.001)	
$\sigma(FS)$	0.030	0.042	0.053	0.064	0.069	-
	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)	

Numbers in parentheses are standard deviations over 1000 simulations. GDP: real GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: final sales; n: employment level of each output producer. Parameter values with a star are benchmark parameter values.

An interesting result is that the value of ε_f should be relatively lower than the value of ε_i in order to match the data. This result implies that intermediate goods buyers are more responsive than finished goods buyers. This is consistent with the empirical facts that most downstream firms sign long term contracts with upstream firms instead of searching for suppliers every period.

Table 7 shows that the predicted correlations are relatively stable in response to a wide range of ξ. The model tends to overestimate the correlations with a low value of ξ, except for the correlation between final sales and the inventory investment, which is underestimated. Moreover, the relative standard deviation of inventory investment is overshot by the model with a large value of ξ, and the relative standard deviation of the inventory-to-sales ratio is overshot with a low value of ξ. Intuitively speaking, as ξ decreases, the matching rates decrease for both sellers and buyers in both goods markets. Because of input inventory the negative effects on final sales is stronger than those on intermediate goods sales; as a result, inventories become too volatile to match the data.

Table 8: Sensitivity analysis: η

η	0.2	0.4	0.8*	2	3	Data
corr(GDP, IS)	-0.857	-0.855	-0.798	0.520	0.685	-0.756
	(0.005)	(0.006)	(0.011)	(0.009)	(0.007)	
corr(GDP, NII)	0.825	0.845	0.882	0.954	0.974	0.789
	(0.012)	(0.011)	(0.008)	(0.003)	(0.002)	
corr(FS, IS)	-0.785	-0.770	-0.666	0.798	0.943	-0.756
	(0.009)	(0.010)	(0.018)	(0.010)	(0.004)	
corr(FS, GDP)	0.978	0.974	0.957	0.871	0.826	0.987
	(0.001)	(0.001)	(0.001)	(0.004)	(0.006)	
corr(FS, NII)	0.691	0.703	0.709	0.686	0.682	0.680
	(0.017)	(0.017)	(0.014)	(0.013)	(0.013)	
$\sigma(FS)/\sigma(GDP)$	0.781	0.752	0.666	0.408	0.299	0.837
	(0.007)	(0.007)	(0.008)	(0.007)	(0.005)	
$\sigma(NII)/\sigma(GDP)$	0.286	0.317	0.412	0.678	0.776	0.220
	(0.004)	(0.004)	(0.005)	(0.004)	(0.003)	
$\sigma(IS)/\sigma(GDP)$	1.210	1.102	0.794	0.606	0.965	0.843
	(0.022)	(0.018)	(0.011)	(0.019)	(0.023)	
$\sigma(GDP)$	0.082	0.087	0.096	0.111	0.117	-
	(0.002)	(0.002)	(0.002)	(0.003)	(0.003)	
$\sigma(NII)$	0.023	0.027	0.039	0.075	0.091	-
	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)	
$\sigma(INV)$	0.011	0.012	0.013	0.018	0.021	-
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	
$\sigma(FS)$	0.064	0.065	0.064	0.045	0.035	-
	(0.002)	(0.002)	(0.002)	(0.001)	(0.001)	

Numbers in parentheses are standard deviations over 1000 simulations. GDP: real GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: final sales; n: employment level of each output producer. Parameter values with a star are benchmark parameter values.

Table 8 shows that the quantitative results are sensitive to changes in the value of relative risk aversion. The model matches the data well with low η. If η were high, (for example, greater than one), both the correlation between inventory-to-sales ratio and GDP (or final sales) and the relative standard deviations would be mismatched. For the cyclical behavior, the model becomes more volatile to shocks with higher η. These results are due to the fact that the motivation for smoothing consumption is strong with high η, therefore final sales respond to the shocks at a lower magnitude. Thus, the relative standard deviation of final sales is much lower than the value observed in the data. Moreover, in order to smooth consumption, households use more material inputs during the transition and hold more inventories at the end of each period. As a result, the response of inventories is too volatile such that the inventory-to-sales ratio is positively correlated with GDP (or final sales) and the relative standard deviation of inventory investment is overestimated.

Table 9: Sensitivity analysis: B_i

B_i	0.1	0.2*	0.4	0.6	0.8	Data
corr(GDP, IS)	-0.797	-0.798	-0.798	-0.798	-0.798	-0.756
	(0.011)	(0.011)	(0.012)	(0.011)	(0.011)	
corr(GDP, NII)	0.882	0.882	0.882	0.882	0.882	0.789
	(0.008)	(0.008)	(0.008)	(0.008)	(0.008)	
corr(FS, IS)	-0.664	-0.665	-0.665	-0.665	-0.665	-0.756
	(0.018)	(0.018)	(0.019)	(0.018)	(0.018)	
corr(FS, GDP)	0.957	0.957	0.957	0.957	0.957	0.987
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	
corr(FS, NII)	0.708	0.708	0.709	0.708	0.709	0.680
	(0.014)	(0.014)	(0.015)	(0.015)	(0.014)	
$\sigma(FS)/\sigma(GDP)$	0.666	0.666	0.665	0.666	0.666	0.837
	(0.008)	(0.008)	(0.008)	(0.008)	(0.008)	
$\sigma(NII)/\sigma(GDP)$	0.412	0.411	0.411	0.411	0.411	0.220
	(0.004)	(0.005)	(0.005)	(0.005)	(0.005)	
$\sigma(IS)/\sigma(GDP)$	0.794	0.794	0.794	0.794	0.794	0.843
	(0.011)	(0.011)	(0.010)	(0.011)	(0.011)	
$\sigma(GDP)$	0.096	0.096	0.096	0.096	0.096	-
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	
$\sigma(NII)$	0.040	0.039	0.040	0.040	0.040	-
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	
$\sigma(INV)$	0.013	0.013	0.013	0.013	0.013	-
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	
$\sigma(FS)$	0.064	0.064	0.064	0.064	0.064	-
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	

Numbers in parentheses are standard deviations over 1000 simulations. GDP: real GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: final sales; n: employment level of each output producer. Parameter values with a star are benchmark parameter values.

Similar to the findings of Wang and Shi (2006), table 9 shows that the inventory regularities are insensitive to changes in B_i, the intermediate goods market's buyer/seller ratio. The intuition is the following. Increased market tightness on one hand generates negative externalities, hence has a negative effect, on buyers' matching probabilities and, on the other hand, it also generates positive externalities, hence has a positive effect, on sellers' matching probabilities. Since positive effects cancel out negative effects, the overall results are insensitive to market tightness.

Table 10 shows that the predicted results are sensitive to the changes in markups, and the model would _t the data more effectively with high markups. If the markup is low, households can consume more in the long run and final sales would respond more strongly to the shocks. More final sales require more material inputs, as a result of which inventories become too volatile and the inventory-to-sales ratio is positively correlated with GDP (or final sales). Moreover, since the effects on GDP are bigger than the effects on inventories, the relative standard deviation of the inventory-to-sales ratio stays within an acceptable region.

