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Foreign Aid, Human Capital Acquisition and Educated Unemployment: Fish or Fishing

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Abstract

By tying aid to the productive purpose of the skilled sector, this paper explores the effects of foreign aid on human capital acquisition and educated unemployment in the recipient economy. Utilizing a search and matching model, a rise in the allocation of aid can increase skilled sector productivity, thereby providing incentives to firms for more job entries and resulting in a lower unemployment rate among skilled workers. However, this result can be mitigated or even overturned when endogenous human capital acquisition is incorporated. We also show that an increase in the portion of foreign aid used for education subsidy can increase the supply but reduce the demand for skilled labor. This thus results in a higher educated unemployment rate in the economy.

JEL classification: E24; F35; J64

Keywords: Foreign Aid; Search; Unemployment

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1. **Introduction**

There is an old proverb, "Give a man a fish and you feed him a day; teach a man to fish and you feed him for a life," saying that the ability to work is of greater benefit than a one-off handout. Foreign aid is commonly used for the donor country to help the recipient country. Nonetheless, in the literature of international trade and development economics, foreign aid is often considered as a one-off handout. The famous debate between Keynes (1929) and Ohlin (1929) indicates that foreign aid can benefit the recipient country for more consumption of goods but recipient immiseration can take place via a deterioration of the terms of trade when distortions exist in the economy.

The research focus on international transfers of income has then shifted towards the welfare effects of aid under distortions. Tied aid imposed by the donor is a type of distortions to the recipient country. In the literature, foreign aid, as a one-off handout, can be tied to: (i) purchases of export goods and services from the donor country (Kemp and Kojima, 1985; Schweinberger, 1990), (ii) supply of public goods or infrastructure (Hatzipanayotou and Michael, 1995), (iii) clean-up of the environment (Chao and Yu, 1999), and (iv) reforms in trade policy (Lahiri and Raimondos, 1995). The welfare effects of tied aid in those studies again rely on the induced changes in the international terms of trade on the economy.

In this paper, rather than treating aid as a one-off handout, we consider foreign aid tied to acquisition of human capital, which is vital to the productivity of the economy. To achieve it, we incorporate foreign aid into a search and matching model. As part of the foreign aid tied to the productive purpose, a rise in the allocation of aid can raise productivity in the skilled sector. This provides an incentive to firms for more job entries which can result in a lower unemployment rate among skilled workers. Nonetheless, this favorable result can be mitigated or even overturned when endogenous human capital acquisition is incorporated. We also show that on the one hand, an increase in the portion of foreign aid used for education subsidy can increase the supply of skilled workers, but on the other hand it leads the demand for skilled labor to decrease. Thus, foreign aid could raise "educated unemployment" in the economy. This unintended consequence is often observed in developing countries. .

This paper is organized as follows. Section 2 examines the effect of tied aid on human capital acquisition in a baseline model, while several extensions are considered in section 3. Section 4 concludes.

2. **Baseline Model**

Consider a developing economy inhabited by a unit mass of workers that are risk neutral and discount the future at a constant rate $r > 0$. Before entering the labor market, each worker decides whether to invest in education and become skilled or remain unskilled. Therefore, workers are either skilled (H) or unskilled (L). We use the index i to distinguish their skill level, i=H,L. Individuals differ in terms of their individual ability h with the cumulative distribution function Ψ on [0,1] and associated density function $\varphi > 0$ and continuous over the unit interval. In the baseline model, we assume that the shares of skilled and unskilled workers in the population are fixed. But in the extension where we allow for human capital acquisition decisions, these proportions will then become endogenously determined.

We assume that labor market fractions only exist in skilled sector, whereas there is full employment in unskilled sector. In the skilled sector, each firm posts a skilled vacancy and incurs a flow cost c_H. Free entry determines endogenously the number of firms in this sector. On the other hand, unemployed workers search for employment.

