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The Impact of Basel Accords on the Lender's Profitability under Different Pricing Decisions

By Bo HUANG and Lyn C. Thomas [1]

Abstract

In response to the deficiencies in financial regulation revealed by the global financial crisis, the Basel Committee on Banking Supervision (BCBS) is proposing to introduce a new capital regulatory standard to improve the banks' ability to absorb shocks arising from financial and economic stress. The regulatory capital requirements in the third of Basel Accords is conceptually similar to the mixture of Basel I (risk-invariant requirements) and Basel II (risk-based requirements), it introduce a non-risk based measure to supplement the risk-based minimum capital requirements and measures.

We look at how the interest rate charged to maximise a lender's profitability is affected by the different versions of the Basel Accord that have been implemented in the last 20 years. We investigate three types of pricing models on a portfolio of consumer loans. These are a fixed price model, a two price model and a variable risk based pricing model. We investigate the result under two different scenarios, Firstly where there is an agreed fixed price the lender has to pay to acquire capital in the market and secondly when the lender decides in advance how much if its equity capital can be used to cover the requirements of a particular loan portfolio. We develop an iterative algorithm for solving these latter cases based on the solution approaches to the former. We also look at the sensitivity of the lending policy not only to the different Basel Accords but also to the riskiness of the portfolio and the costs of capital and loss given default values.

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1 Introduction

Pricing consumer loans is implemented mainly through the interest rate charged, though in some cases there are also fees involved in setting up and operating the loans. In setting the interest rate， lenders consider the cost of capital needed to finance the loan, the probability of a borrower will repay all the loan and if they do not repay, what percentage of the defaulted amount will eventually be lost – the loss given default. They also need to consider the take rate which is what percentage of the population will accept a loan offered at a particular interest rate. However over the last twenty years the lenders, if they are covered by the banking regulations, also need to consider the regulatory capital they need to set aside to cover the unexpected losses on their loans.

Before 1988, there were no requirements for regulatory capital and we denote this requirement as Basel 0 (B0). Between 1988 and 2006, the regulations in the first Basel Accord (B1) required that banks set aside a fixed percentage of equity capital to cover all their risks in lending (Basel 1). For most lending this was 8% of the value of the loan. Kirstein (2001) pointed out this might result in adverse incentive influences. Although the cost of regulatory capital is the same for high risk and low risk borrowers, the bank will charge higher interest rates for more risky loans to compensate for the higher expected losses. So introducing such a regulatory capital requirement may encourage banks to replace low risk low profit customers with high risk high profit customers since they both require the same level of regulatory capital. So the Basel committee introduced a new capital requirement , denoted by Basel 2 (B2) where the capital requirements was sensitive to the credit risk inherent in bank loan portfolio. The Basel 2's proposal required the bank to set different regulatory capital ratios for borrowers with different default risks. The third of the Basel Accords (Basel 3) was recently developed in a response to the deficiencies in financial regulation revealed by the global financial crisis. Basel 3 tightens up what is required as capital, introduces liquidity requirements, and increases the capital requirement by factoring up the capital requirements of Basel 2. The capital requirement in Basel 2 was taken as 8% of the risk weighted assets to have the same terminology as Basel 1, where the risk weighted assets were given by a function of the probability of default of the loans. Basel 3 requires the capital requirement to be again 8.0% of risk weighted assets but then adds botha capital conservation buffer and a counter cyclical buffer. The mandatory capital conservation buffer is 2.5% of risk weighted assets, and its objective is to ensure that banks maintain a buffer of capital that can be used to withstand future periods of financial and economic stress. The discretionary countercyclical buffer, allows regulators to impose up to further capital up to 2.5% of risk weighted assets during periods of high credit growth. Thus we take the Basel 3 regulatory capital to be 13/8 ((8%+2.5%+2.5%)/8%) times the equivalent Basel 2 capital requirement. Table 1 shows the major differences between Basel 2 and 3.

The Basal Accords require the banks to set aside regulatory capital to cover unexpected losses on a loan. If the chance of the loan defaulting l_D is Loss Given Default (LGD), the fraction of the defaulted amount that is actually lost. The minimum capital requirement (MCR) per unit of loan with a probability p of being good is defined as $l_D \cdot K(p)$. This is given under four different MCRs:

Basel 0: Describes the situation pre-1998 when there were no regulatory capital requirements so $K(p) = K_0 = 0$

Basel 1: Describes the MCR under the first Basel Accord where

$$K(p) = K_1 = \frac{0.08}{l_D}$$

Basel 2: Describes the MCR under the second Basel Accord where

$K_2(p) = N \cdot \left[(\frac{1}{1-R})^{1/2} \cdot N^{-1} \cdot (1-p) + (\frac{R}{1-R})^{1/2} \cdot N^{-1} \cdot (0.999) \right] - (1-p)$, where $R = 0.04$ (for credit cards), $N(\cdot)$ is the cumulative normal distribution and $N^{-1}(\cdot)$ is the inverse cumulative normal distribution.

Basel 3: the MCR for the third of Basel Accord can be written as:

$$K_3(p) = \left(\frac{13}{8}\right) \cdot K_2(p) = 1.625 \cdot K_2(p)$$

Basel 2	Basel3
a. Tier 1 Capital	
Tier 1 capital ratio = 4% Core Tier 1 capital ratio = 2% The total capital requirement is 8.0%.	Tier 1 Capital Ratio = 6% Core Tier 1 Capital Ratio (Common Equity after deductions) = 4.5% The total capital requirement is 8.0%.
b. Capital Conservation Buffer	
There no capital conservation buffer is required.	Banks will be required to hold a capital conservation buffer of 2.5% to absorb losses during periods of financial and economic stress.
c. Countercyclical Capital Buffer	
There no Countercyclical Capital Buffer is required.	A countercyclical buffer within a range of 0% – 2.5% of common equity or other fully loss absorbing capital will be implemented.

Table 1: Differences between Basel 2 and Basel 3

Kashyap and Stein (2003) pointed out that there are many potential benefits to risk-based capital requirements, as compared to the "one-size-fits-all" approach embodied in the Basel 1 regulation. The objective of this paper is to understand the influence of different Basel regulatory capital requirements (Basel 1, Basel 2 and Basel 3) on the lender's profitability under different pricing decisions (such as fixed (one) price model, a two price model, and a variable pricing model) in portfolio level.

Fixed-rate pricing was the dominant form of pricing of loans until the early 1990s. More recently the development of the internet and the telephone as new channels for loan applications has made the offer process more private to each individual (Thomas 2009), and developments in credit scoring and response modelling have assisted the banks in marketing their products more efficiently, and in increasing the size of their portfolios of borrowers (Chakravoriti and To 2006). The banks are able to "price" their loan products at different interest rates by adopting methods such as channel pricing, group pricing, regional pricing, and product versioning. Variable pricing, therefore, can improve the profitability of the lender by individual bargaining and negotiation. Since the lender can segment the applicants depending on their default risk and offer different loan terms to each segments, so the simplest variable pricing model is actually two prices model where we assume there are only two types of borrowers which differ in their ability to repay a loan (default risk).

There is a limited literature on the impact of the Basel Accords on consumer loan pricing. Allen DeLong and Saunders (2004) outline the relationship between the Basel accord and credit scoring, and they observe how corporate credit models are modified to deal with small business lending. Ruthenberg and Landskroner (2008) analyze the possible effects of Basel 2 regulation on the pricing of bank loans related to the two approaches for capital requirements (internal and standardized). They indicate that big banks might attract good quality firms since the reduction in interest rates produced by adopting the IRB approach. On the other hand lower quality firms will be benefit by borrowing from small banks, which are more likely to adopt the standardized approach. Perli and Najda (2004) suggest an alternative approach to the Basel capital allocation. They offer a model for the profitability of individual borrower revolving loan they use it to imply that the regulatory capital should be some percentile of the profitability distribution of the loan, but there is no reference to the effect on operating decision. Oliver and Thomas (2005) analyze the changes in the operating decision of which potential consumer borrowers to accept and which to reject because of the effects of different Basel regulations imposed on the retail bank in portfolio level with predetermined capital. Based on the model suggested in that paper, we analyze the impact of different Basel regulations on pricing decisions under the three pricing strategies. We do this both in the case when the lender has an agreed cost of equity for each unit of equity needed to cover the regulatory capital requirement and when the lender decides in advance how much of its equity capital can be set aside to cover the requirements of this loan portfolio.

This paper is organised as follows. Section 2 looks at the profitability model of different pricing decisions (fixed price, two prices and variable pricing)on consumer loans. Section 3 use several numerical examples to investigate the impact of the Basel Accords on different pricing models associated with a portfolio of such loans. The objective each time is to maximise the expected profitability of the portfolio and we report the optimal interest rates that do this. This is under the case when there is an agreed cost of equity. In Section 4 we extend the models by assuming the bank decides in advance how much of its equity capital can be set aside to cover the required regulatory capital of a loan portfolio. Section 5 draws some conclusions from this work.

2 Pricing Models at portfolio level

To consider the pricing models at the portfolio level one needs first to define the profit model for an individual loan. Consider a loan of one unit offered by a lender to a borrower whose probability of being good – that is of repaying the loan in full- is p. Assume the rate charged on the loan is r and if the loan defaults, it does so before there are any repayments and the fraction of the loan that is fiannly lost at the end of the collections process is l_D. Let r_Q be the required return on equity capital, which must be achieved to satisfy shareholders and let the risk free rate at which the lender can borrower the money be r_F. Let B denote the sum of the cost of the regulatory capital $r_Q l_D K(p)$ and the risk free rate r_F, which is the cost of lending 1 unit. If the lender offers lonas at an interest rate r to a borrower whose probability of being Good and repaying is P, we assume the chance the borrower will take the loan – the take probability is $q(r,p)$. With these assumptions the expected profit from making an offer of a loan of 1 unit to an individual borrower is

$$E(Profit) = q(r,p) \cdot [(r-B) \cdot p - (l_D + B) \cdot (1-p)] \qquad \text{Eq.1}$$

where $B = r_Q l_D K(p) + r_F$.

