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# Inflation and Economic Growth: A Hump-shaped Relationship

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**Abstract**

Existing empirical evidence suggests that the relationship between inflation and long-run growth can be hump-shaped. However, the theoretical literature has mainly focused on either a negative or a positive impact of inflation on growth. Our paper revisits the relationship using Barro (1990) framework and finds that the nonlinear hump-shaped relation can be achieved.

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## 1. Introduction

The nature of effects of inflation on long-run growth has long been a subject of debate. The theoretical literature has produced mixed results. On the one hand, Tobin (1965) argues that inflation can enhance accumulation of capital because it lowers interest rate. Therefore, inflation is beneficial to growth. On the other hand, Stockman (1981) proposes that with a binding cash-in-advance constraint on both consumption and private investment, inflation causes physical capital to decline, which in turn depresses the economic growth. More recently, subsequent research has examined the effects of inflation through distorting the accumulation of human and physical capital, through inserting real money balances into production function, and through credit market. Examples of this include Gillman and Kejask (2005), Gylfason and Herbertsson (2001), and Bose (2002). However, most existing models deliver either a positive or a negative effect of inflation on growth.[1]

On the empirical side, there has been substantial empirical work on the relationship between inflation and growth. Many studies (see, among others, Gylfason 1991; Fischer 1993; Ghosh and Philips 1998; and Burdekin *et al*. 2004) find a threshold effect of inflation on growth. When inflation is lower than the threshold, inflation produces a positive effect on growth, whereas the relationship becomes negative when money growth is above it. Therefore, inflation has a nonlinear (hump-shaped) effect on growth.

The objective of this paper is to provide a number of forces that work in opposite directions to determine the effects of inflation. To achieve this, the current study generalizes the productive government expenditure model la Barro (1990) by allowing government expenditure to be financed by both income tax and seigniorage. We propose that using seigniorage as a means of financing government expenditure is important for many underdeveloped countries.[2] At the same time, the threshold effect tends to be more evident in those countries. With a binding cash-in-advance constraint as in Stockman (1981), money growth affects the long-run growth in two ways. First, it decreases the economic growth by reducing the accumulation of physical capital. Second, money growth adds to productive government expenditure that is growth-enhancing. Therefore, the effect of inflation on growth is determined by the relative magnitude of these two forces which work in opposite directions. We find that money growth can have a hump-shaped relationship with the long-run growth rate. Faster money growth raises economic growth if money growth rate is lower than the threshold, whereas it retards economic growth if its growth rate is higher than the threshold.

## 2. The Model

[1] A few exceptions include Paal and Smith (2001) and Vaona (2012). Paal and Smith (2001) emphasize the role of credit market in determining the hump-shaped relation between money growth and economic growth, whereas Vaona (2012) explores this relationship by merging a growth model with a New Keynesian one with sticky wage. However, the mechanisms at work in these studies are quite di¤erent from the ones that we consider in our paper.

[2] Cukierman et al. (1992) show that the fraction of seigniorage in government revenue in some countries can be as large as 28%.

## 2.1 Households

We consider an economy that consists of a unit measure of identical and infinitely-lived households and a government. Each household derives utility from consumption, and maximizes its lifetime utility:

$$\sum_{t=0}^{\infty} \beta^t \ln C_t, \ 0 < \beta \leq 1,$$

where $C_t$ is the individual household s consumption, denotes discount factor, and labor supply $L_t$ is inelastic and normalized to unity. The budget constraint faced by the representative household is:

$$C_t + \frac{M_{t+1}}{P_t} + K_{t+1} - (1-\delta)K_t = (1-\tau)(w_t L_t + R_t K_t) + \frac{M_t}{P_t}, \ 0 < \tau, \delta \leq 1, \tag{1}$$

where $K_t$ denotes the stock level of physical capital, $M_t$ is the nominal money supply, $P_t$ represents the aggregate price level, denotes the capital depreciation rate, $w_t$ is the real wage rate, $R_t$ denotes the rate of return on capital and is the income tax rate. Denote real money holdings and the inflation rate by $m_t = M_t/P_t$, and $_{t+1} = P_{t+1}/P_t 1$, respectively. Following Stockman (1981), the representative household also faces a cash-in-advance constraint as follows:

$$C_t + K_{t+1} - (1-\delta)K_t \leq \frac{M_t}{P_t}. \tag{2}$$

The first-order conditions for the representative household and the associated transversaility conditions are:

$$C_t: \ C_t^{-1} = \lambda_t + \varphi_t, \tag{3}$$

$$M_{t+1}: \ \frac{\lambda_t}{P_t} = \beta\left(\frac{\lambda_{t+1}}{P_{t+1}} + \frac{\varphi_{t+1}}{P_{t+1}}\right), \tag{4}$$

$$K_{t+1}: \ \lambda_t + \varphi_t = \beta\left[(\lambda_{t+1} + \varphi_{t+1})(1-\delta) + \lambda_{t+1}(1-\tau)R_{t+1}\right], \tag{5}$$

$$TVC_1: \ \lim_{t\to\infty} \beta^T \lambda_{t+T} K_{t+T+1} = 0,$$

$$TVC_2: \ \lim_{t\to\infty} \beta^T \varphi_{t+T} \frac{M_{t+T+1}}{P_{t+T+1}} = 0,$$

where $\lambda_t$ and $\varphi_t$ are the Lagrangian multipliers associated with the budget constraint and the cash-in-advance constraint, respectively. Eq. (3) is the standard condition which equates the marginal benefits of consumption to its marginal costs. Eq. (4) states that the marginals values of real money holdings are equal to their marginal costs. Eq. (5) reveals that the rate of return on consumption equals the after-tax rate of return on capital.

## 2.2 Production

There is a large number of identical firms in this economy. In line with Barro (1990), in each period, each firm is assumed to produces output using capital $K_t$, labor $L_t$ 1; and the government s expenditure $G_t$. We further assume that government s expenditure is labor augmenting and the production function is given by

$$Y_t = AK_t^{1-\alpha}(G_t L_t)^{\alpha}. \tag{6}$$

Taking factor prices and the government s expenditure as given, the representative firm chooses $L_t$ and $K_t$ to maximize its profits. Interior solutions of the firm s problem are characterized by the first-order conditions as follows:

$$w_t = \alpha A K_t^{1-\alpha} L_t^{\alpha-1} G_t^{\alpha},$$

$$R_t = (1-\alpha) A K_t^{-\alpha} L_t^{\alpha} G_t^{\alpha}.$$

### 2.3 Government

In each period, the government expenditure is financed by the income tax as well as seigniorage. Therefore, we express the government s budget constraint as follows:

$$G_t = \tau(w_t L_t + R_t K_t) + \frac{M_{t+1} - M_t}{P_t}. \tag{7}$$

On the monetary side of the economy, nominal money supply is assumed to grow at a constant rate and thus money supply evolves according to

$$M_{t+1} = (1+\mu) M_t. \tag{8}$$

## 3. Balanced Growth Path

In this section, we examine the economy s balanced growth path, along which output, consumption, physical capital, government expenditure and real money balances all grow at a common constant rate denoted by $\gamma$: Based on Eqs. (1), (2) and (3), we can have

$$C_t^{-1} = \beta \left[ C_{t+1}^{-1}(1-\delta) + \beta \frac{C_{t+2}^{-1}}{1+\pi_{t+2}} (1-\tau)(1-\alpha) \frac{Y_{t+1}}{K_{t+1}} \right]. \tag{9}$$

Dividing both sides of Eq. (9) by $C_{t+1}^{-1}$ gives rise to the common growth rate as

$$1+\gamma = \beta \left[ (1-\delta) + \beta \frac{(1-\tau)(1-\alpha)}{(1+\gamma)(1+\pi)} \frac{Y_{t+1}}{K_{t+1}} \right]. \tag{10}$$

Combining Eqs (1), (2), (6), (7) and (8), we derive the expression for $Y_{t+1}$=$K_{t+1}$ as follows:

$$\frac{Y_{t+1}}{K_{t+1}} = A^{\frac{1}{1-\alpha}} \left[ \tau + (1-\tau) \frac{\mu}{1+\mu} \right]^{\frac{\alpha}{1-\alpha}} \equiv \frac{Y}{K}. \tag{11}$$

Similarly, combining Eq. (8) and the definition of inflation rate gives the following relationship

$$1+\mu = (1+\gamma)(1+\pi). \tag{12}$$

Substituting Eqs. (11) and (12) into Eq. (10), the expression of the common growth rate is given by

$$\gamma = \beta(1-\delta) + \beta^2 \frac{(1-\tau)(1-\alpha)}{1+\mu} A^{\frac{1}{1-\alpha}} \left[ \tau + (1-\tau) \frac{\mu}{1+\mu} \right]^{\frac{\alpha}{1-\alpha}} - 1.$$