Table 10: Sensitivity analysis: *markup*

markup	0.2	0.4	0.5	0.6	0.7*	Data
corr(GDP, IS)	0.7692	-0.183	-0.648	-0.766	-0.798	-0.756
	(0.005)	(0.029)	(0.021)	(0.015)	(0.011)	
corr(GDP, NII)	0.931	0.921	0.905	0.892	0.882	0.789
	(0.005)	(0.005)	(0.006)	(0.007)	(0.008)	
corr(FS, IS)	0.981	0.171	-0.403	-0.591	-0.666	-0.756
	(0.001)	(0.035)	(0.031)	(0.023)	(0.018)	
corr(FS, GDP)	0.855	0.902	0.924	0.943	0.957	0.987
	(0.005)	(0.002)	(0.002)	(0.001)	(0.001)	
corr(FS, NII)	0.664	0.665	0.675	0.691	0.709	0.680
	(0.014)	(0.014)	(0.014)	(0.015)	(0.014)	
$\sigma(FS)/\sigma(GDP)$	0.365	0.516	0.575	0.624	0.666	0.837
	(0.006)	(0.008)	(0.008)	(0.009)	(0.008)	
$\sigma(NII)/\sigma(GDP)$	0.797	0.584	0.519	0.462	0.412	0.220
	(0.014)	(0.004)	(0.005)	(0.005)	(0.005)	
$\sigma(IS)/\sigma(GDP)$	0.527	0.262	0.402	0.596	0.794	0.843
	(0.012)	(0.008)	(0.006)	(0.008)	(0.011)	
$\sigma(GDP)$	0.549	0.174	0.134	0.111	0.096	-
	(0.014)	(0.004)	(0.003)	(0.003)	(0.002)	
$\sigma(NII)$	0.437	0.102	0.070	0.051	0.039	-
	(0.016)	(0.003)	(0.002)	(0.001)	(0.001)	
$\sigma(INV)$	0.058	0.020	0.016	0.014	0.013	-
	(0.002)	(0.006)	(0.001)	(0.001)	(0.001)	
$\sigma(FS)$	0.200	0.090	0.077	0.069	0.064	-
	(0.005)	(0.002)	(0.001)	(0.002)	(0.002)	

Numbers in parentheses are standard deviations over 1000 simulations. GDP: real GDP; IS: inventory-to-sales ratio; NII: net inventory investment; FS: final sales; n: employment level of each output producer. Parameter values with a star are benchmark parameter values.

But, the inventory investment is much more volatile than the inventories because it is a flow concept, thus the relative standard deviation of inventory investment is overestimated by the model.

5. Conclusion

In this paper, we study both the long run and the short run effects of monetary policy on input inventories. In particular, money growth has nonmonotonic real effects on the steady state input inventory investment, inventory-to-sales ratio and final sales. By calibrating to quarterly US data, we showed that the model is able to replicate the stylized facts on input inventory movements well over the business cycle. And it predicted that input inventories are procyclical, which are different from output inventories. Such procyclical input inventories induce positive responses of employment, since input inventories have positive effects on revenues.

Finally, our model shows that input inventories amplify aggregate fluctuations. This destabilizing role of input inventories attributes to the positive intensive effect. We also conducted a sensitivity analysis of some parameters relative to the baseline calibration. In order to match the data, the model requires that finished goods buyers are less responsive to the shock in comparison to intermediate goods buyers in order to keep the relative standard deviation of the inventory-to-sales ratio (relative to GDP) within a reasonable range. Nevertheless, search frictions in the labor market and the goods markets matter both qualitatively and quantitatively for matching the data.

To conclude, our paper sheds light on the importance of monetary policy to inventories.

References

Berentsen, Aleksander, Guido Menzio, and Randall Wright. 2011. “Inflation and Unemployment in the Long Run." American Economic Review, 101(1): 371-398.

Bils, Mark. 2004. “Studying Price Markups from Stockout Behavior." Unpublished.

Bils, Mark, and James A. Kahn. 2000. “What inventory behavior tells us about business cycles." American Economic Review, 90(3): 458-481.

Blanchard, Oliver, and Peter Diamond. 1989. “The Beveridge Curve." Brookings Papers on Economic Activity, 20(1): 1-76.

Blinder, Alan S., and Louis J. Maccini. 1991. “Taking Stock: A Critical Assessment of Recent Research on Inventories." Journal of Economic Perspectives, 5(1): 73-96.

Chang, Yongsung, Andreas Hornstein, and Pierre Daniel Sarte. 2006. “Understanding how employment responds to productivity shocks in a model with inventories." Unpublished.

Christiano, Lawrence J. 1988. “Why does inventory investment fluctuate so much?" Journal of Monetary Economics, 247-280.

Coen-Pirani, Daniele. 2004. “Markups, Aggregation, and Inventory Adjustment." American Economic Review, 94(5): 1328-53.

Fisher, Jonas D. M., and Andreas Hornstein. 2000. “(S, s) Inventory Policies in General Equilibrium." Review of Economic Studies, 67(1): 117-45.

Herrera, Ana, and Elena Pesavento. 2005. “The Decline in U.S. Output Volatility: Structural Changes and Inventory Investment." Journal of Business & Economic Statistics, 23(4): 462-472.

Herrera, Ana, Irina Murtazashvili, and Elena Pesavento. 2008. “The Comovement in Inventories and in Sales: Higher and Higher." Economics Letters, 99(1): 155-158.

Hosios, Arthur. 1990. “On the Efficiency of Matching and Related Models of Search Unemployment." Review of Economic Studies, 57: 279-298.

Humphreys, Brad R., Louis J. Maccini, and Scott Schuh. 2001. “Input and Output Inventories." Journal of Monetary Economics, 47(2): 347-375.

Iacoviello, Matteo, Fabio Schiantarelli, and Scott Schuh. 2011. “Input and Output Inventories in General Equilibrium." International Economic Review, 52(4): 1179-1213.

Jung, Yongseung, and Tack Yun. 2006. “Monetary Policy Shocks, Inventory Dynamics, and Price-setting Behavior." Unpublished.

Kahn, James A. 1987. “Inventories and the Volatility of Production." American Economic Review, 77(4): 667-79.

Kahn, James A., Margaret M. McConnell, and Gabriel Perez-Quiros. 2002. “On the Causes of the Increased Stability of the U.S. Economy." Economic Policy Review, 183-202.

Khan, Aubhik, and Julia K. Thomas. 2007a. “Explaining Inventories: A Business Cycle Assessment of the Stockout Avoidance and (S,s) Motives." Macroeconomic