2.1 Fixed (one) price model in portfolio level

For the fixed price model, we assume the lender only offers one interest rate r to all potential borrowers. We will assume throughout that the take rate or response rate function is the linear function :

$$q(r,p) = min\{max[0, 1 - b \cdot (r - r_L) + c \cdot (1-p), 1]\}, \text{ for } r \geq r_L > 0. \qquad \text{Eq. 2}$$

This means the borrower's response rate is dependent on their riskiness as well as on the interest rate charged. Throughout the paper we compare the results by looking at a class of numerical examples. In these we assume $r_L = 0.04, b = 2.5$, and $c = 2$. This implies that an 1% increase in interest rate drops the take probability by 0.025 while if the default probability of the borrower goes up by 0.01 (Good drops by this amount) the take probability goes up by 0.02 . For borrower with a default rate of 0.01, 100% of them would take a loan of rate 6%, while only 50% of them would take a loan with interest rate 25%. To maximise the profit over a portfolio, the lender must accept the loans that have positive expected profit, but reject all the loans that are unprofitable. This defines a cut-off probability (or cut off point p_c) which is the probability of being Good at which the expected profit from the borrower is zero. Thus the lender should accept all the customers with probability of being good above the cut off point $p_c = \frac{l_D + B}{r + l_D}$, and reject all the applications with probability of being good $of\ p_c$ or lower. In our numerical examples, we assume the probability of the borrowers being good has a uniform distribution. The probability density function is $f(p) = \frac{1}{1-a}, a \leq p \leq 1$, so it means no one with probability of being good less than a is in the potential borrowers' population. Therefore, the cut off point for a portfolio actually is max (a, p_c).

If we define $EP(r,p)$ to be the expected profit from a portfolio with cut off point max (a, p_c), we find

$$EP(r,p) = \underset{r, p_c}{\text{Max}} \int_{\max(a, p_c)}^{1} q(r,p) \cdot [(r-B) \cdot p - (l_D + B) \cdot (1-p)] \cdot f(p) \cdot dp \qquad \text{Eq. 3}$$

where $B = r_Q l_D K(p) + r_F$.

2.2 Two Price Model at portfolio level

In this case, we assume the lender has two different rates that can be offered to potential borrowers. Suppose the rates are r_1 and r_2, $r_1 < r_2$, the lender's strategy is given by two values (a segmentation point p^* and a cut off point $\max(a, p_c)$). Rate r_1 is made to the borrowers whose probability p of being good is $p \geq p^*$; rate r_2 is made to those who probability of being good is p, $p^* > p \geq max(a, p_c)$, where again it is assumed the probability of the borrowers being good in the portfolio has a uniform distribution over [a,1].

In the two price model, the optimal cut off point depends on the higher rate r_2, so $p_c = \frac{l_D+B}{r_2+l_D}$ since that rate gives higher profit than the other one. In this case $max\ (a, p_c)$ is 'first round of screening' to determine the range of potential borrowers' population.

We believe the expected profit of the lender with inferior rate r_2 is always equal to or less than the expected profit of the lender with superior rate r_1, but the chance of borrowers accepting such a loan is always less. So the probability of being good p^*, at which the lender start to offer r_1 rather than r_2, can be achieved from the equality

$$q(r_1) \cdot [(r_1 - B) \cdot p - (l_D + B) \cdot (1 - p)] = q(r_2) \cdot [(r_2 - B) \cdot p - (l_D + B) \cdot (1 - p)]$$

Eq. 4

Using the take function given in (2) this results in $p^* =$

$$\frac{\left[1 + \frac{1}{c} - \frac{b}{c} \cdot l_D - \frac{b}{c} \cdot r_1 - \frac{b}{c} \cdot r_2 + \frac{b}{c} \cdot r_L\right] + \sqrt{\left(1 + \frac{1}{c} - \frac{b}{c} \cdot l_D - \frac{b}{c} \cdot r_1 - \frac{b}{c} \cdot r_2 + \frac{b}{c} \cdot r_L\right)^2 + 4 \cdot \frac{b}{c} \cdot (l_D + B)}}{2}$$

Eq. 5

So p^* is the segmentation point which divides those who passed the lender's 'first round of screening' into the two groups to be offered different interest rates.

The choice between lender and borrowers is a typical interactive decision-making. The lender, therefore, has to consider whether the rate offered is attractive to any borrowers. This lead to two constraints –one for each interest rate- that says there is an upper limit on the goodness of the borrowers if at least some of them to want to accept loans at that interest rate, namely :

$q(r_1) = 1 - b \cdot (r_1 - r_L) + c \cdot (1 - p) > 0$, which results into $p_1 = 1 + \frac{1}{c} - \frac{b}{c} \cdot (r_1 - r_L)$

and

$q(r_2) = 1 - b \cdot (r_2 - r_L) + c \cdot (1 - p) > 0$, which leads to $p_2 = 1 + \frac{1}{c} - \frac{b}{c} \cdot (r_2 - r_L)$

So if one offers r_1 only the borrowers who probability of being good is below p_1 will accept it, and only those whose probability of being good is below p_2 will accept r_2.

Hence the expected value of lender's total profit is showed by following equation,

$$EP(r,p) = \underset{r_1,r_2,p^*,p_c,}{Max} \left\{ \int_{\max(a,p_c)}^{\min(p^*,p_2)} q(r_2,p) \cdot [(r_2 - B) \cdot p - (l_D + B)(1-p)] \cdot f(p) \cdot dp \right.$$
$$\left. + \int_{p^*}^{\min(1,p_1)} q(r_1,p) \cdot [(r_1 - B) \cdot p - (l_D + B)(1-p)] \cdot f(p) \cdot dp \right\}$$

Eq.6

where $B = r_Q l_D K(p) + r_F$.

2.3 Variable Pricing Model in portfolio level

Variable pricing (risk based pricing) means that the interest rate charged on a loan to a potential borrower depends on the lender's view of the borrower's default risk and can any value..
If the lender believes the borrower has a probability p of being Good, then the lender believes the expected profit if a rate $r(p)$is charged to be

$$EP(r,p) = \underset{r,p}{\text{Max}} \int_{a}^{1} q(r,p) \cdot [(r - B) \cdot p - (l_D + B) \cdot (1-p)] \cdot f(p) \cdot dp$$

Eq.7

where $B = r_Q l_D K(p) + r_F$.

In order to find the optimal interest rate for a certain probability p of being Good, we differentiate this equation with respect to r and set the derivate to zero, to find when the profit is optimised. This gives a risk based interest rate of

$$r^*(p) = B + (l_D + B)\frac{1-p}{p} - \frac{q(r,p)}{\frac{\partial q(r,p)}{\partial r}}$$ Eq.8

This calculations of risk based interest rate can be found in the book by Thomas (2009). Note in this case there is no cut off probability of being Good below which one will not an applicant. Instead one offers such applicants such a high interest rate that it is highly unlikely that they will accept the offer. This occurs at probability levls of p where *q(r*(p),p) =0.*

3 The impact of the Basel Accords on the different pricing models at portfolio level

We calculate the various business measures such as the expected profit of lender, optimal interest rate and optimal cut off on numerical examples under the different Basel Accords. This allows us to see the impact of the changes in the Accord on profit and on who is likely to get loans.

3.1 Example for one price model

Consider the situation where $r_Q = 0.05$, $r_F = 0.05,$ and $l_D = 0.5$ so that equity holders expect a return of 5% and the rate at which the lender can borrow money t subsequently lend out is also 5%. We also assume that for loans that default 50% of the original amount lent will be recovered by the end of the collections process. With this, regulatory capital for each Basel rule leads respectively to:.

Basel 0: since $K(p) = K_0 = 0$, we have $= r_Q l_D K(p) + r_F = 0.05$.

Basel 1: $K(p) = K_1 = \frac{0.08}{l_D} = 0.16$, hence we have B= $r_Q l_D K(p) + r_F = 0.054$.

Basel 2: since $K_2(p) = N \cdot \left[(\frac{1}{1-R})^{1/2} \cdot N^{-1} \cdot (1-p) + (\frac{R}{1-R})^{1/2} \cdot N^{-1} \cdot (0.999) \right] - (1-p)$, where $R = 0.04$ (credit cards), then we have $B = r_Q l_D K_2(p) + 0.05$.

Basel 3: we assume $K_3(p) = 1.625 \cdot K_2(p)$, we get $B = 1.625 \cdot r_Q l_D K_2(p) + 0.05$

We now consider the portfolio where 1 unit is available to be lent over the whole portfolio. The portfolio has a distribution over the riskiness of the potential borrowers so that their chance of each being good is uniform over the region [a,1] where a can be 0.6,0.7,0.8 or 0.9. The optimal interest rates and the expected profits achieved under different Basel Accords are obtained by solving (2) and are shown in Table 2.