Job seekers and vacant jobs are matched randomly in a pair-wise fashion. The mass of successful job matches in skilled labor market is determined by the matching function $M(u_H, v_H)$, where u_H and v_H denote respectively the number of unemployed workers and vacancies of skilled worker. We define the labor market tightness in skilled labor market as $\theta_H = v_H / u_H$. The rate at which skilled vacancies are filled is $q(\theta_H) = M / v_H$, where $q'(\theta_H) < 0$. The rate at which unemployed skilled workers find jobs is $m(\theta_H) = \theta_H q(\theta_H)$, where $m'(\theta_H) > 0$. We also assume that matches dissolve at a rate s_H.

2.1. *Asset Value Functions*

Let Π_H be the present discounted value associated with a filled vacancy created by a skilled firm matched with a skilled worker and V_H the expected income streams accrued to unfilled vacancy. And let E_H and U_H the values associated with an employed and an unemployed skilled worker, respectively. Then in steady state, we have:

$$r\Pi_H(h) = y_H(h) - w_H(h) - s_H[\Pi_H(h) - V_H], \tag{1}$$

$$rV_H = -c_H + q(\theta_H)[\Pi_H(h) - V_H], \tag{2}$$

where we denote $w_H(h)$ as the wage rate of a skilled worker, and the flow of output is $y_H(h)$. Note that in the baseline model, to simplify our analysis, we assume that the flow of output does not depend on individual's ability, i.e., $y_H(h) = P_H$, where P_H is the skilled sector productivity. Therefore, in the baseline model, $\Pi_H(h) = \Pi_H$ and $w_H(h) = w_H$. In the extension, we will relax this assumption to allow the flow of output to be a function of both individual ability and skilled sector productivity.

The expected income streams accrued to skilled employed workers is:

$$rE_H(h) = w_H(h) + T - s_H[E_H(h) - U_H], \tag{3}$$

The value associated with skilled unemployed workers is:

$$rU_H(h) = T + m(\theta_H)[E_H(h) - U_H], \tag{4}$$

where T is the untied foreign aid distributed to individuals as a lump-sum transfer. Again in the baseline model, $E_H(h) = E_H$ and $U_H(h) = U_H$.

Free entry implies that, in equilibrium, the expected payoff of posting a vacancy is equal to zero, that is,

$$V_H = 0. \tag{5}$$

The unskilled sector is assumed to be fully employed. The present discounted value associated with an unskilled worker E_L is:

$$E_L = \int_0^{\infty} e^{-rt}(w_L + T) = \frac{w_L + T}{r} \tag{6}$$

where w_L is the wage rate for unskilled workers which we assume is taken as exogenously given.

2.2. *Foreign Aid Allocation*

Similar to Chatterjee and Turnovsky (2007), we assume that the recipient economy receives total amount of A foreign aid and λ is an aid-tying ratio.

Moreover, we suppose that λ is required by the donor to be used on public inputs to improve skilled sector productivity $P_H = \lambda A$, while the $T = (1-\lambda)A$ is a lump-sum transfer to individuals.

2.3. Wage Determination

Once a worker meets a firm, they bargain over the wage rate. They solve a generalized Nash bargaining problem given by

$$\underset{w_H}{Max}(E_H - U_H)^{\beta}(\Pi_H - V_H)^{(1-\beta)},$$

where $\beta \in (0,1)$ represents the worker's bargaining strength. The solution to this problem gives:

$$(1-\beta)(E_H - U_H) = \beta(\Pi_H - V_H), \tag{7}$$

where $E_H - U_H$ and $\Pi_H - V_H$ are the worker's and the firm's surplus from the match, respectively.

Substituting for $E_H - U_H$ and Π_H, using equations (1) to (4), in equation (7) and noting that $V_H = 0$ (equation 5), we find:

$$w_H = \frac{r + s_H + m(\theta_H)}{r + s_H + \beta m(\theta_H)}\beta P_H, \tag{8}$$

2.4. Steady-State Equilibrium

Using the free-entry condition (equation 5), we derive the following system:

$$\frac{c_H}{q(\theta_H)} = \frac{1-\beta}{r + s_H + \beta m(\theta_H)}\lambda A, \tag{9}$$

Equation (9) defines a unique market tightness θ_H^* for skilled labor market. The following proposition is immediate:

Proposition 1. (Existence and Uniqueness) *A steady-state equilibrium exists and is unique.*