Dynamics, 11(5): 638-665.
Khan, Aubhik, and Julia K. Thomas. 2007b. "Inventories and the Business Cycle: An Equilibrium Analysis of (S,s) Policies." American Economic Review, 97(4): 1165-1188.
Kryvtsov, Oleksiy, and Virgiliu Midrigan. 2010a. "Inventories and real rigidities in New Keynesian business cycle models." Journal of the Japanese and International Economies, 24(2): 259-281.
Kryvtsov, Oleksiy, and Virgiliu Midrigan. 2010b. "Inventories, Markups, and Real Rigidities in Menu Cost Models." Unpublished.
Maccini, Louis J., Bartholomew J. Moore, and Huntley Schaller. 2004. "The Inter-est Rate, Learning, and Inventory Investment." American Economic Review, 94(5): 1303-27.
McConnell, Margaret M., and Gabriel Perez-Quiros. 2000. "Output Flfluctuations in the United States: What Has Changed since the Early 1980's?" American Economic Review, 90(5): 1464-1476.
Menner, Martin. 2006. "A Search-Theoretic Monetary Business Cycle Model with Capital Formation." The B.E. Journal of Macroeconomics, 0(1): 11.
Ramey, Valerie A, and Daniel J Vine. 2004. "Why Do Real and Nominal Inventory-Sales Ratios Have Different Trends?" Journal of Money, Credit and Banking, 36(5): 959-63.
Ramey, Valerie A., and Kenneth D. West. 1999. Inventories. Vol. 1 of Handbook of Macroeconomics. 1 ed., J. B. Taylor and M. Woodford.
Rogerson, Richard, and Robert Shimer. 2011. Search in Macroeconomic Models of The Labor Market. Vol. 4 of Handbook of Labor Economics. 1 ed., Elsevier.
Shimer, Robert. 2005. "The Cyclical Behavior of Equilibrium Unemployment and Vacan-cies." The American Economic Review, 95(1): 25-49.
Shi, Shouyong. 1998. "Search for a Monetary Propagation Mechanism." Journal of Eco-nomic Theory, 81: 314-352.
Shi, Shouyong. 2006. "Viewpoint: A Microfoundation of Monetary Economics." Canadian Journal of Economics, 39: 643-688.
Stock, James H., and Mark W. Watson. 2002. "Has the Business Cycle Changed and Why?" NBER Macroeconomics Annual 2002, 17: 159-230.
Wang, Weimin, and Shouyong Shi. 2006. "The Variability of the Velocity of Money in a Search Model." Journal of Monetary Economics, 53: 537-571.
Wen, Yi. 2011. "Input and Output Inventory Dynamics." American Economic Journal: Macroeconomics, 3(4): 181-212.

A. Appendix A

In this section, we prove that the model economy exists at least one steady state, which satisfie $(\lambda^f;\Omega_a) > 0$. Denote the steady state values with an asterisk, which can be rewritten by the dynamic system:

$$z^f(s^{f*})^\xi = \frac{\gamma - \beta}{\beta} \frac{\omega^{f*}}{U'(c^*) - w^{f*}}, \tag{45}$$

$$\Omega_i^* = \beta\varphi'(q^{i*}), \tag{46}$$

$$\frac{(1-\beta(1-\delta_n))}{\beta}k(v^*) = \{\sigma z^f B^f (s^{f*})^\xi \omega^{f*} - \sigma\varphi^f \tag{47}$$

$$+ [(1 - z^f B^f (s^{f*})^\xi)\sigma(1-\delta_i)\beta - \sigma]\varphi'(q^{i*})\},$$

$$\Phi^{f\prime}(s^{f*}) = z^f (s^{f*})^{\xi-1}[U'(c^*) - \omega^{f*}]q^{f*}, \tag{48}$$

$$v^*\mu(v^*) = \delta_n q^{f*}, \tag{49}$$

$$q^{i*} = \{1 - (1-\delta_i)[1 - z^f B^f (s^{f*})^\xi]\}q^{f*}, \tag{50}$$

$$c^* = a_p^f B^f z^f (s^{f*})^\xi q^{f*}. \tag{51}$$

As demonstrated in section II, the steady state system can be reduced to two equations which are repeated here for future use:

$$z^f[s^f(\omega^{f*}, q^{f*})]^\xi = \frac{\gamma-\beta}{\beta}\frac{\omega^{f*}}{U'(c(\omega^{f*}, q^{f*})) - w^{f*}},$$

$$(1-\beta(1-\delta_n))k(v(q^{f*})) + \beta\sigma\varphi^f = \beta\{\sigma z^f B^f (s^f(\omega^{f*}, q^{f*}))^\xi \omega^{f*}$$
$$+[(1 - z^f B^f (s^f(\omega^{f*}, q^{f*}))^\xi)\sigma(1-\delta_i)\beta - \sigma]\varphi'(q^i(\omega^{f*}, q^{f*}))\}.$$

Similarly to Shi (1998), above two equations give a relationship between ω^f and q^f, denote $q^{f*} = q^f 1(\omega^{f*})$ and $q^{f*} = q^f 2(\omega^{f*})$. The steady state value ω^{f*} is a solution to $Q^f 1(\omega^{f*}) = Q^f 2(\omega^{f*})$. To ensure $\lambda^f > 0$, the solution must satisfy $U'(c^*) \geq \omega^{f*} + \Delta$, where $\Delta > 0$ is an arbitrarily small number. That is, we require $q^{f*} \leq q^f(\omega^{f*}, \Delta)$ [15], where $q^f(\omega^{f*}, \Delta)$ is defined by:

$$U'(c(\omega_f, q^f(\omega^f, \Delta))) = \omega^f + \Delta. \tag{52}$$

Using Lemma 3.2 as explained in Shi (1998), we can prove that the function Q^{f}(!^{f}; _) is well defined and has the following properties for sufficiently small $\Delta > 0$: $Q_{wf}^f(\omega^f, \Delta) < 0, Q^f(\infty, \Delta) = 0$; and $\lim_{\Delta\to 0} Q^f(0, \Delta) = \infty$. The function $q^f 1(\omega^f)$ satisfies $q^f 1'(\omega^f) < 0$; $q^f 1(0) = \infty$ and $q^f 1(\infty) = 0$. Furthermore, the two curves $q^f 1(\omega^f)$ and $Q^f(\omega^f, \Delta) < 0$ have a unique intersection at a level denoted $\omega_1^f(\Delta)$ which satisfies $\lim_{\Delta\to 0}\omega_1^f(\Delta) = 0$ [16]

In order to prove the uniqueness, we also need to know the properties of q^{f}2. Although the properties of q^{f}2 are the same as what is described in Lemma 3.3 in Shi (1998), the proof is not the same due to different function forms.

We are going to prove that q^{f}2 has the following properties: $q^f 2(0) = 0, q^f 2(\infty) = 0$, and $q^f 2'(\omega^f) < 0$ for sufficiently large ω^f. The two curves $q^f 2(\omega^f)$ and $Q^f(\omega^f, \Delta)$ have

---

[15] First, positive nominal interest rate implies $\gamma > \beta$ and is enough to ensure $\lambda > 0$. If q^{f}– > Q^{f}(!^{f}–; _), U^{00}(c) < 0 implies U^{0}(c(!^{f}–; q^{f}–)) < U^{0}(c(!^{f}–; Q^{f}(!^{f}–; _))), which violates U^{0}(c–) _ !^{f}– + _.

[16] Since equation (35) is identical to the steady state equation (3.4) in Shi (1998), we omit the proof here.

a unique intersection at a level denoted $\omega^f 2(\Delta)$ which approaches infinity when Δ approaches zero.