We can see from Table 2 that optimal interest rates for Basel 1, Basel 2 and Basel 3 are always bigger than optimal interest rates achieved under Basel 0. This is because the costs of regulatory capital need to be covered. The optimal interest rates charged drop but the expected portfolio profits rise up as the potential borrower portfolio become less risky. Obviously the expected portfolio profits for all Basel regulations are less than that of made with no regulatory capital requirements case. Comparing column 5 and column 8, one sees that the expected portfolio profits under Basel 2 is smaller than that achieved under Basel 1 until one reaches the a=0.9 portfolio. This is because Basel 2 regulation requires the lender to set flexible regulatory capital ratios for different risk types, the equity capital request decreases with increasing of the quality of the borrowers. We can see that in column 7 which shows the regulatory capital needed under Basel 2 (for the riskiest who are accepted in the portfolio) is larger than regulatory capital ratio of Basel 1 (which is fixed at K_1 =0.16) except when the minimum good rate a reaches 0.9. The expected portfolio profits for Basel 3 is always smaller than that for Basel 1 and 2 capital requirements since Basel 3 regulation set much tighter capital restriction on equity capital than Basel 1 and 2. This can be seen from regulatory capital ratios of Basel 3 showed in column 10 where the ratios of regulation are extremely high by comparing with that for Basel 1 and 2. The profit though under Basel 3 only decreases between 1% and 3.5% compared with Basel 2. The change in the optimal interest rate charged between the different Basel Accords is far less than the change that occurs when the portfolio gets less risky as it moves from 0.6 to 0.9.

a	r B0	E(P) B0	r B1	E(P) B1	r B2	K(2)	E(P) B2	r B3	K(3)	E(P) B3
0.6	0.3768	0.0618	0.3778	0.0599	0.3782	0.2447	0.0589	0.3933	0.3983	0.0571
0.7	0.35	0.0778	0.3523	0.0757	0.3535	0.2380	0.0747	0.3556	0.3868	0.0728
0.8	0.3140	0.0886	0.3162	0.0865	0.3169	0.2097	0.0859	0.3188	0.3408	0.0842
0.9	0.2791	0.0942	0.2812	0.0922	0.2810	0.1491	0.0923	0.2823	0.2423	0.0911

Table 2: Optimal Expected profit and optimal interest rates under fixed price

3.2 Example for two prices model

In this example we take the same borrowing rates and the LGD as in Example 2.1, namely, $r_Q = 0.05$, $r_F = 0.05$, and $l_D = 0.5$ and we keep the same linear response rate function with $r_L = 0.04, b = 2.5$, and $c = 2$. The results under the different Basel regulations are given in Tables 3 to 6

Table 3: two price model under Basel 0: no requirement for regulatory capital

a	r_2	$\max(a, P_c)$	r_1	p^*	Expected Profit
0.6	0.485568	0.6	0.317348	0.790765	0.07506936
0.7	0.415754	0.7	0.297977	0.845737	0.08508152
0.8	0.353758	0.8	0.279661	0.898441	0.09168503
0.9	0.297323	0.9	0.262075	0.94968	0.09493876

When the value of minimum good rate a increase that means the borrowers become less risky, the optimal cut off point and segment point increase and the expected portfolio profits increase as well, but the optimal interest rates (r_1, r_2) to charge decrease. Note that in all four portfolios we make offers to all the potential borrowers and make the offer of the better interest rate to just over half of them. The expected profit increases as the portfolio becomes less risky but of course both interest rates decrease in order to be able to attract sufficient of the higher quality applicants.

Table 4: Two price model under Basel 1:$K_1(p) = \frac{0.08}{l_D} = 0.16$

a	r_2	max (a, P_c)	r_1	p^*	Expected Profit
0.6	0.488475	0.6	0.319604	0.790703	0.07305781
0.7	0.418356	0.7	0.300156	0.845704	0.08304529
0.8	0.356118	0.8	0.281772	0.898428	0.0896575
0.9	0.299488	0.9	0.264128	0.949676	0.09294516

There is a small increase in the optimal interest rates being charged by comparing with the solutions for Basel 0 shown in Table 3, while the profits are slightly lower.. This is because there are now regulatory capital costs to be covered. Again offers are made to everyone in the portfolio, but the p value at which one switches to the lower interest rate is fractionally lower under this set of regulations than under Basel 0.s needed to be considered in no regulatory requirement case.

Basel 2:

The Basel Accord 2's regulatory capital ratio varies with p, and it is difficult to integrate the expression $l_D \cdot K_2(p) = l_D \cdot N \cdot \left[(\frac{1}{1-R})^{1/2} \cdot N^{-1} \cdot (1-p) + (\frac{R}{1-R})^{1/2} \cdot N^{-1} \cdot (0.999) \right] - l_D \cdot (1-p)$ over the whole portfolio. We take two approximations to ease this calculation. In the first conservative approximation we set the probability of the borrowers being good p in equation of $l_D \cdot K_2(p)$ to equal with p_c if they take the higher interest rate r_2 and p^* if they take the lower interest rate r_1 . The regulatory capital ratio for all borrowers is always larger than the true regulatory capital required. We also build a model assuming that for the group who have the higher interest rate they all have regulatory capital corresponding to a probability of being good which is the average value between $Max(p_c, a)$ and min $\{p^*, p_2\}$. For the higher quality group we assume the regulatory capital is taken as if they all have the average good probability between p^* and $\min\{1, p_1\}$ We found there is no significant difference in these two approximations so report just one set of results.

Table 5:Two price model under Basel 2 regulations

a	r_2	max (a, P_c)	r_1	p^*	Expected Profit
0.6	0.488913	0.6	0.31957	0.791461	0.072208
0.7	0.418609	0.7	0.299716	0.846327	0.082389
0.8	0.355927	0.8	0.280833	0.899	0.089405
0.9	0.298533	0.9	0.262549	0.950295	0.093397

It can be seen from Table 5 interest rate r_1 being charged on good quality borrowers under Basel 2 is always smaller than that for Basel 1, but the inferior rate r_2 being charged on risky borrowers is higher than that for Basel 1 if a=0.6 or 0.7 but lower than it if a=0.8 or 0.9. The expected profit is lower under Basel 2 than Basel 1 except in the case when a=0.9 when the potential set of borrowers is the least risky of the four cases. Thus under the two price model Basel 2 does what it was expected to do and encourages high quality loans, even if the effect is quite small. Again all the borrowers are made an offer of one of the two interest rates in all four cases. However the Basel 2 segment point is higher than the Basel 1 segment point, and in fact higher than the Basel 0 segment point. So under the Basel 2 regulations less of the portfolio is given the lower interest rate but one should treat this result with some caution because of the approximations concerning the regulatory capital.

Table 6: Two price model under Basel 3: $K_3(p) = 1.625 \cdot K_2(p)$

a	r_2	max (a, P_c)	r_1	p^*	Expected Profit
0.6	0.491035	0.6	0.320974	0.79186	0.070447
0.7	0.420426	0.7	0.30082	0.846656	0.080727
0.8	0.357323	0.8	0.281591	0.899298	0.087995
0.9	0.299371	0.9	0.262907	0.950565	0.092441

The optimal interest rates and expected portfolio profits for Basel 3 is given in Table 6 where we can see the expected portfolio profits for Basel 3 is always smaller than that for Basel 1 and 2 as a result of Basel 3 regulation setting much tighter capital restriction than Basel 1 and 2. The segment probability where one switches rates is always higher than that for Basel 2 and Basel 1. Both of optimal superior and inferior rates ($r_1$and r_2) are always higher than that for Basel 2. Moreover, the rate r_2 charged to riskier borrowers under Basel 3 is higher than that for Basel 1 in the three cases a+0.6,0.7 and 0.8 and only drops below the Basel 1 rate with the least risky portfolio a=0.9. The rate r_l charged to the less risky borrowers is higher than the Basel 1 case if a=0.6 but then drops below the Basel 1 rate for "better" portfolios. So although Basel 3 still encourages lenders to seek out less risky portfolios the rewards compared with Basel 1 are non existent in profit and only the highest quality customers benefit from reduced rates.

3.3 Example for variable pricing model

We assume there is no adverse selection in this variable pricing model whereas in Huang and Thomas (2009) it is assumed that the fact the borrower accepts the loan affects his estimate of being good. Using the cost structure of the pervious example with $r_Q = 0.05$, $r_F = 0.05$ and $l_D = 0.5$, and assuming again the parameters for the linear response rate function are $r_L = 0.04$, $b = 2.5$, and $c = 2$, we get the numerical results in the following tables. .The optimal interest rates offered by lender under Basel Accords are shown in Table 7

Table 7: optimal interest rates under variable pricing

p	$r(p)$ B0	$r(p)$ B1	$r(p)$ B2	$r(p)$ B3
0.35	1.015714	1.021428	1.022753	1.027153
0.4	0.8975	0.9025	0.904159	0.908321
0.5	0.72	0.724	0.725898	0.729584
0.6	0.588333	0.591667	0.59344	0.596632
0.7	0.482857	0.485714	0.487108	0.489765
0.8	0.39375	0.39625	0.397027	0.399076
0.9	0.315556	0.317777	0.317626	0.318922
0.95	0.279474	0.281579	0.280755	0.281555
0.96	0.272458	0.274542	0.27355	0.274233
0.97	0.265506	0.267567	0.266391	0.266945
0.98	0.258612	0.260654	0.259268	0.259678
0.99	0.251778	0.253798	0.252164	0.252406

Obviously the optimal interest rates charged decrease as the probability of the borrowers being good increases under all Basel regulation rules. Since there is no requirement for regulatory capital for Basel 0, the optimal interest rates for B0 are always less than those that for Basel 1, 2 and 3 regulations. We can see from Table 7 that the interest rates for Basel 2 are higher than those required under Basel 1 until the probability of the borrowers being good is over 0.9. The optimal interest rates for Basel 3 also becomes smaller than that for Basel 1 as the probability of the borrowers being good is high enough. However, the tighter capital restriction in Basel 3, means this only happens when the probability of the borrower being good rises above 0.95.