Proof. The proof of Proposition 1 is presented in the Appendix. ■

At the steady-state equilibrium, the flow into unemployment equals the flow out of unemployment for skilled workers. The steady-state unemployment rate for skilled workers is:

$$u_H^* = \frac{s_H}{s_H + m(\theta_H^*)}. \tag{10}$$

2.5. Labor Market Effects of Foreign Aid

We examine the effects of allocations of foreign aid on labor market tightness, wage rate and unemployment rate for skilled workers. We have the following proposition:

Proposition 2. *If the share of foreign aid used to improve skilled sector productivity becomes higher, then we have:*

$$\frac{d\theta_H^*}{d\lambda} > 0,\ \frac{dw_H^*}{d\lambda} > 0,\ and\ \frac{du_H^*}{d\lambda} < 0.$$

Proof. The proof of Proposition 2 is presented in the Appendix. ■

For skilled workers, an increase in the foreign aid used to improve skilled sector productivity results in a higher profit to firms, which encourages entries of skilled jobs and increases the tightness of labor market for skilled workers. With the raise in the tightness of labor market for skilled workers, skilled worker's bargaining position improves and their wage rate becomes higher as well. Meanwhile, the finding rate of skilled jobs for these workers goes up and thus their unemployment rate goes down.

3. Extension

In the extension, we modify our baseline model in three ways. First, we allow individuals to make human capital acquisition decisions and thus the ratios of skilled and unskilled workers become endogenously determined. Second, the total foreign aid now can be divided into three portions: an investment in skilled sector productivity that aims to improve level of output $P_H = \lambda A$, or as an investment in the education system that aims to improve level of education $S = (1-\lambda)\delta A$ or as a lump sum transfer distributed to individuals $T = (1-\lambda)(1-\delta)A$. Last, the flow of output in skilled sector now also depends on each individual's ability, i.e., $y_H(h) = P_H h$. Note that equation (2) will now become:

$$rV_H = -c_H + q(\theta_H)\{E[\Pi_H(h)] - \}. \tag{11}$$

This implies that a vacancy can be randomly matched with unemployed skilled workers that possess different levels of ability. Therefore, we use $E[\Pi_H(h)]$ to denote the expectation of the value of a filled job.

To make the portion of skilled workers endogenous, individuals need to decide whether to invest in education and become skilled or remain unskilled before entering the labor market. As mentioned above, we assume that individuals differ with respect to their ability. We denote the cost of acquiring training by Z. As individuals enter the labor market in the state of unemployment, they compare the values of unemployment for skilled and employment for unskilled workers when making their decisions on human capital acquisition. Recall that we assume that in unskilled sector, individuals are fully employed. An individual will invest in education if the benefit from this decision exceeds the cost, that is, a worker will invest in education if

$$U_H(h) - E_L \geq Z - S,$$

Thus, all individuals with ability h higher than some cutoff value $\chi \in (0,1)$ will invest in education. Plugging the expressions of $U_H(h)$ and E_L yields χ as

$$\chi = \frac{(r + s_H + \beta m(\theta_H))\{w_L + [Z - (1-\lambda)\delta A]r\}}{\beta m(\theta_H)\lambda A}. \tag{12}$$

3.1. Steady-State Equilibrium

Using the free-entry condition and equation (11), we derive the following system:

$$\frac{c_H}{q(\theta_H)} = \frac{1-\beta}{r + s_H + \beta m(\theta_H)} \lambda A \Lambda(\chi), \tag{13}$$

Equation (13) defines unique market tightness $\theta_H^* = \Omega(\chi^*)$ for skilled labor market. We use $\Lambda(\chi)$ to denote average ability of workers in skilled sector. It is a function of the cutoff ability χ and satisfies

$$\Lambda(\chi) = \int_{\chi}^{1} \frac{\varphi(h)}{1-\Psi(\chi)} h dh, \tag{14}$$

By differentiating equation (14) with respect to χ, we obtain:

$$\frac{d\Lambda(\chi)}{d\chi} = \Lambda'(\chi) = \frac{\varphi(\chi)}{1-\Psi(\chi)}[\Lambda(\chi) - \chi] > 0. \tag{15}$$

This result implies that when χ rises, the least able individual of the skilled workers becomes the ablest worker of the unskilled workers. Therefore, the average productivity in skilled sector rises. This induces more job entries in the skilled sector, thereby leading the skilled labor market tightness to increase. See equation (A4) in Appendix for details.

