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Equilibrium-Informed Trading with Relative Performance Measurement

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Abstract

This article analyzes the informative trading of professional money managers within a rational-expectations equilibrium model in which managers care about their performance relative to their peer group. I find that the existence of uninformed managers causes informed managers with relative performance concerns to trade less informatively, engendering less informative prices. When managers are differentially informed, they need to forecast the average performance based on private signals, and each manager may place more weight on the private signal if the signal provides good information about the average performance. The price aggregates those signals and thus becomes more informative.

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1. Introduction

In modern financial markets, institutional investors such as mutual funds, pension funds, and investment banks actively trade a large portion of assets.[1] Thus, it is important to understand how professional money managers' behavior affects asset prices. Individual investors delegate their money to fund managers and pay the managers for their superior skills or information. Thus, the pay structure affects fund managers' trading behavior. Money managers are normally remunerated on the basis of their performance relative to the chosen benchmark, which is generally the average level of the entire industry. Hence, fund managers care about their position relative to their peer group.

Relative performance compensation affects money managers' portfolio decisions in diverse ways; however, to date, the theoretical literature has focused primarily on risk sharing. By contrast, few studies have investigated the effects of relative performance compensation on money managers' other trading motivations. Given the extensive literature on information trading and price information efficiency, the trading behaviors of informed fund managers should not be ignored. Market efficiency (with respect to information) is one of the most important issues in modern finance theory, and with the substantial changes in the financial market structure, the effect of institutional investors on price information efficiency should be analyzed. Moreover, money managers are likely to hold private information because individuals delegate portfolio management to money managers. Thus, it is important to understand informational trading if the agents are professional money managers. This article proposes a rational-expectations equilibrium (REE) model of delegated portfolio management to study the informative trading of fund managers and examine the informational efficiency of asset prices.

My model has two dates, 0 and 1, and two assets, one risky and one risk-free, are traded in the market. Risk-averse money managers maximize their utility over their remuneration in period 1 by optimally choosing their portfolios. Moreover, there are two types of managers, informed and uninformed. Informed managers receive different but correlated signals and thus trade conditionally with regard to their private information. By contrast, uninformed managers observe only the price and extract information from it. At equilibrium, the price aggregates in-formed managers' information and partially reveals this information because of the presence of noise.

The managers' remuneration takes a linear form with regard to both absolute and relative performance. Relative performance is defined as the difference between the fund performance and the average level, and fund managers earn a bonus if their performance is above the average performance and incur a penalty if their performance is below the average performance. Compensation takes a linear form for the relative performance evaluation, providing symmetric payoffs when managers' performance is below or above the benchmark. Such a payoff structure is also called "fulcrum" compensation, which was proposed by the U.S. Congress in 1970 as the amended Investment Advisors Act.

Relative performance remuneration links managers' payoff to the average performance of all managers, which is uncertain because managers have different information sets. In my model, each mean–variance manager needs to hedge the additional uncertainty from the relative performance compensation. In other words, each manager wants to reduce the tracking errors relative to the bench-mark. Thus, the

[1] According to Allen (2001), by 2000, less than 40% of U.S. corporate equities were directly owned by individuals, with even lower rates of individual ownership in France (24%) and the United Kingdom (21%).

main mechanism of price determination examined in the model is hedging, and the hedging component in the optimal demand takes the form of the conditional covariance of the risky-asset payoff and the average performance of managers. For the purpose of hedging, uninformed managers effectively change their trading aggressiveness, whereas informed managers forecast the conditional covariance based on the private signals and the price. These behaviors change the weights that informed and uninformed managers place on their signals, which affects the price informativeness.

In the model, I show that uninformed managers' hedging behavior induces them to trade more aggressively as a result of their use of the price to forecast the average performance. Informed managers always trade less aggressively on their private information if the benchmark for the compensation is exogenous because they tilt the portfolio toward the benchmark. In my model, however, the bench-mark is endogenously determined and uncertain to each manager. Each informed manager's hedging behavior can induce the manager to weigh the signal more heavily if the price is a relatively bad predictor of the benchmark. If the percent-age of informed managers is relatively high, I can determine the conditions under which the price can become more informative. I show that when noise traders are very noisy or when managers are relatively risk averse, the price is likely to be more informative.