The expected individual profits for the Basel Accords are given in Table 8

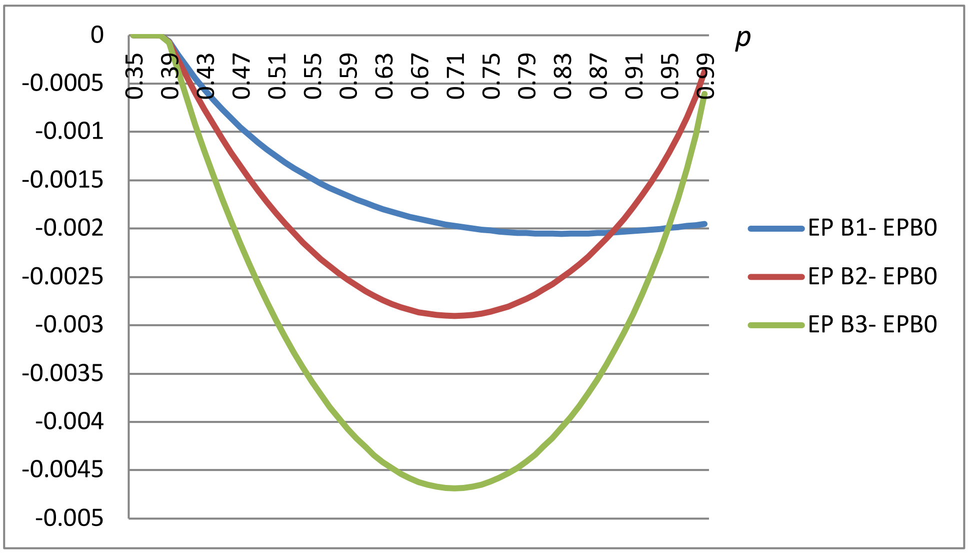
Table 8: Expected individual borrower profits under variable pricing

p	$E(P)$ B0	$E(P)$ B1	$E(P)$ B2	$E(P)$ B3
0.35	0	0	0	0
0.4	0.000506	0.000306	0.00025093	0.000136
0.5	0.018	0.01682	0.016274144	0.01524
0.6	0.044204	0.042504	0.04161319	0.040034
0.7	0.068014	0.066057	0.06511253	0.063331
0.8	0.085078	0.083028	0.082395783	0.080741
0.9	0.094044	0.092011	0.092148375	0.090973

0.95	0.095501	0.093506	0.094284804	0.093529
0.96	0.095561	0.093576	0.094517747	0.093869
0.97	0.095545	0.093571	0.094694659	0.094165
0.98	0.095457	0.093493	0.094823436	0.094429
0.99	0.095295	0.093343	0.094920163	0.094686

In Table 8 one sees that the expected profits under all the Basel Accords increase as the borrowers become less risky. Since the regulatory capital rules in Basel 2 and Basel 3 depend on probability of the borrowers being good, the expected profit under both regulations becomes larger than that for Basel 1 when the probability of the borrowers being good is high enough . This is obvious if one plots the difference in profitability between each Basel regulation and Basel 0.

Figure 1: Difference in profitability between Basel Accords (Basel 1, Basel 2, Basel 3) and Basel 0



The above figure show the expected profits for Basel 1, Basel 2 and Basel 3 minus the profit achieved under Basel 0. Note in all cases there is no profit until the probability of being good is at least 0.35. The difference between Basel 2 and Basel 0 reaches its largest value when the probability of the applicant being good equals to 0.71, and thereafter the difference starts to lessen until it becomes 0. A comparison of curve for Basel 2 and that for Basel 3 shows the same thing happens to the difference in the profitability of Basel 3 and Basel 0. However Figure curve shows difference on profitability between Basel 1 and Basel 0 shows how different is the impact of Basel 1 where the difference curve does not diminish much as the probability of the applicant being good increases to 1. This is because Basel 1 requires a fixed amount of equity set aside (0.08) for all risk types (even those whose probability of being good is 1).

4 How the Basel Accords affects the different pricing models when there is a predetermined amount of equity capital

The previous model looked at what impact the Basel Accord requirements for regulatory capital have on the performances of lender's different pricing decisions assuming a known required rate of ROE at the portfolio level. A more realistic model is to consider the portfolio has a predetermined amount of equity capital available and so there is no known acceptable ROE. Instead the lender decides in advance how much of its equity capital can be used to cover the minimum capital requirements of a particular loan portfolio.

Initially we assume that the funds for portfolio are raised by borrowing from the market at the risk free rate r_F and loss given default on any loan is l_D irrespective of the rate charged. Ignoring the regulatory capital, the profit from a loan is modified from Eq.1 to

$$MaxE(Profit) = q(r,p) \cdot [(r - r_F) \cdot p - (l_D + r_F) \cdot (1 - p)] \quad \text{Eq. 9}$$

where the linear take probability function $q(r,p)$ is

$$min\{max[0, 1 - b \cdot (r - r_L) + c \cdot (1 - p), 1]\}.$$

We assume the probability of these portfolio loans being good also has a uniform distribution in the range [a,1]. If we define $EP(r,a)$) to be the expected profit from a portfolio where the pricing regime is to charge an interest rate r(p) and the potential portfolio only has borrowers with an a chance of being good, then

$$EP(r(.)) = \underset{r,}{\text{Max}} \int_a^1 q(r(p),p) \cdot [(r(p) - r_F) \cdot p - (l_D + r_F) \cdot (1 - p)] \cdot f(p) \cdot dp \quad \text{Eq.10}$$

The regulatory capital required for such a portfolio cannot exceed the equity capital Q set aside. Thus, given the limit on the equity capital provided, we need to solve the following constrained optimization problem in order to maximize the expected profit from the portfolio.

$$EP(r(.)) = \underset{r,}{\text{Max}} \int_a^1 q(r(p),p) \cdot [(r(p) - r_F) \cdot p - (l_D + r_F) \cdot (1 - p)] \cdot f(p) \cdot dp \quad \text{Eq. 11}$$

subject to

$$Q(r,a) = \int_{a,}^1 l_D \cdot k(p) \cdot f(p) \cdot q(r(p),p) \cdot dp \leq Q \quad \text{Eq. 12}$$

This is equivalent (Thomas 2009) to solving an unconstrained optimisation problem with a Lagrangian function that must be maximised namely

$$EP(r,a) - \lambda(Q(r,a) - Q) \qquad \text{Eq 13}$$

Under the Basel 2 and Basel 3 regulatory requirements where

$$k(p) = N \cdot \left[\left(\frac{1}{1-R}\right)^{1/2} \cdot N^{-1} \cdot (1-p) + \left(\frac{R}{1-R}\right)^{1/2} \cdot N^{-1} \cdot (0.999) \right] - l_D \cdot (1-p)$$

it is not feasible to get a closed form solution to the integration in (12) which has to be calculated in the fixed price and two price cases; the variable price case reduces to a knapsack problem and so the integrand is not required. So we undertake a numerical approximation to obtain (12). Given that we have a uniform distribution of borrowers over the probability of being good and that for any r we are only interested in a region where the take probability of the borrowers is larger than zero ($1 - b \cdot (r - r_L) + c \cdot (1-p) \geq 0$), Eq.12 can be rewritten into

$$\frac{l_D}{1-a} \int_{,a}^{\min\left(1, \frac{1-b(r-r_L)}{c} + 1\right)} K(p) \cdot q(r,p) \cdot dp \leq Q \qquad \text{Eq 14}$$

If we define $I[r,w] = \int_{w}^{\min\left(1, \frac{1-b(r-r_L)}{c} + 1\right)} K(p) \cdot q(r,p) \cdot dp$

Then (14) becomes $\frac{l_D}{1-a} \cdot I[r,a] \leq Q$

We choose a set of points, a+kh, k=0,1,2, n where b=a+nh and then approximate by

$$I(r,a,b) = \sum_{i=0}^{n-1} K(a+hi) \cdot q(r, a+hi)h,$$

For example, if an interest rate r is being charged to borrowers whose probability of being good is not less than 0.7, so the equity capital set aside is given by following calculations:

$$I(r,.7,1) = \sum_{i=0}^{29} K(.7+.01i) \cdot q(r,.7+.01i)0.01;$$

Unlike the earlier part of this paper there is no market price of equity. We first check if the unconstrained solution to (11) satisfies the constraint (12). If so, that solution which is the solution under Basel 0 will also be the solution under the relevant Basel requirement. If it does not satisfy the contraint then there is a positive shadow price and we wish to solve (13). However for a fixed shadow price finding the optimal rate r in (13) is the same as the problems solved in section 3 where there was a fixed cost of equity. So one finds the optimal price and then checks whether with that price the constraint is exactly satisfied. One can then apply the bisection method or other iterative routines to the shadow prices until one finds the shadow price and the corresponding optimal related price function that exactly satisfies constraint (12).

4.1 Numerical Examples

We use numerical examples to explore the impact of the various Basel Accord requirements on different pricing models when there is a predetermined amount of equity capital set aside to cover the regulatory requirements of the portfolio, We compare the optimal interest rate, optimal cut off and expected portfolio profits under the different Basel regulations.