Differentiating the common growth rate with respect to money growth rate gives rise to the following:

$$\frac{\partial \gamma}{\partial \mu} = \beta^2 A^{\frac{1}{1-\alpha}} (1-\tau)(1-\alpha) \frac{\left(1 - \frac{1-\tau}{1+\mu}\right)^{\frac{\alpha}{1-\alpha}}}{(1+\mu)^2} \left[ \frac{\alpha}{1-\alpha} \frac{1-\tau}{(1+\mu)(1-\frac{1-\tau}{1+\mu})} - 1 \right] \gtrless 0, \text{ only if } \mu \lesseqgtr \frac{\alpha - \tau}{1-\alpha}.$$

Clearly, an increase in money supply exhibits a nonlinear effect on economic growth. On the one hand, it adds to productive government expenditure which is growth-enhancing. On the other hand, an increase in money growth reduces long-run growth. As in Stockman (1981), with a binding cash-in-advance constraint on both consumption and private investment, inflation causes physical capital to decline, which in turn depresses the economic growth. The results in here show that the positive force dominates when money growth is small so that long-run growth increases as increases. However, when money supply continues to grow, the negative force will eventually override the positive one so that long-run growth is decreasing with.

**Proposition 1** *When productive government expenditure is financed by income tax and seigniorage, money growth/inflation can have a hump-shaped relationship with the long-run growth rate. Faster money growth raises economic growth if money growth rate is lower than the threshold* $_1$*, whereas it retards economic growth if its growth rate is higher than the threshold.*

## 4. Conclusion

Our paper reexamines the relationship between money growth and economic growth using Barro (1990) framework in which productive government expenditure is financed by income tax and seigniorage. We find that money growth affects the long-run growth in two ways. First, it decreases the economic growth by reducing the accumulation of physical capital. Second, money growth adds to productive government expenditure that is growth-enhancing. Furthermore, we show that money growth can have a hump-shaped relationship with the long-run growth rate. Faster money growth raises economic growth if money growth rate is lower than the threshold, whereas it retards economic growth if its growth rate is higher than the threshold.

## References


Barro, R.J., 1990. "Government Spending in A Simple Model of Endogenous Growth". *Journal of Political Economy*, 98, 103-125.

Bose, N., 2002. "Inflation, the Credit Market and Economic Growth." *Oxford Economic Papers* 54, 412 434.

Burdekin, K., Arthur, D., Manfred, K., Thitithep, S., and Thomas, W., 2004. "When Does Inflation Hurt Economic Growth? Different Non-linearities for Different Countries". *Journal of Macroeconomics* 26, 519 532.

Cukierman, A., Edwards, S., and Tabellini, G., 1992. "Seigniorage and Political Instability". *Amercian Economic Review*, 82, 537-555.

Fischer, S., 1993. "The Role of Macroeconomic Factors in Growth". *Journal of Monetary Economics*, 41, 485 512.

Ghosh, R., and Steven, P., 1998. "Inflation, Disinflation and Growth". *Working paper* 98/68, International Monetary Fund.

Gillman, M., and Michal, K., 2005. “Contrasting Models of the Effect of Inflation on Growth”. *Journal of Economic Surveys* 19, 113 136.

Gylfason, T., 1991. “Inflation, Growth and External Debt: A View of the Landscape.” *World Economy* 14, 279 297.

Gylfason, T., and Tryggvi, H., 2001. “Does Inflation Matter for Growth?” *Japan and the World Economy* 13, 405 428.

Paal, B., and Bruce, S., 2001. “The Sub-optimality of the Friedman Rule and the Optimum Quantity of Money”. *IEHAS Discussion Paper* 0113.

Stockman, Alan C., 1981. “Anticipated Inflation and the Capital Stock in a Cash-in-advance Economy.” *Journal of Monetary Economics* 8, 387 393.

Tobin, J., 1965. “Money and Economic Growth”. *Econometrica* 33, 671 684.

Vaona, A., 2012. “Inflation and Growth in the Long Run: A New Keynesian Theory and Further Semi-parametric Evidence.” *Macroeconomic Dynamics*, 16, 94-132.