First, let us show $q^f 2(0) = 0$ by rearranging equation (36):

$$\begin{aligned} \varphi'(q^i(\omega^{f*}, q^{f*})) &= \frac{\beta[\sigma z^f B^f (s^f(\omega^{f*}, q^{f*}))^\xi \omega^{f*} - \sigma\varphi^f] - (1-\beta(1-\delta_n))k(v(q^{f*}))}{\beta[1-(1-z^f B^f(s^f(\omega^{f*}, q^{f*}))^\xi)\sigma(1-\delta_i)\beta]\sigma} \\ &\le \frac{\sigma z^f B^f (s^f(\omega^{f*}, q^{f*}))^\xi \omega^{f*}}{1-(1-z^f B^f(s^f(\omega^{f*}, q^{f*}))^\xi)\sigma(1-\delta_i)\beta]\sigma} \\ &\le \frac{\sigma z^f B^f \omega^{f*}}{\{[1-\sigma(1-\delta_i)\beta]/(s^f(\omega^{f*}, q^{f*}))^\xi + z^f B^f \sigma(1-\delta_i)\beta\}\sigma} \end{aligned} \quad (53)$$

The right-hand side of (A9) approaches zero as ω^f approaches zero, becuase $\lim_{\omega^f \to 0} numerator = 0$ and $\lim_{\omega^f \to 0} denominator = z^f B^f \sigma(1-\delta_i)\beta$. Since $\varphi(') > 0$, equation (A9) implies $\lim_{\omega^f \to 0} q^i(\omega^{f*}, q^{f*}) = 0$. Finally $\lim_{\omega^f \to 0} q^f 2(\omega) = 0$, is implied by the steady state equation $q^{i*} = \{1-(1-\delta_i)[1-z^f B^f (s^{f*})^\xi]\}q^{f*}$

Second, let us prove that the two curves $q^f 2(\omega^f)$ and $Q^f(\omega^f, \Delta)$ have a unique intersection. By plugging the equation of c- into the f~~ourth~~ equation of the steady state system, we can get a useful equation:

$$\Phi^{f'}(s^{f*})s^{f*} = [U'(c^*) - \omega^{f*}]\frac{c^*}{a_p^f B^f}. \quad (54)$$

As the definition of $Q^f(\omega^f, \Delta)$, set $\omega^f = u'(c) - \Delta$. Then equation (A10) implies that s^f is a function of $(c, \Delta): \Phi^{f'}(s^{f*})s^{f*} = \Delta c^* / (a_p^f B^f)$. Denote the solution for s^f as $s^f(c, \Delta)$. Because $\Phi^{f'}(0) = 0, \Phi^{f'}(.) > 0$ and $\Phi^{f''}(.) > 0$, we can get $s^f(c, 0) = 0$, $s^f(c, \infty) = \infty$, $s^f(0, \Delta) = 0$ and $s^f(c, \Delta) > 0$. By rearranging equation (A10), we can prove that $c/(s(c, \Delta))^\xi$ is an increasing function of c.

Rearranging the steady state equation of c*, it is easy to see that $q^f(c, \Delta)$ is also an increasing function of $c: q^f(c, \Delta) = c / a_p^f B^f z^f [s^f(c, \Delta))^\xi]\}q^f(c, \Delta)$. Similarly, q^i can be rewritten as a function of $c: q^f(c, \Delta) = c / a_p^f B^f z^f [s^f(c, \Delta))^\xi]\}q^f(c, \Delta)$. Since both $s^f(c, \Delta)$ and $q^f(c, \Delta)$ are increasing in $c, q^i(c, \Delta)$ is an increasing function of c.

Now we are ready to prove that the two curves $q^f 2(\omega^f)$ and $Q\ (\omega^f, \Delta)$ have a unique intersection. Rewrite equation (36) in terms of (c, Δ):

$$\begin{aligned} \mathrm{LHS}(36) &= (1-\beta(1-\delta_n))k(v(q^f(c, \Delta))) + \beta\sigma\varphi^f && (55) \\ \mathrm{RHS}(36) &= \beta\{\sigma z^f B^f (s^f(c, \Delta))^\xi \omega^{f*} \\ &+ [(1-z^f B^f(s^f(c, \Delta))^\xi)\sigma(1-\delta_i)\beta - \sigma]\varphi'(q^i(c, \Delta))\} && (56) \end{aligned}$$

The left-hand side of (36) is an increasing function of c, and the right-hand side of (36) is a decreasing function of c, because $s^f(c, \Delta)$, $q^f(c, \Delta)$ and $q^i(c, \Delta)$ are increasing in c, $k'(v) > 0$ and $\varphi'(q^i) > 0$. Moreover, since $q^f(0, \Delta) = 0$,

$k(v(q^f(0,\Delta)))=0$, $q^f(\infty,\Delta)=\infty$ and $k(v(q^f(\infty,\Delta)))=\infty$ it is easy to see that $\lim_{c\to 0} RHS(36)=\infty$ and $\lim_{c\to\infty} RHS(36)=-\infty$. Similarly, the right-hand side has the following properties. $\lim_{c\to 0} RHS(36)=\infty$, because $q^f(0,\Delta)=0$, $s^f(\infty,\Delta)=\infty$ and $\lim_{c\to 0} cu'(c)=\infty$. And $\lim_{c\to\infty} RHS(36)=-\infty$. because $q^i(\infty,\Delta)=\infty$, $q^f(\infty,\Delta)=\infty$ and $\lim_{c\to\infty} cu'(c)=0$.

Given these properties of (36), there is a unique solution for c to (36). Denote this solution by $c(\Delta)$, then $\omega^f 2(\Delta)=u'(c(\Delta))-\Delta$ is unique. Thus there must be a unique intersection between the two curves $q^f 2(\omega^f)$ and $Q^f(\omega^f,\Delta)$.

Third, we are going to prove that $\lim_{\Delta\to 0}\omega^f 2(\Delta)=0$, $q^f 2'(\omega^f)<0$ and $q^f 2(\infty)=0$. For fixed c, $\lim_{\Delta\to 0} LHS(36)=\infty$ and $\lim_{\Delta\to 0} RHS(36)=-\infty$, because $\lim_{\Delta\to 0} s(c,\Delta)=0$ and $\lim_{\Delta\to 0} q^f(c,\Delta)=\infty$. Thus (36) is satisfied only when $\lim_{\Delta\to 0} c(\Delta)=0$ and $\lim_{\Delta\to 0}\omega^f 2(\Delta)=0$. Next, $q^f 2'(\omega^f)<0$ since $q^f 2(c,\Delta)$ is an increasing function of c and $\omega^f 2'(c(\Delta))<0$. This can be proved by plugging $\omega^f 2'(c(\Delta))$ into $q^f 2(\omega^f)$ and analyzing $q^f 2(\omega^f 2(c(\Delta)))$.

Now, we are going to prove $q^f 2(\infty)=0$. Because $Q^f(0,\Delta)$ is a positive constant and $q^f 2(0)=0$ is proven, $q^f 2(0)<Q^f(0,\Delta)$ and the curve $q^f 2(\omega^f)$ must cross the curve $Q^f(\omega^f,\Delta)$ from below if the two have a unique intersection. Moreover, because $Q^f(\infty,\Delta)=0$ is proven in Lemma 1 and $0\le q^f 2(\omega^f)\le Q^f(\omega^f,\Delta)$ for $\omega^f<\omega^f 2(\Delta)$, $0\le q^f 2(\infty)<0$ or $\omega^f<\omega^f 2(\Delta)$. Then $q^f 2(\infty)=0$ since $q^f 2(\omega^f)$ is continuous and only has one intersection with $Q^f(\omega^f,\Delta)$.

Finally, given the properties of equations (35) and (36) proven, there exists at least one steady state for the model.