4.1.1 Example for one price model

Using the cost structure of the examples from earlier part of this paper with $r_F = 0.05$ and $l_D = 0.5$, assume again the parameters for the linear response rate function are $r_L = 0.04, b = 2.5,$ and $c = 2$. We also assume the loan portfolio is of credit cards and so under the Basel 1 requirements the regulatory capital is $K(p) = K_1 = {}^{0.08}/_{l_D}$ while the correlation in the Basel 2 and Basel 3 capital requirement is $R = 0.04$. We can now apply the values in Eq. 11 and Eq. 13 under the three regulatory regimes (Basel 1, Basel 2, Basel3). Firstly the optimal interest rate, optimal cut off (max(a,p_c) and expected portfolio profits for Basel 1 is shown in Table 9

Table 9: Output of fixed price model with fixed quantity Q of equity to cover capital requirements

a	0.6		
Q	r	$\max(a, P_c)$	$E(P)$
0.01	0.401949	0.816895	0.032002
0.02	0.386739	0.740423	0.050027
0.03	0.37903	0.679048	0.059286
0.04	0.376854	0.627242	0.061889
a	0.7		
Q	r	$\max(a, P_c)$	$E(P)$
0.01	0.407457	0.841031	0.034311
0.02	0.393195	0.775777	0.056396
0.03	0.384206	0.724084	0.070762
0.04	0.36	0.7	0.0776
a	0.8		
Q	r	$\max(a, P_c)$	$E(P)$
0.01	0.414136	0.869762	0.036977
0.02	0.401949	0.816897	0.064002
0.03	0.37	0.8	0.081575
0.04	0.32	0.8	0.088533
a	0.9		
Q	r	$\max(a, P_c)$	$E(P)$
0.01	0.43	0.9	0.040137
0.02	0.380001	0.9	0.070034

0.03	0.330001	0.9	0.088055
0.04	0.279998	0.9	0.094201

In each of the Tables 9-11 we stop increasing the Q values when we reach the unconstrained solution, i.e. when no further equity capital is required

Table 10: Optimal interest rate, optimal cut off and expected portfolio profits under Basel 2

a	0.6		
Q	r	$\max(a, P_c)$	$E(P)$
0.01	0.374611	0.82688	0.034369
0.02	0.38006	0.759399	0.047545
0.03	0.37953	0.708077	0.055931
0.04	0.377708	0.664649	0.060537
0.05	0.377513	0.629911	0.061878
a	0.7		
Q	r	$\max(a, P_c)$	$E(P)$
0.01	0.369972	0.854468	0.038429
0.02	0.378746	0.787716	0.056217
0.03	0.380226	0.744864	0.066864
0.04	0.379595	0.70992	0.074234
0.05	0.351949	0.7	0.077804
a	0.8		
Q	r	$\max(a, P_c)$	$E(P)$
0.01	0.363238	0.884763	0.045531
0.02	0.374369	0.828519	0.068081
0.03	0.370705	0.8	0.081397
0.04	0.315316	0.8	0.088609
a	0.9		
Q	r	$\max(a, P_c)$	$E(P)$
0.01	0.353655	0.918846	0.064084
0.02	0.329441	0.9	0.088189

It is seen from these tables that the expected portfolio profits always increase as the borrowers become less risky. Meanwhile, the larger amount Q of equity set aside for a portfolio the lender have, the more profits one can achieve. The optimal interest rate to charge drops as increasing of amount Q of equity set aside for a portfolio under Basel 1 regulation.

Table 11: Optimal interest rate, optimal cut off and expected portfolio profits under Basel 3

a	0.6		
Q	r	$\max(a, P_c)$	$E(P)$
0.01	0.366959	0.868989	0.025911
0.02	0.376899	0.808587	0.038035
0.03	0.379852	0.766591	0.046211
0.04	0.380205	0.736586	0.05156
0.05	0.379504	0.707384	0.056026
0.06	0.378389	0.680736	0.059173
0.07	0.37736	0.655069	0.061134
0.08	0.37686	0.630074	0.061881
a	0.7		
Q	r	$\max(a, P_c)$	$E(P)$
0.01	0.362857	0.88628	0.029951
0.02	0.374217	0.829535	0.045115
0.03	0.378328	0.793269	0.054766
0.04	0.37979	0.768332	0.061179
0.05	0.380211	0.748692	0.065968
0.06	0.380007	0.724143	0.07143
0.07	0.379273	0.701342	0.07577
0.08	0.355765	0.7	0.077742
a	0.8		
Q	r	$\max(a, P_c)$	$E(P)$
0.01	0.354892	0.914746	0.033662
0.02	0.367484	0.866581	0.052787
0.03	0.373242	0.835957	0.065086
0.04	0.375172	0.822824	0.070368
0.05	0.36719	0.8	0.082265
0.06	0.332626	0.8	0.087838
a	0.9		
Q	r	$\max(a, P_c)$	$E(P)$
0.01	0.349461	0.931685	0.053989
0.02	0.356492	0.909443	0.071506
0.03	0.340678	0.9	0.085203
0.04	0.292207	0.9	0.093795

It is interesting to note that under the Basel 2 and Basel 3 regulations the optimal interest rate increases as the equity available increases when there is very little equity available, However interest rates then decrease as the equity increases when there is more equity available. This is because increasing the interest rate has two counterbalancing effects. Firstly it increases the profit on each of the borrowers who accept the loan but it diminishes the chance an individual borrower will take the loan. When there is very little equity available, that equity constraint means one cannot take a large number of borrowers anyway and so it is more profitable to increase the interest rate. When the amount of equity is still constraining but quite large then one wants to make sure one has enough borrowers to "use" all the equity and hence one starts to decrease the interest rate.

Figures 2 to 5 show the expected profits for the 4 Basel rules (regulatory requirements, Basel 1, Basel 2, and Basel 3) under the one price model.

Figure 2: Expected profit using fixed price under the Basel regulations with a=0.6.

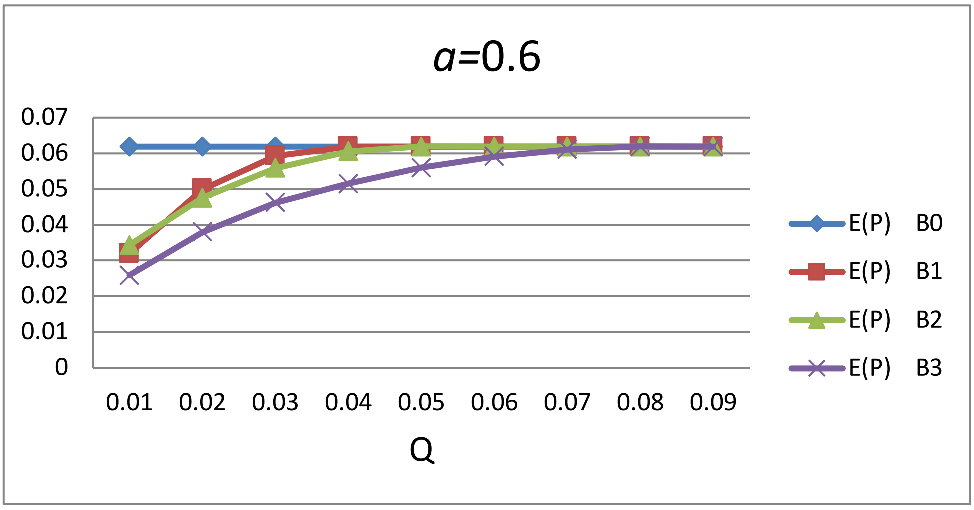


Figure 3: Expected profit using fixed price under the Basel regulations with a=0.7.

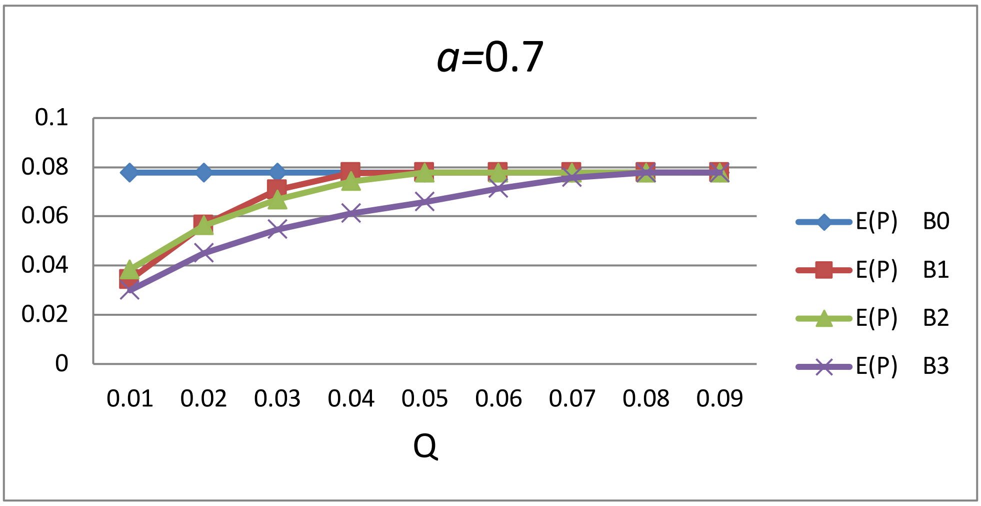


Figure 4: Expected profit using fixed price under the Basel regulations with a=0.8.

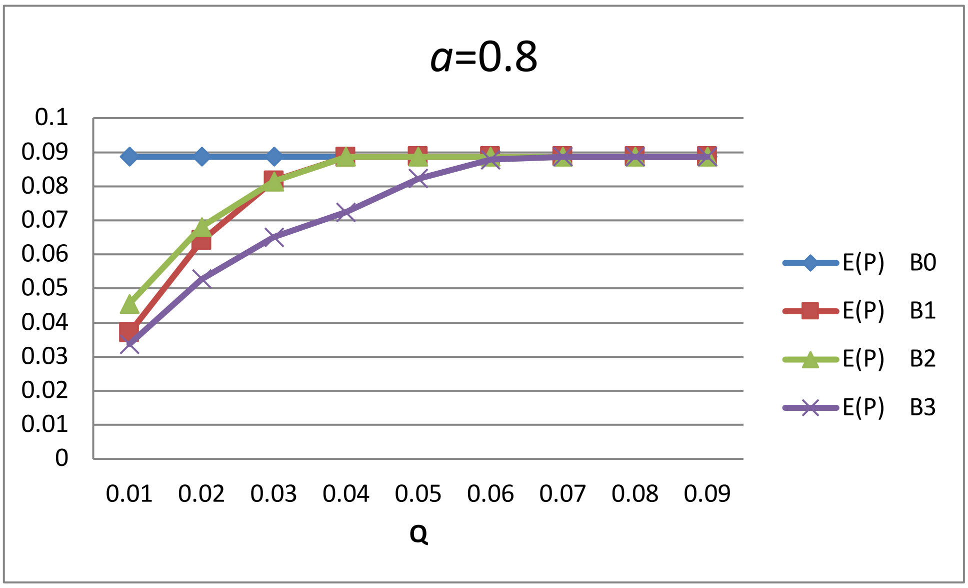
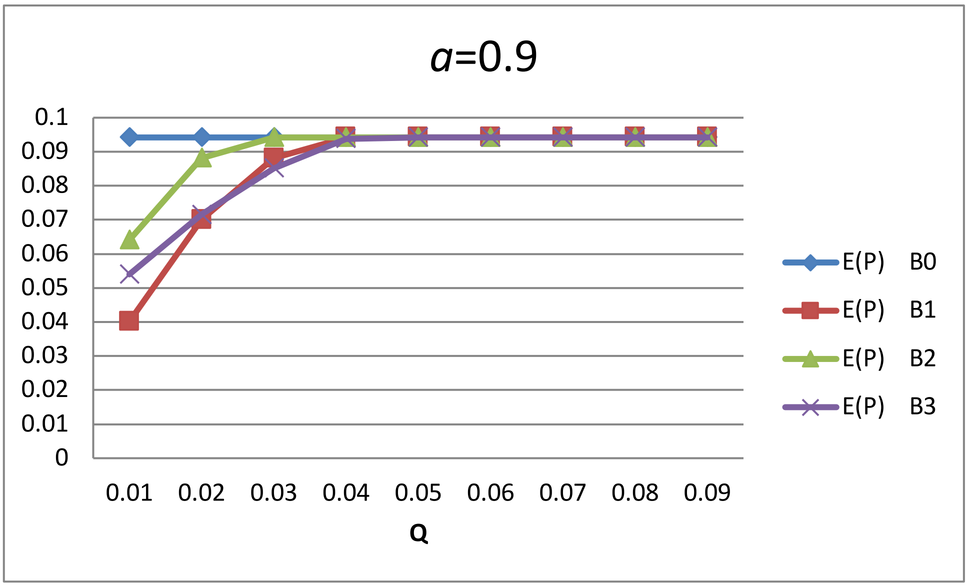