Proposition 3. (Existence and Uniqueness) *A steady-state equilibrium exists and is unique.*

Proof. The proof of Proposition 3 is presented in the Appendix. ■

At the steady-state equilibrium, the flow into unemployment equals the flow out of unemployment for skilled workers. The steady-state unemployment rates for skilled workers is

$$u_H^* = \frac{s_H}{s_H + m(\theta_H^*)}. \tag{16}$$

3.2. Labor Market Effects of Foreign Aid

We examine the effects of allocations of foreign aid on the share of the educated, labor market tightness, wage rate, and unemployment rate among skilled workers.

We first examine the effects of an increase in δ. Differentiating χ^* with respect to δ and evaluating it at the steady state, we have:

$$\frac{d\chi^*}{d\delta} = -\frac{(1+r)(1-\lambda)(r + s_H + \beta m(\theta_H^*))}{A\beta m(\theta_H^*)\lambda^2} < 0.$$

Thus, an increase in the share of education subsidy leads to a rise in the fraction of skilled workers. A reduction in χ^* will lower the average ability of workers in the skilled sector. Therefore, an increase in education subsidy discourages entry of skilled jobs and reduces the tightness of labor market for skilled workers. With the reduction in the tightness of labor market for skilled workers, skilled worker's bargaining position decreases and their wage rate becomes lower. Moreover, the finding rate of skilled jobs for these workers goes down and thus their unemployment rate goes up. These results are summarized in Proposition 4.

Proposition 4. *If the share of foreign aid used for education subsidy becomes higher, then we have:*

$$\frac{d\chi^*}{d\delta} < 0, \quad \frac{d\theta_H^*}{d\delta} < 0, \quad \frac{dw_H^*}{d\delta} < 0, \text{ and } \frac{du_H^*}{d\delta} > 0.$$

Proof. The proof of Proposition 4 is presented in the Appendix. ■

Similarly, we examine the effects of an increase in λ.

$$\frac{\partial\chi^*}{\partial\lambda} = \frac{(r + s_H + \beta m(\theta_H))[\delta Ar - Zr - w_L]}{A\beta m(\theta_H)\lambda^2} \gtrless 0.$$

An increase in λ implies not only a rise in skilled sector productivity, but also a reduction in education subsidy. These two effects work in opposite directions to determine the share of skilled workers. Only when δ is sufficiently large, i.e., $\delta < \frac{Zr + w_L}{Ar}$, we have $\frac{\partial \chi^*}{\partial \lambda} < 0$. This indicates that the positive effect of an increase in skilled sector productivity on χ dominates the negative effect caused by the reduction of education subsidy. As a result, the share of skilled workers increases.

Proposition 5. *If the share of foreign aid used for public inputs to improve skilled sector productivity becomes higher, then we have:*

$$\frac{d\chi^*}{d\lambda} \gtrless 0, \quad \frac{d\theta_H^*}{d\lambda} \gtrless 0, \quad \frac{dw_H^*}{d\lambda} \gtrless 0, \textit{ and } \frac{du_H^*}{d\lambda} \gtrless 0.$$

Proof. The proof of Proposition 5 is presented in the Appendix. ■

On the one hand, an increase in the share of public inputs to improve skilled sector productivity leads more firms to entry into the skilled labor market. On the other hand, in this case, the fraction of foreign aid used for education subsidy decreases, which raises the average ability of skilled workers and encourages job entries. Therefore, these two forces work together to improve skilled workers' labor market outcomes. However, an increase in foreign aid used for production purpose also induces individuals to invest in education and become skilled. This will lower the average ability of skilled sector, discourage job entries and cause opposite effects.

4. Conclusions

In this paper, we consider foreign aid tied to the productive purpose of the skilled sector of the recipient country. By employing a search and matching model with endogenous human capital acquisition, we have examined the effects of foreign aid on human capital acquisition and educated unemployment in the recipient economy. In the absence of endogenous human capital acquisition, an increase in the foreign aid used for the productive purpose can lower the unemployment rate among skilled workers. However, this result can be mitigated or even overturned when endogenous human capital acquisition is incorporated. We also show that an increase in foreign aid used for education subsidy can induce more educational investment but with a higher educated unemployment rate of the economy. This unintended consequence is often observed in developing countries.