B. Appendix B

B.1. Proof of proposition 1

Now we are going to prove that the long run effect of money growth on q^f is not monotonic. Since Equation (36) is independent of γ, while equation (35) will be shifted to the right as $\gamma\to\beta$ and to the left as $\gamma\to\infty$. Since $q^f 2(0)=0$, $q^f 2(\infty)=0$ and $q^f 2'(\omega^f)<0$ for sufficiently large ω^f, equation (36) is hump-shaped. Thus steady state q^f decreases with if γ is high, but increases with γ if it γ is low.

To prove the production of intermediate goods $a_b^i q^i$ is nonmonotonic, we take derivative of that:

$$\frac{\partial i}{\partial\gamma}=\frac{\partial i(q^f(\omega^f))}{\partial q^f}\frac{\partial q^f}{\partial\omega^f}\frac{\partial\omega^f}{\partial\gamma}, \tag{57}$$

where $\partial\omega^f/\partial\gamma<0$, and $\partial q^f/\partial\omega^f$ first increases with γ then decreases with large value of γ. $\partial i/\partial q^f>0$ will be proved in the next proposition. Since $n=q^i$ with Leontief production function, employment also increases with γ if it is low, but decreases with γ if it is high.

Take derivative of the final sales $(s^{f\zeta} a_b^f q^f)$:

$$\frac{\partial FS}{\partial \gamma} = z^f \xi a_b^f q^f s^{f\xi-1} \frac{\partial s^f}{\partial \gamma} + z^f a_b^f s^{f\xi} \frac{\partial q^f}{\partial \gamma}. \tag{58}$$

Since $\partial s^f / \partial \gamma > 0$, final sales are not monotonic in γ as q^f does.

B.2. Proof of proposition 2

Since the difference between steady state net inventory investment and steady state inventory level is just a constant multiplier $a_p^f \delta_i$, we only prove the long run response of inventory investment. Equation (37) implies that the steady state net inventory investment is the difference between the quantity of goods per match and the final sales discounted at a proper rate, namely,

$$NII^* = a_p^f \delta_i i = (1 - \delta_i)\delta_i a_p^f q^f [1 - z^f B^f (s^f)^\xi]. \tag{59}$$

The derivative of i with respect to q^f can be derived from this equation:

$$\begin{aligned} \frac{\partial i}{\partial q^f} &= (1 - \delta_i)[1 - z^f B^f (s^f)^\xi] - (1 - \delta_i) q^f \xi z^f B^f (s^f)^{\xi-1} \frac{\partial s^f}{\partial q^f} \\ &> 0. \end{aligned} \tag{60}$$

$\partial i / \partial q^f > 0$ because $\partial s^f / \partial q^f > 0$. Since i is a function of $q^f(\omega^f)$, the effects of money growth on input inventory investment can be studied by taking the derivative of $a_p^f \delta_i i(q^f(\omega^f))$ with respect to γ :

$$\begin{aligned} a_p^f \delta_i \frac{\partial i(q^f(\omega^f))}{\partial \gamma} &= \frac{\partial i}{\partial q^f} \frac{\partial q^f}{\partial \omega^f} \frac{\partial \omega^f}{\partial \gamma} \\ &> 0, \quad \text{if } \gamma \text{ if low;} \\ &< 0, \quad \text{if } \gamma \text{ if high.} \end{aligned} \tag{61}$$

We can conclude that $\partial NII^* / \partial \gamma > 0$ if γ is low and $\partial NII^* / \partial \gamma < 0$ if γ is high. This is because that $\partial q^f / \partial \omega^f < 0$ when γ is low, $\partial q^f / \partial \omega^f > 0$ when γ is high as implied by proposition 1. Moreover, $\partial i / \partial q^f > 0$ and $\partial \omega^f / \gamma < 0$.

Since both final sales and NII are nonmonotonic in , it is clear that GDP is also increases with γ if it is low, and decreases with γ if it is high.

B.3. Proof of proposition 4

By rearranging equation (39), we can get a expression for steady state inventory-to-sales ratio:

$$
\begin{aligned}
IS &= \frac{a_p^f i^*}{a_p^f B^f z^f (s^{f*})^{\xi} q^{f*}} \\
&= (1-\delta_i)[\frac{a_p^f (q^{f*})}{a_p^f B^f z^f (s^{f*})^{\xi} q^{f*}} - 1], \\
&= (1-\delta_i)[\frac{1}{B^f z^f (s^{f*})^{\xi}} - 1] \qquad (62)
\end{aligned}
$$

The effects of money growth on the inventory-to-sales ratio can be studied by taking the derivative with respect to:

$$
\begin{aligned}
\frac{\partial IS^*}{\partial \gamma} &= -(1-\delta_i)\frac{B^f z^f \xi s^{f^{\xi-1}}}{[B^f z^f (s^{f*})^{\xi}]^2}\frac{\partial s^f}{\partial q^f}\frac{\partial q^f}{\partial \omega^f}\frac{\partial \omega^f}{\partial \gamma}, \qquad (63)\\
&> 0, \quad \text{if } \gamma \text{ if low;} \\
&< 0, \quad \text{if } \gamma \text{ if high.}
\end{aligned}
$$

Evaluating at $\omega = \omega^*(\gamma)$, the inventory-to-sales ratio has a hump-shaped long run response to the money growth rate across steady states, because $\partial q^f / \partial \omega^f < 0$ when γ is low; and $\partial q^f / \partial \omega^f > 0$ when γ is high as implied by proposition 1. Moreover, $\partial \omega^f / \gamma < 0$ and $\partial s^f / \partial q^f > 0$.

C. Appendix C

The household's new decision problem is altered as follows. The representative house-hold taking the sequence $\{\hat{q}_t^i, \hat{m}_t^i, \hat{q}_t^f, \hat{m}_t^f, \hat{W}_t\}_{t\geq 0}$ and initial conditions $\{M_0, i_0, n_0\}$ as given,

chooses $\{C_t, a_t, s_t^i, s_t^f, \Delta_{t+1}, M_{t+1}, i_{t+1}, v_t, n_{t+1}\}_{t\geq 0}$ to maximize its expected lifetime utility:

$$
\max \sum_{t=0}^{\infty} \beta^t \mathbb{E}_{-1}[U(c_t) - g_{st}^i a_p^i \varphi(\hat{q}_t^i) - a_p \hat{n}_t \varphi^f - a_b^i \Phi^i(s_t^i) - a_b^f \Phi^f(s_t^f) - a_p^f K(v_t)] \qquad (64)
$$

subject to the following constraints for all $t \geq 0$:

$$
\begin{aligned}
c_t &\leq (1-FI) s_t^f g_{bt}^f a_b^f \hat{q}_t^f, \qquad (65)\\
\frac{(1-\Delta_{t+1})M_{t+1}}{a_b^f} &\geq \hat{m}_{t+1}^f, \quad \forall F_{bt+1}{}^* \qquad (66)\\
q_t^f &= A_t a_t^{\alpha} n_t^{1-\alpha}, \qquad (67)\\
a_t &\leq i_t + \frac{1}{a_p^f} s_t^i g_{bt}^i a_b^i \hat{q}_t^i, \quad \forall F_{pt}{}^* \qquad (68)\\
q_t^f &\geq \hat{q}_t^f, \quad \forall F_{pt}{}^* \qquad (69)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\Delta_{t+1}M_{t+1}}{a_b^i} &\geq \hat{m}_{t+1}^i, \quad \forall I_{bt}{}^* && (70)\\
M_{t+1} &\leq M_t + \tau_t - s_t^i g_{bt}^i a_b^i \hat{m}_t^i + g_{st}^i a_p^i \hat{m}_t^i + a_p^f \hat{n}_t \hat{P}_t \hat{W}_t && (71)\\
&- s_t^f g_{bt}^f a_b^f \hat{m}_t^f + g_{st}^f a_p^f \hat{m}_t^f - \hat{P}_t a_p^f \hat{W}_t n_t, &&\\
0 &\leq a_p^f[(1-\delta_n)n_t + v_t\mu_t - n_{t+1}], && (72)\\
a_p^f i_{t+1} &\leq (1-\delta_i)[a_p^f i_t + s_t^i g_{bt}^i a_b^i \hat{q}_t^i - g_{st}^f a_p^f a_t]. && (73)
\end{aligned}
$$