Figure 5: Expected profit using fixed price under the Basel regulations with a=0.9.



These Figures describe what happens to the expected profit as the equity capital set aside for a portfolio is increased from 0.1. As equity capital increases, the expected portfolio profits increase first. However, it is obvious to see the expected portfolio profits do not depend on the size of the regulatory capital. Once the expected portfolio profits reach the largest values which are the profits achieved under the Basel 0 regulation, the lender should stop increasing the equity capital. In these Figures if the lender has enough equity one ends up in the Basel 0 case regardless of which Basel requirement is used. Moreover, if the lender only has a little equity then the lender will take the "best" borrowers first and so the lender makes more profit with Basel 2 than Basel 1, since it requires less regulatory capital for these very good borrowers. However when the lender has more equity then they take more of the risky borrowers and so now Basel 1 leads to more profit than Basel 2. For example, when $a = 0.7$ with $Q = 0.01$ the Basel 2 curve is above the Basel 1 curve. When the quality of the applicants rise, this trend will be more obvious so the Basel 2 curve shown in Figure 5 when $a = 0.9$ is always above the Basel 1 curve. Basel 3 regulation set much tighter capital restriction on equity capital than Basel 1 and 2, thus we can see from the Figures for value of a respectively equals to 0.6, 0.7 and 0.8 that the expected portfolio profit for Basel 3 is always lower than that for Basel 1and 2 capital requirements before the profit reaches the Basel 0 solution. With higher quality of borrowers such as $a = 0.9$, the expected profit for Basel 3 lies between the Basel 1 and Basel 2 if the capital restriction of Q is small.

4.1.2 Example for two prices model

For the two price model, the customers are segmented into two groups by the lender. Thus the expected profit from the portfolio given the limit on the equity capital provided is modified from Eqs 11 and 12 to ,

$$EP(r_1, r_2, a) = \left\{ \int_{\max(a, p_c)}^{p^*} q(r_2, p) \cdot [(r_2 - r_F) \cdot p - (l_D + r_F)(1 - p)] \cdot f(p) \cdot dp \, r_2 + \int_{p^*}^{1} q(r_1, p) \cdot [(r_1 - r_F) \cdot p - (l_D + r_F)(1 - p)] \cdot f(p) \cdot dp \right\}$$

Eq. 15

subject to

$$Q(r_1, r_2, a) = \int_{\max(a, p_c)}^{p^*} l_D \cdot K(p) \cdot f(p) \cdot q(r_2, p) \cdot dp + \int_{p^*}^{1} l_D \cdot K(p) \cdot f(p) \cdot q(r_1, p) \cdot dp \leq Q$$

Eq. 16

Where $p_c = \frac{l_D + r_F}{r_2 + l_D}$, and p* is defined by equation (4)

This is again a constrained optimisation problem and so we can solve using the Lagrangian approach namely to optimise

$$EP(r_1, r_2, a) - \lambda(Q(r_1, r_2, a) - Q)$$

Note again that this is equivalent to the problem in section 3 if we assume the cost of equity is λ and so for any λ we can solve to find the optimal rates that maximise the Lagrangian given that λ. So we solve the unconstrained problem which is equivalent to solving the problem under Basel 0. If this does not satisfy constraint (16) then we use a bisection approach to find the λ that when we solve to find the corresponding optimal rates r_1 and r_2 the constraint is exactly satisfied.

We take the same values used in the one price model example so that the risk free rate is 0.05 and the loss given default value is set as 0.5. The response rate function is also the same as in that example with $r_L = 0.04, b = 2.5,$ and $c = 2$. In this model, the calculations applied in Eqs 15 and 16 give the following results under the different regulatory regimes

Table 12:Expected portfolio profits, optimal rates and cut off points for two rate pricing under Basel 1

	$a = 0.6$			$a = 0.7$			$a = 0.8$			$a = 0.9$		
$Q = 0.01$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$
	0.540563	0.380708	0.034998	0.525008	0.390639	0.036354	0.511123	0.399903	0.040778	0.442378	0.39549	0.042145
	$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*	
	0.739984	0.874296		0.776579	0.89374		0.810881	0.911098		0.9	0.970296	
$Q = 0.02$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$
	0.561394	0.357092	0.055255	0.542857	0.362344	0.061962	0.44645	0.356033	0.068351	0.381178	0.366709	0.070305
	$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*	
	0.657626	0.829161		0.7	0.857562		0.8	0.934588		0.9	0.990736	
$Q = 0.03$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$
	0.55149	0.341818	0.069192	0.452909	0.326015	0.078915	0.379451	0.325686	0.083967	0.33	0.33	0.088054
	$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*	
	0.604856	0.800944		0.7	0.88912		0.8	0.959833		0.9	1	
$Q = 0.04$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$
	0.485568	0.317348	0.075069	0.415754	0.297977	0.085082	0.353758	0.279661	0.091685	0.297324	0.262075	0.094939
	$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*	
	0.6	0.790765		0.7	0.845737		0.8	0.898441		0.9	0.94968	

Table 13:Expected portfolio profits, optimal rates and cut off points for two rate pricing under Basel 2

	$a=0.6$			$a=0.7$			$a=0.8$			$a=0.9$		
$Q = 0.01$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$
	0.512181	0.336644	0.039017	0.490955	0.332085	0.041492	0.474815	0.327972	0.052215	0.436327	0.31658	0.068005
	$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*	
	0.767836	0.881632		0.812638	0.905259		0.842484	0.921008		0.9	0.951657	
$Q = 0.02$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$
	0.538626	0.339834	0.054335	0.523695	0.338564	0.059216	0.469353	0.32357	0.073331	0.330684	0.276542	0.091269
	$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*	
	0.688585	0.840198		0.738898	0.866423		0.8	0.906057		0.9	0.974729	
Q=0.03	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$
	0.546024	0.338367	0.06472	0.505931	0.329949	0.074665	0.387419	0.294519	0.087642	0.297324	0.262075	0.094939
	$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*	
	0.636884	0.81346		0.7	0.855445		0.8	0.931816		0.9	0.94968	
Q=0.04	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$
	0.52187	0.328799	0.073186	0.424307	0.303237	0.083454	0.353758	0.279661	0.091685	0.297324	0.262075	0.094939
	$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*	
	0.6	0.802316		0.7	0.885065		0.8	0.898441		0.9	0.94968	
Q=0.05	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$
	0.485568	0.317348	0.075069	0.415754	0.297977	0.085082	0.353758	0.279661	0.091685	0.297324	0.262075	0.094939
	$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*	
	0.6	0.790765		0.7	0.845737		0.8	0.898441		0.9	0.94968	

Table 14:Expected portfolio profits, optimal rates and cut off points for two rate pricing under Basel 3

	$a = 0.6$			$a = 0.7$			$a = 0.8$			$a = 0.9$		
$Q = 0.01$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$
	0.505367	0.335298	0.036265	0.48452	0.3305	0.038758	0.453045	0.321934	0.040009	0.424242	0.31297	0.051087
	$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*	
	0.78312	0.889688		0.824767	0.911675		0.879701	0.940438		0.924366	0.963391	
$Q = 0.02$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$
	0.520485	0.338083	0.042788	0.505367	0.335298	0.048353	0.483184	0.330188	0.057242	0.436559	0.316977	0.063034
	$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*	
	0.747472	0.870922		0.78312	0.889688		0.82738	0.913031		0.905909	0.953956	
$Q = 0.03$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$
	0.535492	0.339766	0.051748	0.522258	0.338353	0.058235	0.479349	0.32715	0.071008	0.368954	0.29095	0.084757
	$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*	
	0.701435	0.846866		0.742773	0.868458		0.8	0.903318		0.9	0.96536	
$Q = 0.04$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$
	0.543507	0.339404	0.05976	0.535627	0.339774	0.069129	0.42524	0.307892	0.082217	0.298939	0.26472	0.094142
	$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*	
	0.661841	0.826372		0.700937	0.846606		0.8	0.919109		0.9	0.983558	
Q=0.05	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$
	0.546287	0.338109	0.065634	0.494346	0.326149	0.076448	0.397333	0.298001	0.086449	0.297324	0.262075	0.094939
	$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*	
	0.632133	0.810989		0.7	0.859248		0.8	0.928337		0.9	0.94968	
Q=0.06	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$
	0.546547	0.336141	0.071043	0.441843	0.308956	0.082376	0.337366	0.276956	0.090662	0.297324	0.262075	0.094939
	$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*	
	0.601671	0.794856		0.7	0.878095		0.8	0.951418		0.9	0.94968	
Q=0.07	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$
	0.516254	0.327149	0.073515	0.383812	0.290058	0.083877	0.353758	0.279661	0.091685	0.297324	0.262075	0.094939
	$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*	
	0.6	0.804298		0.7	0.902796		0.8	0.898441		0.9	0.94968	
Q=0.08	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$	$r(2)$	$r(1)$	$E(P)$
	0.485568	0.317348	0.075069	0.415754	0.297977	0.085082	0.353758	0.279661	0.091685	0.297324	0.262075	0.094939
	$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*		$\max(a, P_c)$	p^*	
	0.6	0.790765		0.7	0.845737		0.8	0.898441		0.9	0.94968	

the expected portfolio profits for all Basel regulations in the two prices model are always greater than those in one price model. In two prices model both of superior rate r_1 and inferior rate r_2 in three Basel regimes decrease as increasing of the capital restriction of Q, and the segment probability in three Basel regimes increase along with increasing of Q. Moreover we can see in three Basel regimes the differences between superior rate r_1 and inferior rate r_2 are likely to be small if the lender has more equity capitals..