To determine the exact channels that affect price informativeness, I consider three simplified information structures. First, I consider a baseline model with symmetric information, in which all managers receive a common signal. This analysis provides a baseline case in which no manager needs to hedge. Second, I examine a model that incorporates asymmetric information, in which only some managers observe a common signal, whereas others do not, and I analyze the interaction between informed and uninformed managers. Third, I analyze a model in which all managers receive "equally accurate" but independent signals, which enables me to discuss informed managers' hedging.

Regarding the first information structure, the results show that relative performance does not matter in a symmetric-information setting, which is not surprising because, given the same information set, all managers submit the same demand and thus achieve the same level of performance. This economy is the same as the benchmark without relative performance.

Under the second information structure, asymmetric information (Grossman and Stiglitz (1980)), uninformed managers need to hedge against the uncertainty generated by the performance of informed managers, which causes uninformed managers to trade more aggressively on information extracted from the price. Informed managers, however, must account for uninformed managers' trading behavior because of the relative performance concerns. Hence, they trade less aggressively on their signals, which results in a less informative price.

For the third information structure, differential information, all managers are "equally" informed with independent signals. In this economy, each manager has an incentive to reduce the tracking error between his or her performance and the benchmark. To do so, the manager needs to forecast the conditional covariance of the risky-asset payoff and the average performance of all managers based on two pieces of information: the private signal and the price. When the signal is a more precise predictor than the price, managers place more weight on the private signal and less weight on the price. The price thus aggregates more information and becomes more informative. When the price is more precise than the private signal, the opposite is true.

In the differential information model, the price not only represents public information but also performs the role of aggregating information. To separate the two effects, I extend the model by adding an exogenous public signal that is observable to all managers. I show that relative performance leads managers to place more weight on the common signal than on private signals. Compared with the differential information setting, where the price is the public signal, if managers place less weight on private signals but more weight on public signals, the price aggregates less information and becomes less informative. As a result, managers place more weight on private signals.

In the context of the mutual fund industry, managers may have relative performance concerns because of fund flows. If fund managers' compensation is a fixed proportion of the assets they manage, relatively good performance leads to a large fund size because it attracts new fund inflows. In this context, relative performance matters even if relative performance is not explicitly set in the con-tract. However, empirical evidence shows that the flow–performance relationship is convex (Chevalier and Ellison (1997)). For the purpose of comparison, I also analyze the case with convex compensation. In particular, I consider a special convex contract, the option contract, and analyze its effect on the information-inference problem under the asymmetric-information structure (Grossman and Stiglitz (1980)).

When the option embedded in the manager's compensation is out of the money, managers do not have relative performance concerns and thus trade based on their own information. When the option is in the money, informed managers trade less informatively, as in the case with the linear contract. Due to the non-linearity of the option contract, the inference problem for uninformed managers becomes complicated because they may be confused regarding when the informed manager's option compensation is in the money. In any circumstance, I show that uninformed managers effectively change their trading aggressiveness.

My article relates to two research strands in the literature. The first strand concerns research using the REE model to study the information content of asset prices. The second strand concerns research on delegated portfolio management to study the role of financial institutions in financial markets and the effects of agency frictions on asset prices. My article builds on the static REE model (e.g., Grossman (1976), Grossman and Stiglitz (1980), and Diamond and Verrecchia (1981)) and analyzes the informative trading of fund managers.

In the literature, certain theories focus on models with a single representative fund manager. If only one manager exists, his or her performance relative to a peer group does not matter. Articles using such a model include those by Vayanos and Woolley (2013) and He and Krishnamurthy (2011), (2013). My article, in contrast, considers heterogeneous managers who care about their performance relative to a peer group.

Among models with multiple traders, some consider an economy with both investors and fund managers in which fund managers care about their performance relative to a passive benchmark (e.g., Standard & Poor's (S&P) 500). Such articles include those by Basak and Pavlova (2013), Cuoco and Kaniel (2011), and Kaniel and Kondor (2013). Other articles with multiple managers, such as my article, consider an endogenous benchmark, which is defined as the average performance of all managers. For example, Kapur and Timmermann (2005) and Basak and Makarov (2012) consider fund managers' performance relative to a peer group. All of these articles consider symmetric information and thus focus on risk sharing, whereas my article considers an REE model and studies informative trading.