Functions $U(\cdot), \Phi^f(\cdot)$ and $K(\cdot)$ have the same properties as in the benchmark model. $\Phi^i(s_t^i)$ is a buyer's disutility of searching in the intermediate goods market. The function Φ^i satisfies $\Phi^{i'} > 0$ and $\Phi^{i''} > 0$ for $s^i > 0$, and $\Phi^i(0) = \Phi^{i'}(0) = 0$. I_{bt}^* (with measure $s_t^i g_{bt}^i a_b^i$) is the set of matched intermediate goods buyers in period t. Moreover, as discussed in Shi (1998)[17]. We modify the model to incorporate fixed investment which is a constant fraction of aggregate sales.

Denote the multipliers of money constraint (C3) and (C7) by Λ_t^f and Λ_t^i respectively. All of the multipliers of the rest conditions are the same as in the benchmark model.

The terms of trade in the intermediate goods market are determined by Nash bargaining. As in the benchmark model, we assume the intermediate goods buyers and sellers have the same bargaining powers, and the terms of trade can be pinned down by the following two equations:

$$
\begin{aligned}
P_t^i(\bar{\Omega}_{Mt} + \bar{\Lambda}_t^i) &= \bar{\Omega}_{at} + (1-\delta_i)\bar{\Omega}_{it}, && (74)\\
\varphi'(q_t^i) &= P_t^i \Omega_{Mt}. && (75)
\end{aligned}
$$

Then, we can write the dynamic system

$$
\begin{aligned}
n_{t+1} &= (1-\delta_n)n_t + v_t\mu(v_t), && (76)\\
a_p^f i_{t+1} &= (1-\delta_i)[a_p^f i_t + s_t^i g_{bt}^i a_b^i q_t^i - g_{st}^f a_p^f a_t], && (77)\\
i_t &= a_t - s_t^i g_{bt}^i a_b^i q_t^i / a_p^f, && (78)\\
\mathbb{E}[\frac{1-\Delta_{t+1}}{\Delta_{t+1}}] &= \mathbb{E}[\frac{\omega_{t+1}^f q_{t+1}^f a_b^f}{\varphi'(q_{t+1}^i) q_{t+1}^i a_b^i}], && (79)\\
0 &= \beta\mathbb{E}\Big[\{\omega_{t+1}^f + z^f(s_{t+1}^f)^\xi[(1-FI)U'(c_{t+1} - \omega_{t+1}^f)]\}\Big] && (80)\\
&- \mathbb{E}\Big[\frac{(1-\Delta_{t+1})\gamma_t \omega_t^f q_t^f}{(1-\Delta_t)q_{t+1}^f}\Big], &&\\
\Omega_{it} &= \beta\mathbb{E}\{(1-\delta_i)\Omega_{it+1} + g_{st+1}^f a_p^f[\varphi'(q_{t+1}^i) && (81)\\
&+ \lambda_{t+1}^i - (1-\delta_i)\Omega_{it+1}]\}, &&\\
k(v_t) &= \beta\mathbb{E}[\sigma g_{st+1}^f A_{t+1} a_{t+1}^\alpha n_{t+1}^{-\alpha} \omega_{t+1}^f (1-\alpha) && (82)\\
&- \sigma\varphi^f + (1-\delta_n)\Omega_{nt+1}], &&\\
\Phi^{i'}(s_t^i) &= g_{bt}^i[g_{st}^f a_p^f \lambda_t^i - (1-g_{st}^f a_p^f)\varphi'(q_t^i) && (83)\\
&+ (1-g_{st}^f a_p^f)(1-\delta_i)\Omega_{it}]q_t^i, &&
\end{aligned}
$$

[17] Also see Wang and Shi(2006)

$$\Phi^{f'}(s_t^f) = z^f(s_t^f)^{\xi-1}[(1-FI)U'(c_t) - w_t^f]q_t^f, \quad (84)$$

$$c_t = (1-FI)a_p^f B^f z^f(s_t^f)^{\xi} q_t^f, \quad (85)$$

$$A_t\alpha(\frac{n_t}{a_t})^{1-\alpha}\omega_t^f = a_p^f[\varphi'(q_t^i) + \lambda_t^i] + (1-a_p^f)(1-\delta_i)\Omega_{it}. \quad (86)$$

D. Appendix D

Now, we are going to describe the calibration procedures. Derive the steady state equations from the dynamic system:

$$v^*\mu(v^*) = \delta_n n^*, \quad (87)$$

$$\delta_i a_p^f i^* = (1-\delta_i)[z^i a_b^i (s^{i*})^{\xi} q^{i*} - z^f B^f (s^{f*})^{\xi} a_p^f a^*], \quad (88)$$

$$a^* = i^* + z^i a_b^i (s^{i*})^{\xi} q^{i*}/a_p^f, \quad (89)$$

$$z^f(s^{f*})^{\xi} = \frac{\gamma-\beta}{\beta}\frac{\omega^{f\star}}{(1-FI)U'(c^*) - w^{f\star}}, \quad (90)$$

$$\Omega_i^* = \frac{\beta z^f B^f (s^{f*})^{\xi} a_p^f[\lambda^{i*} + \varphi'(q^{i*})]}{1-\beta(1-\delta_i)(1-z^f B^f (s^{f*})^{\xi} a_p^f)} \quad (91)$$

$$k(v^*) = \beta[(1-\alpha)\sigma z^f B^f (s^{f*})^{\xi} A^*(a^*)^{\alpha}(n^*)^{-\alpha}\omega^{f*} - \sigma\varphi^f] + \beta(1-\delta_n)k(v^*), \quad (92)$$

$$\Phi^{i'}(s^{i*}) = g_b^{i*}\{[g_s^{f*}a_p^f\lambda^{i*} - (1-g_s^{f*}a_p^f)[\varphi'(q^{i*}) - (1-\delta_i)\Omega_i^*]\}q^{i*}, \quad (93)$$

$$\Phi^{f'}(s^{f*}) = z^f(s^{f*})^{\xi-1}[(1-FI)U'(c^*) - \omega^{f*}]q^{f*}, \quad (94)$$

$$A^*\alpha(\frac{n^*}{a^*})^{1-\alpha}\omega^{f*} = a_p^f[\varphi'(q^{i*}) + \lambda^{i*}] + (1-a_p^f)(1-\delta_i)\Omega_i^*, \quad (95)$$

$$c^* = (1-FI)a_p^f B^f z^f(s^{f*})^{\xi} q^{f*}. \quad (96)$$

The average labor participation rate (LP = 0:6445), the average unemployment rate (UR = 0:061) and the assumption $a_p^i = a_p^f$ can be used to pin down parameters (u, a_p^i, a_p^f):

$$u = LP \cdot UR = 0.0393. \quad (97)$$

Since the households have measure one, the labor participation rate equals $u + a_p^f(1+n) + a_p^i$. Using the assumption $a_p^i = a_p^f$, the number of sellers in the intermediate goods market a_p^i is equate to its counterparts in the finished goods market a_p^f, and the steady state vacancies can be calculated as the following:

$$a_p^f = (LP-u)/(2+n) = 0.2017, \quad (98)$$

$$a_p^i = LP - u - a_p^f(1+n) = 0.2017, \quad (99)$$

$$v^* = \left[\frac{\delta_n \cdot n}{\bar{\mu}(a_p^f/u)^{\phi-1}}\right]^{1/\phi}. \quad (100)$$