Compare between expected profits for 4 regulation rules (with no regulatory requirements, Basel 1, Basel 2, and Basel 3) under two prices model.

Figure 6: Expected profit using fixed equity capital under the Basel regulations with a=0.6.

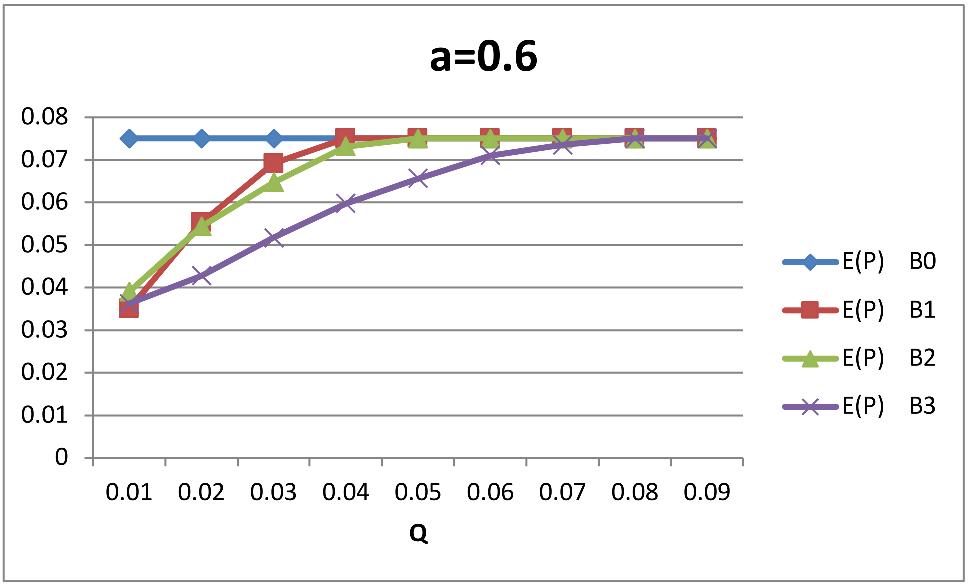


Figure 7: Expected profit using fixed equity capital under the Basel regulations with a=0.7.

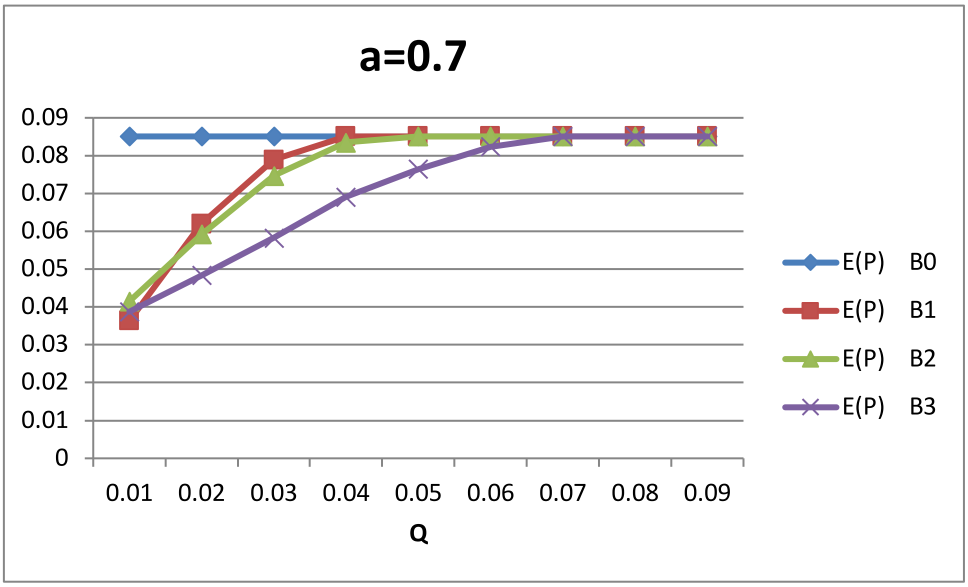


Figure 8: Expected profit using fixed equity capital under the Basel regulations with a=0.8.

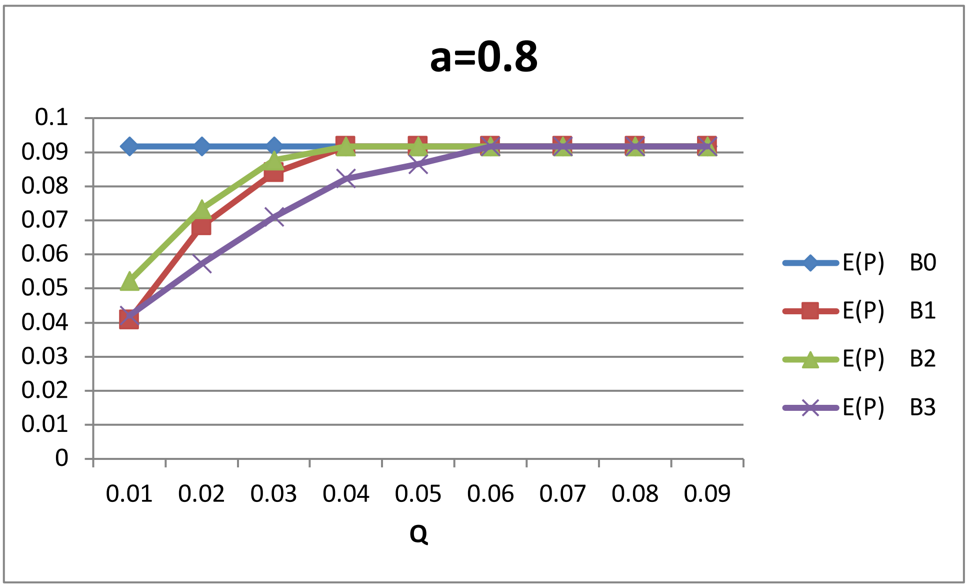
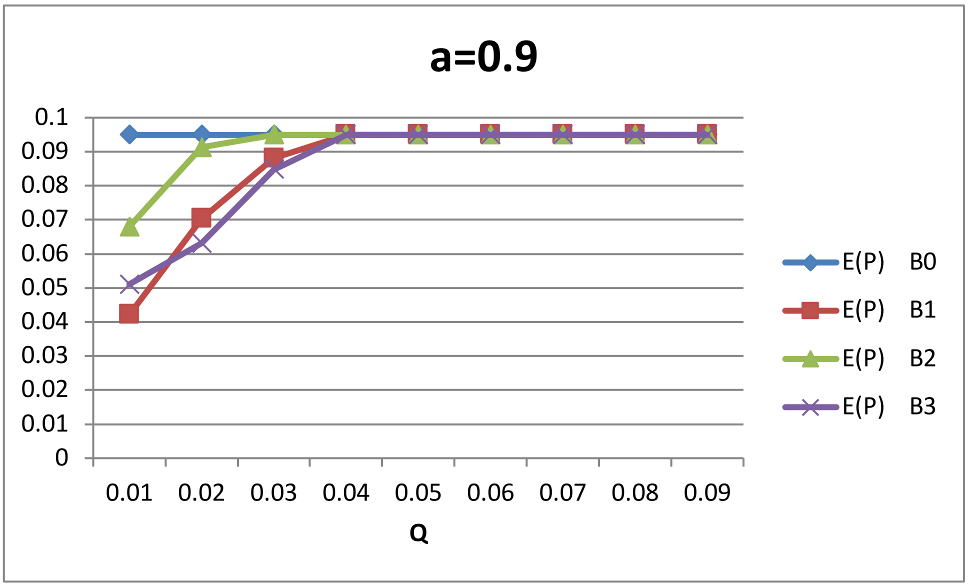


Figure 9: Expected profit using fixed equity capital under the Basel regulations with a=0.9.



These Figures describe what happens to the expected profit as the equity capital set aside for a portfolio is increased from 0.01. We can see the expected portfolio profits increase first as equity capital increases. If the lender has enough equity one ends up in the Basel 0 case regardless of which Basel requirement is used. As we know Basel 2and Basel 3 requires less regulatory capital for these very good borrowers, so if the lender only has a little equity then the lender will take the "best" borrowers first and so the lender makes more profit with Basel 2and Basel 3 than Basel 1, and the expected portfolio profits for Basel 3 lies between that of Basel 2 and Basel 1 as the amount of Q equity capital set aside equals to 0.01.