Certain articles in the literature use an asset pricing model that incorporates both asymmetric information and career concerns. Dasgupta and Prat (2006) demonstrate that career concerns can cause more uninformative trading. In a sense, relative performance concerns are close to reputation concerns. However, my model differs from these previous models in many respects. For instance, Dasgupta and Prat present a game theoretical model with asymmetric information, whereas my model is a more standard asset pricing model that allows for richer information structures. A common result of these articles and mine is that prices can be less informative when career or relative performance concerns exist; however, I also show that prices can be more informative, which is a novel finding in this strand of the literature.

In general, my article is also related to the literature on relative wealth concerns. In this literature stream, Bakshi and Chen (1996) examine the effect of concerns regarding social status on portfolio and consumption choices. In their article, the average wealth level of the society is exogenously given, whereas it is endogenous in my model. Furthermore, DeMarzo, Kaniel, and Kremer (2007), (2008) show that endogenous relative wealth concerns lead to overinvestment and bubbles; however, their analysis does not consider information asymmetry.

Garcia and Strobl (2011) present an article close to mine that examines how relative wealth concerns affect information acquisition within a rational-expectations paradigm. Although Garcia and Strobl propose a modeling motivation and framework that are similar to those of my article, their article is different from mine in many aspects. For instance, Garcia and Strobl adopt a "catching up with the Joneses" type of utility function and focus on the information acquisition of agents. My article, instead of being utility driven, begins with explicit compensation and focuses on managers' behavior in hedging the additional uncertainty generated by the endogenous benchmark. The price informativeness with different information structures is the core concept of my analysis. Thus, my article analyzes a different problem in a different context and hence complements Garcia and Strobl's article.

The rest of the article is organized as follows: I first introduce the model in Section II and then solve the model in Section III. Section IV considers specific information structures, namely, symmetric, asymmetric, and differential in-formation structures. Section V presents a model with convex compensation. Finally, I conclude in Section VI. All lengthy proofs and figures are provided in the Appendix.

2. Model

There are two dates, t =0, 1, and two assets, a risky asset and a risk-free asset, are traded in the market. The risky asset pays the liquidating dividend d at t =1, whereas the risk-free asset has a constant return r between t =0 and 1. I normalize r =0 for simplicity. d has a normal distribution with mean $\overline{d}$ and variance σ_d^2. The price of the risky asset, P, is determined in equilibrium at t =0, and the supply of the risky asset is S +u, where u is normally distributed with mean 0 and variance σ_u^2.

A. Fund Managers and Compensation

I consider a competitive market that is populated by many fund managers, who form a continuum of measure 1. Each manager is endowed with an initial wealth W_0 (the initial fund under management). Moreover, the individual investors (households) do not have access to the market, so they can only delegate the management of their money to fund managers. Therefore, fund managers construct portfolios by investing their initial wealth W_0 between the risky and risk-free assets.

Fund managers are remunerated by compensation for both absolute and relative performance. To be specific, the remuneration for fund manager i is linear

$$F_{1,i} = I + aW_{1,i} + b(W_{1,i} - \overline{W}), \quad (1)$$

where $W_{1,i}$ is the final fund size at period 1 for manager i, and $\overline{W}$ is the average final fund size (or performance) of all managers:

$$\overline{W} = \int_0^1 W_{1,i} di. \tag{2}$$

In equation (1), I is a fixed component,[2] a >0 is the parameter associated with the fund's absolute performance, and b >0 is the coefficient of the fund's performance relative to the average level. Equation (1) links managers' payoff to the performance of their peers, which engenders relative performance concerns among the managers. Because all managers have the same initial fund W_0 at t =0, the fund size at t =1, $W_{1,i}$, represents the performance of each manager.

Equation (1) takes a linear form with respect to the relative performance evaluation, providing symmetric payoffs when managers' performance is below or above the benchmark. Proposed by the U.S. Congress in 1970 as the amended Investment Advisors Act, this fulcrum compensation payoff structure motivates me to choose a linear compensation contract. Moreover, Kapur and Timmermann (2005) show that a linear contract form is optimal under certain information structures, which also justifies my modeling choice of linear compensation.

Fund managers' preference is given by a simple mean–variance utility:

$$U(w) = \mathrm{E}(F_1) - \frac{\tau}{2}\mathrm{var}(F_1),[3] \tag{3}$$

where τ measures the degree of risk aversion. Therefore, fund managers maximize the utility of their compensation at t =1 by optimally choosing a portfolio at t =0.