The depreciation rate of input inventory can be pinned down by matching the average inventory to output ratio and the average inventory investment to output ratio, which are $a_p^i i^* / GDP$ and $a_p^i \delta_i i^* / GDP$ respectively in the model.

$$\delta_i = \frac{NII/GDP}{INV/GDP} = 0.0038, \tag{101}$$

where GDP=NII+FS.

By matching the average velocity of M2 money stock (v_c^{f*} = 1:836), we can get $z^f (s^{f*})^{\zeta}$ = 2:4947 which can be used later:

$$\begin{aligned} v_c^{f*} &= p^f c^{f*}/m^f \\ &= c^{f*}/(a_b^f q^{f*}) \\ &= (1-FI) z^f (s^{f*})^{\xi} \end{aligned} \tag{102}$$

Similarly, $v_c^{i*} = z^f (s^{i*})^{\zeta}$. We need two more targets to pin down B^f: the average input inventory to final sales ratio (ISR) and the intermediate inputs to final sales ratio (IPS). In this model:

$$ISR = a_p^i i^*/c^{f*}, \tag{103}$$

$$IPS = \frac{z^f B^f (s^{f*})^{\xi} a_p^f a^*}{B^f v_c^f a_p^f q^f} \frac{P_i}{P^f}, ; \tag{104}$$

$$\Rightarrow \frac{z^f B^f (s^{f*})^{\xi} a_p^f a^*}{B^f v_c^f a_p^f q^{f*}} = IPS(1 + markup). \tag{105}$$

By plugging equation (D3) into equation (D2), We can get $i^* / a^* = (1-\delta_i)[1 - z^f B^f (s^{f*})^{\zeta}]$. We also can rewrite i^* / a^* in terms of ISR and IPS:

$$\begin{aligned} \frac{i^*}{a^*} &= \frac{a_p^i i^*}{c^f} \frac{B^f v_c^{f*} a_p^f q^{f*}}{z^f B^f (s^{f*})^{\xi} a_p^f a^*} z^f B^f (s^{f*})^{\xi}, \\ &= \frac{ISR \cdot B^f v_c^f}{(1-FI) IPS(1+markup)} \quad . \end{aligned} \tag{106}$$

After equalizing the two equations of i^* / a^*, B^f can be calculated in terms of ISP, IPS, v_c^{i*} and the markup:

$$\begin{aligned} B^f &= \frac{(1-\delta_i)}{\{(1-\delta_i) + ISR/[IPS(1+markup)]\} v_c^{f*}/(1-FI)} \quad , \\ &= 0.1947. \end{aligned} \tag{107}$$

Next, equations (D4)-(D6) are plugged into equation (D9) to get rid of ω^f; λ^i and Ω_i; we can then pin down the parameter . Rearranging equations (D1)-(D10), we can get:

$$\omega^{f*} = \frac{\beta v_c^{f*}}{\gamma^* - \beta + \beta v_c^{f*}/(1-FI)} U'(c^*), \tag{108}$$

$$
\begin{aligned}
&\equiv E\cdot U'(c^*),\\
\lambda^i &= \frac{\gamma^*-\beta}{\beta z^i(s^{i*})^{\xi}}\frac{\omega^{f*}}{1+markup},\\
&= \frac{\gamma^*-\beta}{\beta v_c^{i*}(1+markup)}E\cdot U'(c^*),\\
&\equiv F\cdot U'(c^*),
\end{aligned}
$$

Where,

$$
\frac{\omega^{f*}}{\omega^{i*}} = \frac{P^{f*}\Omega_M^*}{P^{i*}\Omega_M^*} \quad \Rightarrow \omega^{i*} = \frac{\omega^{f*}}{1+markup} \quad , \tag{109}
$$

and,

$$
\begin{aligned}
\Omega_i^* &= \frac{\beta B^f v_c^{f*} a_p^f/(1-FI)}{1-\beta(1-B^f v_c^{f*} a_p^f/(1-FI))(1-\delta_i)}[\frac{\omega^{f*}}{1+markup}+\lambda^{i*}],\\
&= \frac{\beta B^f v_c^{f*} a_p^f/(1-FI)}{1-\beta(1-B^f v_c^{f*} a_p^f/(1-FI))(1-\delta_i)}[\frac{E}{1+markup}+F]U'(c^*),\\
&\equiv G\cdot U'(c^*).
\end{aligned} \tag{110}
$$

Then α can be calculated by plugging the above equations into equation (D9):

$$
\begin{aligned}
\alpha &= \frac{(a_p^f[E/(1+markup)+F]+(1-a_p^f)(1-\delta^i)G}{E}\frac{a^*}{q^*} \quad , \\
&\equiv H\frac{a^*}{q^{f*}} \quad , \\
&= 0.4156,
\end{aligned} \tag{111}
$$

where $a^*/q^{f*} = 0.6822$ can be calculated by rearranging $i^*/a^* = (1-\delta_i)[1-z^f B^f (s^{f*})^{\zeta}]$:

$$
\begin{aligned}
a_p^f i^* &= (1-\delta_i)[1-z^f B^f(s^{f*})^{\xi}]a_p^f a^*,\\
\Rightarrow \frac{a_p^f i^*}{B^f v_c^{f*} a_p^f (q^{f*})} &= (1-\delta_i)[1-z^f B^f(s^{f*})^{\xi}]\frac{a_p^f a^*}{B^f v_c^{f*} a_p^f q^{f*}},\\
\Rightarrow ISR &= (1-\delta_i)\frac{1}{B^f v_c^{f*}}\frac{a^*}{q^{f*}} - (1-\delta_i)z^f B^f(s^{f*})^{\xi}\frac{a_p^f a^*}{B^f v_c^{f*} a_p^f q^{f*}},\\
\Rightarrow a^*/q^{f*} &= [ISR+(1-\delta_i)IPS(1+markup)]B^f v_c^{f*}/(1-\delta_i).
\end{aligned} \tag{112}
$$

Since α is known, we can calculate a^* = 0:5198 by rearranging the production function $q^{f*} = A^* a^{*\alpha} n^{*(1-\alpha)}$:

$$
\begin{aligned}
\frac{q^{f*}}{a^*} &= A^* a^{*(\alpha-1)} n^{*(1-\alpha)},\\
\Rightarrow a^* &= [\frac{q^{f*}}{a^*}A^* n^{*(1-\alpha)}]^{1/(\alpha-1)}.
\end{aligned} \tag{113}
$$