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Appendix

This appendix provides the mathematical proofs of Propositions 1 to 5.

Proof of Proposition 1. An interior equilibrium is θ_H^* which satisfies equation (9). Differentiating both side of equation (9) with respect to θ_H leads to:

$$\frac{\partial[c_H / q(\theta_H)]}{\partial\theta_H} = -\frac{(1-\beta)\beta\lambda A}{[r+s_H+\beta m(\theta_H)]^2} m'(\theta_H), \tag{A1}$$

Since the left-hand side (LHS_{A1}) of equation (A1) increases in θ_H, and right hand-side (RHS_{A1}) of equation (A1) decreases in θ_H. Thus, a sufficient condition for the existence and uniqueness of solution to θ_H^* is $\frac{c_H}{q(0)} \le RHS_{A1}\big|_{\theta_H=0}$ and $\frac{c_H}{q(+\infty)} \ge RHS_{A1}\big|_{\theta_H=+\infty}$. Since $q(\theta)'<0$, $q(0)=+\infty$, $q(+\infty)=0$, $m(\theta)'>0$, $m(+\infty)=+\infty$, $m(0)=0$, the above conditions always hold. Q.E.D.

Proof of Proposition 2. Recall that from Proposition 1, there is a unique θ_H^* which satisfies equation (5). Rewrite equation (9), we have:

$$F_1(\theta_H) = \frac{c_H}{q(\theta_H)} - \frac{(1-\beta)\lambda A}{r+s_H+\beta m(\theta_H)}, \tag{A2}$$

Thus, by using equation (8) and equation (10), we have:

$$\frac{\partial\theta_H^*}{\partial\lambda} = -\frac{\partial F_1(\lambda,\theta_H^*)/\partial\lambda}{\partial F_1(\lambda,\theta_H^*)/\partial\theta_H} > 0,$$

$$\frac{\partial u_H^*}{\partial\lambda} = \frac{\partial u_H^*}{\partial\theta_H}\frac{\partial\theta_H^*}{\partial\lambda} < 0,$$

$$\frac{\partial w_H^*(\theta_H^*,\lambda)}{\partial\lambda} = \frac{\partial w_H^*(\theta_H^*,\lambda)}{\partial\lambda} + \frac{\partial w_H^*(\theta_H^*,\lambda)}{\partial\theta_H}\frac{\partial\theta_H^*}{\partial\lambda} > 0. \qquad \text{Q.E.D.}$$

Proof of Proposition 3. An interior equilibrium is a vector (θ_H^*, χ^*) which satisfies equation (13); and exists $\chi^* \in (0,1)$ satisfies equation (12), where $\theta_H(\chi^*) = \Omega(\chi^*)$.

From equation (13), we have:

$$F_2(\theta_H,\chi) = \frac{r+s_H+\beta m(\theta_H)}{q(\theta_H)} - \frac{(1-\beta)\lambda A\Lambda(\chi)}{c_H} \tag{A3}$$

First, determine the existence and uniqueness of function $\Omega(\chi)$.

$$\lim_{\theta_H\to 0} F_2(\theta_H,\chi) = -\frac{(1-\beta)\lambda A\Lambda(\chi)}{c_H} < 0,$$

$$\lim_{\theta_H\to\infty} F_2(\theta_H,\chi) = \infty,$$

$$\frac{\partial F_2(\theta_H,\chi)}{\partial\theta_H} > 0.$$

Therefore, there exists a unique function $\Omega(\chi^*)$ for equation (12). And differentiating $\Omega(\chi)$ with respect to $\Lambda(\chi)$ and χ, we have:

$$\frac{\partial\Omega(\chi)}{\partial\Lambda(\chi)} = -\frac{\partial F_2(\Omega(\chi),\Lambda(\chi))/\partial\Lambda(\chi)}{\partial F_2(\Omega(\chi),\chi)/\partial\Omega(\chi)} = \frac{(1-\beta)\lambda Aq(\theta_H)^2}{\partial F_2(\Omega(\chi),\chi)/\partial\Omega(\chi)} > 0, \qquad \text{(A4)}$$