4.1.3 Example for variable pricing model

Variable pricing means that the interest rate charged on a loan to potential borrower depends on the default risk of individuals. Thus in that case the expected profit is

$$EP(r(p),a) = \underset{r(p)}{\text{Max}} \int_a^1 q(r(p),p) \cdot [(r(p) - r_F) \cdot p - (l_D + r_F)(1-p)] \cdot f(p) \cdot dp$$

Eq. 17

Subject to $Q(r(p),a)=\int_a^1 1_D K(p)q(r(p),p)f(p)\, dp \le Q$ Eq. 18

Using the Lagrangian approach this is equivalent to finding the $r(p)$ that maximises $EP(r(p),a) - \lambda \cdot Q(r(p),a)$ where λ is chosen so that the constraint is satisfied. So initially one solves the unconstrained problem, which gives the solution under B0, where there is no Basel requirements and the variable rate function is $r_0(p)$ where $r_0(\cdot)$is the interest rate in (8) when there is no Basel requirement and so $B = r_F$. If there is insufficient equity available for this solution to hold we know that the shadow price of equity is positive and that the equity constraint is exactly satisfied. We find the solution by adjusting the shadow price of equity λ until the constraint is exactly satisfied. Solving the problem to find the optimal interest rates for a given λ is equivalent to solving the variable pricing problem in the previous section with the cost of equity being λ. So initially if there is no equity available obviously we want to accept no one. This is done by making λ very large which leads to extremely high interest rates, which no one will accept. As equity increases and so λ starts to fall we make lower interest rate offers to all borrowers. Eventually we will start making the optimal interest rate offer under Basel 0 to some of the potential borrowers. Since this is the one that maximises the profit form this group of borrowers we do not make lower interest rate offers to those even if λ has been further reduced. The point at which this happens for borrowers whose probability of being good is p is when their expected profitability per unit of regulatory capital $\theta(p)$ is equal to λ. $\theta(p)$ is the ratio

$$\theta(p) = \frac{((r_0(p)-r_F)p-(l_D+r_F)(1-p))}{l_D \cdot K(p)}$$ Eq. 19

Further discussion on the use of this marginal return on equity (ROE) can be found in the book by Thomas (2009).

Using the cost structure of all the previous examples in this section, we show the results when $r_L = 0.04, b = 2.5,\ c = 2, r_F = 0.05$ and $l_D = 0.5$.

Optimal interest rates with Basel 1 is shown in **Table 14**

Q	r(P) B1	r(P) B1	r(P) B1	r(P) B1	r(P) B1	r(P) B1	r(P) B1	r(P) B1	r(P) B1
0.01	0.778333	0.645714	0.53625	0.442222	0.399474	0.391208	0.383031	0.374939	0.366929
0.02	0.713333	0.59	0.4875	0.398888	0.358421	0.350583	0.342825	0.335143	0.327535
0.03	0.65	0.535714	0.44	0.356667	0.318421	0.311	0.303649	0.296367	0.289151
0.04	0.588334	0.482858	0.393751	0.315556	0.279474	0.272458	0.265506	0.258612	0.251778

Expected profits with Basel 1 is shown in **Table 15**

p	0.6	0.7	0.8	0.9	0.95	0.96	0.97	0.98	0.99
Q	E(P) B1	E(P) B1	E(P) B1	E(P) B1	E(P) B1	E(P) B1	E(P) B1	E(P) B1	E(P) B1
0.01	0	0.0216	0.044466	0.057945	0.061301	0.061717	0.062051	0.062304	0.062477
0.02	0.020767	0.047925	0.0675	0.07842	0.080698	0.080912	0.081048	0.081107	0.081091
0.03	0.0385	0.063125	0.0808	0.090242	0.091898	0.091995	0.092017	0.091964	0.091838
0.04	0.044204	0.068014	0.085078	0.094044	0.095501	0.095561	0.095545	0.095457	0.095295

It is obvious to see that the expected profit goes up as the riskiness of the borrower decreases, and expected profit also increases if the capital restriction of Q increases.

Optimal interest rates with Basel 2 is given in **Table 16**

p	0.6	0.7	0.8	0.9	0.95	0.96	0.97	0.98	0.99
Q	r(P) B2	r(P) B2	r(P) B2	r(P) B2	r(P) B2	r(P) B2	r(P) B2	r(P) B2	r(P) B2
0.01	0.825808	0.680541	0.546147	0.411877	0.33902	0.323222	0.306694	0.289109	0.269755
0.02	0.749203	0.616772	0.496987	0.380806	0.319811	0.306846	0.293407	0.279271	0.263956
0.03	0.68792	0.565757	0.457659	0.355948	0.304445	0.293746	0.282777	0.271401	0.259317
0.04	0.628679	0.516443	0.419642	0.33192	0.289591	0.281083	0.272503	0.263793	0.254833
0.05	0.588334	0.482858	0.393751	0.315556	0.279474	0.272458	0.265506	0.258612	0.251778

Expected profits with Basel 2 is given in **Table 17**

p	0.6	0.7	0.8	0.9	0.95	0.96	0.97	0.98	0.99
Q	E(P) B2	E(P) B2	E(P) B2	E(P) B2	E(P) B2	E(P) B2	E(P) B2	E(P) B2	E(P) B2
0.01	0	0	0.03863	0.07317	0.08708	0.08938	0.09143	0.09318	0.0945
0.02	0.00539	0.03663	0.06376	0.08447	0.09164	0.09272	0.09366	0.09441	0.09493
0.03	0.02933	0.05599	0.07691	0.09037	0.09402	0.09447	0.09482	0.09506	0.09516
0.04	0.04176	0.06604	0.08374	0.09344	0.09526	0.09538	0.09543	0.09539	0.09527
0.05	0.0442	0.06801	0.08508	0.09404	0.0955	0.09556	0.09555	0.09546	0.0953

We found that the expected portfolio profits for Basel 2 are smaller than that for Basel 1 when the probability of the borrowers being default is high. Though Basel 2 regulatory capital requirements are flexible, the costs of equity capital request decreases with decreasing of the riskiness of the borrowers.

Hence the expected portfolio profit for Basel 2 is greater than that achieved under Basel 1 once the probability of the borrowers being good reaches 0.9.

Optimal interest rate with Basel 3 is shown in **Table 18**

p	0.6	0.7	0.8	0.9	0.95	0.96	0.97	0.98	0.99
Q	r(P) B3	r(P) B3	r(P) B3	r(P) B3	r(P) B3	r(P) B3	r(P) B3	r(P) B3	r(P) B3
0.01	0.874643	0.721194	0.577487	0.431686	0.279474	0.272458	0.265506	0.258612	0.251778
0.02	0.801613	0.660401	0.530621	0.402064	0.284676	0.276894	0.269104	0.261277	0.253349
0.03	0.762609	0.627932	0.505589	0.386243	0.294041	0.284877	0.275581	0.266073	0.256176
0.04	0.722775	0.594772	0.480027	0.370086	0.303821	0.293215	0.282346	0.271082	0.259128
0.05	0.68543	0.563685	0.456061	0.354939	0.313185	0.301197	0.288824	0.275878	0.261956
0.06	0.646425	0.531216	0.43103	0.339118	0.323173	0.309712	0.295732	0.280993	0.264971
0.07	0.609081	0.500128	0.407064	0.323971	0.332953	0.31805	0.302497	0.286002	0.267924
0.08	0.588334	0.482858	0.393751	0.315556	0.351266	0.33366	0.315164	0.295381	0.273453

Expected profits with Basel 3 is shown in **Table 19**

p	0.6	0.7	0.8	0.9	0.95	0.96	0.97	0.98	0.99
Q	E(P) B3	E(P) B3	E(P) B3	E(P) B3	E(P) B3	E(P) B3	E(P) B3	E(P) B3	E(P) B3
0.01	0	0	0.01756	0.063701	0.08326	0.086571	0.089565	0.092145	0.094133
0.02	0	0.012851	0.047611	0.077206	0.088708	0.090572	0.092227	0.093619	0.09465
0.03	0	0.031183	0.060062	0.082802	0.090965	0.09223	0.09333	0.09423	0.094864
0.04	0.017092	0.046096	0.070191	0.087354	0.092802	0.093578	0.094227	0.094726	0.095039
0.05	0.030063	0.056581	0.077313	0.090555	0.094093	0.094527	0.094858	0.095076	0.095162
0.06	0.039142	0.063922	0.082298	0.092795	0.094997	0.09519	0.095299	0.09532	0.095247
0.07	0.043558	0.067492	0.084724	0.093885	0.095436	0.095513	0.095514	0.095439	0.095289
0.08	0.044204	0.068014	0.085078	0.094044	0.095501	0.095561	0.095545	0.095457	0.095295

Though the Basel 3 requires a much tighter capital restriction than Basel 2, the expected profits for Basel 3 are smaller than that for Basel 2.

5 Conclusion

Our analysis began with looking at the model of probability of a single loan, then we expend this to a particular portfolio loan related to different pricing decisions, which allows us to use several numerical examples to explore what the Basel Accords will effect various business measures including expected profit, optimal interest rate, and optimal cut off.

From the results yielded from those numerical examples, we find that using variable pricing the resultant profit is always the highest of the three, and profit achieved by using two prices model is always higher than that of using one price model. The profit made with no regulatory capital requirements case is always the greatest among the four cases (Basel 0, Basel 1, Basel 2, and Basel 3) if there is no any impact of adverse selection. If the borrower has a good quality, then Basel 2 will give more profit to the lender than Basel 1. And if the borrower is more risky, then the lender make more profit with Basel 1. The expected portfolio profits for Basel 3 is always smaller than that for Basel 1 and Basel 2. For predetermined equity capital, if the lender only has a little equity then the lender can make more profit with Basel 2 than Basel 1, but if the lender has more equity then they are taking more of the risky applicants and so Basel 1 gives more profit than Basel 2. With higher quality borrowers, the expected profit for Basel 3 lies between the Basel 1 and Basel 2 if the capital restriction of Q is very small.

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