Then q^{f*}; c^{f*}; i^* and a_b^f can be calculated:

$$
\begin{aligned}
q^{f*} &= A^*(a^*)^\alpha (n^\alpha)^{1-\alpha} = 0.7619, \\
c^{f*} &= (1-FI)a_p^f B^f v_c^{f*} q^{f*} = 0.0546, \\
i^* &= i^*/a^* \cdot a^* = 0.2662, \\
a_b^f &= B^f a_p^f = 0.0393.
\end{aligned}
$$

Now, we can calculate ω^f ; λ^i and Ω_i using the value of c^{f*} :

$$
\begin{aligned}
\omega^{f*} &\equiv E \cdot U'(c^*) = 7.4215, \\
\lambda^i &\equiv F \cdot U'(c^*) = 0.4760, \\
\Omega_i^* &\equiv G \cdot U'(c^*) = 4.4572.
\end{aligned}
$$

Using the seventh target, which is that the shopping time of the population is 11:17% of the working time and the working time is 30% of agents discretionary time, we can calculate the buyer's search intensity in the finished goods market s^{f*}. Once s^{f*} is known, z^f and z_1^f can be determined as follows:

$$
\begin{aligned}
s^{f*} &= 0.1117 * 0.3(a_p^f(1+n) + a_p^i)/a_b^f = 0.5162, \\
z^f &= z^f (s^f)^\xi / ((1-FI)(s^{f*})^\xi), \\
z_1^f &= z^f * (B^f)^{1-\xi} = 3.0524.
\end{aligned}
$$

Since we assume the intermediate goods market and the finished goods market are sym-metric, a_b^i ; s^{i*}, and z_i^i can be determined in a similar way:

$$
\begin{aligned}
a_b^i &= B^i a_p^i = 0.0393, \\
s^{i*} &= 0.1117 * 0.3(a_p^f(1+n) + a_p^i)/a_b^i = 1.7207, \\
z^i &= z^i (s^i)^\xi / (s^i)^\xi = v_c^i/(s^i)^\xi, \\
z_1^i &= z^i (B^i)^{1-\xi} = 0.0934.
\end{aligned}
$$

Now the quantity of intermediate goods per trade (q^i), the constant in the disutility function of producing intermediate goods (b) and the constant in the disutility of posting vacancies (K_0) can be calculated by using equation (D3), the function of $\varphi^i(q^i)$ and the last target ($K = 3.72 * 10^{-4}$):

$$
\begin{aligned}
q^{i*} &= \frac{a^* - i^*}{v_c^{i*} a_b^i} = 6.5096, \\
b &= \varphi^{i\prime}(q^{i*})/q^{i*}, \\
&= \omega^{i*}/q^{i*} = 0.6706, \\
K_0 &= K/v^{*2} = 5.9501e-004.
\end{aligned}
$$

Finally, the parameters $(\varphi^i, \varphi_0^i, \varphi^f, \varphi_0^f)$ can be determined by using the steady state relations:

$$
\varphi^i = \frac{b}{2}(q^i)^2 = 14.2091,
$$

$$
\begin{aligned}
\varphi^f &= [\beta(1-\alpha)\sigma B^f v_c^{f*}\omega^{f*}q^{f*}/((1-FI)n^*) - [1-\beta(1-\delta_n)]\Omega_n]/(\beta\sigma), \\
&= 1.6025, \\
\varphi_0^i &= \left\{\frac{z^i(s^{i*})^{\xi-1}[B^f a_p^f v_c^{f*}\lambda_i^*/(1-FI) + (1-B^f a_p^f v_c^{f*}/(1-FI))[(1-\delta_i^*)\Omega_i^* - bq^{i*}]]q^{i*}}{\varphi^i(1+1/\epsilon_i)(s^{i*})^{1/\epsilon_i}}\right\}^{\frac{\epsilon_i}{1+\epsilon_i}} \\
&= 0.1104, \\
\varphi_0^f &= \left(\frac{z^f(s^{f*})^{\xi-1}((1-FI)(c^{f*})^{-\eta} - \omega^{f*})q^{f*}}{(\varphi^f(1+1/\epsilon_f)(s^{f*})^{1/\epsilon_f})}\right)^{\epsilon_f/(1+\epsilon_f)} \\
&= 1.8043.
\end{aligned}
$$

E. Appendix E: Data Sources

1. Underlying Detail - NIPA Tables, The Bureau of Economic Analysis

- Table 1AU. Real Manufacturing and Trade Inventories, Seasonally Adjusted, End of Period [Chained 1996 dollars, 1967-96, SIC] (Q)
- Table 1AU2. Real Manufacturing and Trade Inventories, Seasonally Adjusted, End of Period [Chained 2005 dollars, 1967-97, SIC] (Q)
- Table 1BU. Real Manufacturing and Trade Inventories, Seasonally Adjusted, End of Period [Chained 2005 dollars, 1997 forward, NAICS] (Q)
- Table 2AU. Real Manufacturing and Trade Sales, Seasonally Adjusted at Monthly Rate [Chained 1996 dollars, 1967-96, SIC] (Q)
- Table 2AUI. Implicit Price Deflators for Manufacturing and Trade Sales [Index base 1996, 1967-96, SIC] (Q)
- Table 2BU. Real Manufacturing and Trade Sales, Seasonally Adjusted at Monthly Rate [Chained 2005 dollars, 1997 forward, NAICS] (Q)
- Table 2BUI. Implicit Price Deflators for Manufacturing and Trade Sales [Index base 2005, 1997 forward, NAICS] (Q)
- Table 4AU1. Real Manufacturing Inventories, by Stage of Fabrication (Materials and supplies), Seasonally Adjusted, End of Period [Chained 2005 dollars, 1967-97, SIC] (Q)
- Table 4AU2. Real Manufacturing Inventories, by Stage of Fabrication, Seasonally Adjusted (Work-in-process), End of Period [Chained 2005 dollars, 1967-97, SIC] (Q)
- Table 4BU1. Real Manufacturing Inventories, by Stage of Fabrication (Materials and supplies), Seasonally Adjusted, End of Period [Chained 2005 dollars, 1997 forward, NAICS] (Q)
- Table 4BU2. Real Manufacturing Inventories, by Stage of Fabrication (Work-in-process), Seasonally Adjusted, End of Period [Chained 2005 dollars, 1997 forward, NAICS] (Q)

2. Databases, the Federal Reserve Bank of St. Louis

- M2 Money Stock, seasonally adjusted, end of period, quarterly
- Velocity of M2 Money Stock, seasonally adjusted, end of period, quarterly

3. Databases, Bureau of Labor Statistics

- Civilian Labor Force (Seasonally Adjusted) - LNS11000000
- Civilian Employment (Seasonally Adjusted) - LNS12000000

- Civilian Unemployment (Seasonally Adjusted) - LNS13000000
- Manufacturing Employment - CES3000000001

4. Manufacturing Industry Productivity Database, The National Bureau of Economic Research

- emp: Total employment in 1000s, 1987 SIC version
- matcost: Total cost of materials in $1,000,000, 1987 SIC version
- pimat: Deflator for MATCOST 1987=1.000, 1987 SIC version