$$\frac{\partial\Omega(\chi)}{\partial\chi} = \Omega'(\chi) = -\frac{\partial F_2(\Omega(\chi),\chi)/\partial\chi}{\partial F_2(\Omega(\chi),\chi)/\partial\Omega(\chi)} = \frac{(1-\beta)\lambda A\Lambda'(\chi)}{\partial F_2(\Omega(\chi),\chi)/\partial\Omega(\chi)} > 0.$$

This monotonicity indicates that there exists a unique vector (θ_H^*, χ^*) that satisfies equation (13).

Then turn to equation (12). Rewrite equation (12), we have:

$$F_3(\chi) = \beta m(\theta_H)\lambda A\chi - (r+s_H+\beta m(\theta_H))[w_L + (Z-(1-\lambda)\delta A)r]. \qquad \text{(A5)}$$

Let $\lim_{\chi\to 0}\Omega(\chi) = \underline{\theta_H}$, $\lim_{\chi\to 1}\Omega(\chi) = \overline{\theta_H}$, then we have:

$$\lim_{\chi\to 0} F_3(\chi) = -(r+s_H+\beta m(\underline{\theta_H}))[w_L + (Z-(1-\lambda)\delta A)r] < 0,$$

$$\lim_{\chi\to 1} F_3(\chi) = \beta m(\overline{\theta_H})\lambda A - (r+s_H+\beta m(\overline{\theta_H}))[w_L + (Z-(1-\lambda)\delta A)r],$$

$$\frac{\partial F_3(\chi)}{\partial\chi} = \beta\{\lambda A m(\theta_H) + \lambda A\chi m'(\theta_H)\Omega'(\chi) - m'(\theta_H)\Omega'(\chi)[w_L + (Z-(1-\lambda)\delta A)r]\}.$$

Thus, to have an unique solution $\chi^* \in (0,1)$ satisfying equation (12), we need $\lim_{\chi\to 1} F_3(\chi) > 0$, and $\frac{\partial F_3(\chi)}{\partial\chi} > 0$. Q.E.D.

Proof of Propositions 4&5. From equation (12) and equation (A3) we have:

$$\frac{\partial\chi(\delta)}{\partial\delta} = -\frac{r(1-\lambda)(r+s_H+\beta m(\theta_H))}{\beta m(\theta_H)\lambda} < 0,$$

$$\frac{\partial\chi(\lambda)}{\partial\lambda} = \frac{(r+s_H+\beta m(\theta_H))[\delta Ar - Zr - w_L]}{A\beta m(\theta_H)\lambda^2} \gtrless 0.$$

In addition, the free-entry condition defines an implicit function $\Omega(\chi)$, whose properties are provided in the Proof of Proposition 3. Thus, from equation (12) and equation (A3) we have:

$$\frac{\partial\theta_H}{\partial\delta} = \frac{\partial\Omega(\delta,\chi)}{\partial\chi}\frac{\partial\chi}{\partial\delta} < 0,$$

$$\frac{\partial\theta_H}{\partial\lambda} = \frac{\partial\Omega(\lambda,\chi(\lambda))}{\partial\lambda} + \frac{\partial\Omega(\lambda,\chi(\lambda))}{\partial\chi}\frac{\partial\chi}{\partial\lambda} \gtrless 0,$$

where $\frac{\partial\Omega(\lambda,\chi(\lambda))}{\partial\lambda} = -\frac{\partial F_2(\theta_H,\lambda)/\partial\lambda}{\partial F_2(\theta_H,\lambda)/\partial\Omega(\lambda)} > 0$. Plus, $\frac{\partial u_H}{\partial\theta_H} < 0$ and $\frac{\partial w_H}{\partial\theta_H} > 0$, we have $\frac{dw_H}{d\delta} < 0$, $\frac{du_H}{d\delta} > 0$, $\frac{dw_H}{d\lambda} \gtrless 0$ and $\frac{du_H}{d\lambda} \gtrless 0$. Q.E.D.