B. Information Structure

Fund managers are heterogeneously informed. Among all managers, a portion λ of managers are informed, and 1−λ are uninformed. Each informed manager i observes a signal $s_i = d + \varepsilon_i$, in which ε_i is independent of d and is normally distributed with a mean of 0 and a variance of σ_ε^2. For any ε_i and ε_j, the two variables have a constant correlation coefficient ρ. There are some noise traders u, who are assumed to be on the supply side. Adding noise to the economy, which is a standard practice in the literature, prevents the price from fully revealing information.

All random variables are described as follows:

$$\begin{pmatrix} d \\ u \\ \varepsilon_i \\ \varepsilon_j \end{pmatrix} \sim \mathrm{N}\left\{ \begin{pmatrix} \overline{d} \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_d^2 & 0 & 0 & 0 \\ 0 & \sigma_u^2 & 0 & 0 \\ 0 & 0 & \sigma_\varepsilon^2 & \rho\sigma_\varepsilon^2 \\ 0 & 0 & \rho\sigma_\varepsilon^2 & \sigma_\varepsilon^2 \end{pmatrix} \right\}.$$

Because the correlation coefficient ρ for any ε_i and ε_j is constant, I apply the Vasicek (2002) 1-factor structure to model the signal noise. Thus, ε_i has the structure of the 1-factor model:

$$\frac{\varepsilon_i}{\sigma_\varepsilon} = \sqrt{\rho}Y + \sqrt{1-\rho}X_i,[4] \tag{4}$$

where Y is the common factor, and X_i is the specific factor. Y and all X_i values follow the standard normal distribution, and X_i, X_j, Y , and d are independent.

Kapur and Timmermann (2005) consider a model with the same type of compensation as mine. However, their information structure is significantly different from mine. In their article, fund managers receive a common signal, which is the

[2] It could be any constant, as it will not affect my result.

[3] An alternative would be the constant absolute risk aversion (CARA) utility function, which may yield a slightly different result. However, using CARA utility, I cannot explicitly calculate the conditional covariance between the asset payoff and the average performance, which is the key driver of the model. Moreover, when I examine a convex contract, I cannot use CARA for the calculation. Thus, I use the mean–variance utility function instead of CARA.

[4] I apply this structure because it can simplify the notations, and it is easy to see that corr$[\varepsilon_i, \varepsilon_j] = \rho$

same as my assumption of ρ =1. However, they also assume that investors receive different signals, which are less precise than those of fund managers. This assumption is imposed to rationalize the delegation relationship between investors and managers. In my article, investors do not play any role in the model, and I focus on the interactions of different managers. Moreover, I consider managers' inference problem arising from the equilibrium price, which is not considered by Kapur and Timmermann.

3. Solving the Model

In this section, I solve the model presented in Section II. The informed man-agers' information set, F_I, consists of a private signal and the price. That is, $F_I = \{s_i, P\}$. Uninformed managers can observe only the price, P; hence, they have the information set $F_u = \{P\}$. With this information structure, the equilibrium in this economy is defined in the following definition:

Definition 1. Subject to the compensation defined by equation (1), the noisy REE is defined as follows:

1. Each informed manager i maximizes his or her utility conditional on F_I:

$$\max_{x_i^I} \mathrm{E}(F_{1,i}^I \mid s_i, P) - \frac{\tau}{2}\mathrm{var}(F_{1,i}^I \mid s_i, P).$$

2. Each uninformed manager i maximizes his or her utility conditional on F_u:

$$\max_{x_i^u} \mathrm{E}(F_{1,i}^u \mid P) - \frac{\tau}{2}\mathrm{var}(F_{1,i}^u \mid P).$$

3. The market clears:

$$\int_0^{\lambda} x_i^I(p)di + \int_0^{1-\lambda} x_i^u(p)di \quad = \quad S + u.$$

The key difference between this equilibrium and the standard REE is the compensation fee. To better understand the model, I analyze the compensation before I solve the equilibrium. The most important component of the fee is the benchmark, which is given in equation (2). With some simple manipulation, I express the benchmark as follows:

$$\overline{W} \quad = \quad W_0 + (d-P)\left(\int_0^{\lambda} x_j^I dj + \int_0^{1-\lambda} x_j^u dj\right),^{5} \tag{3}$$

where x_j^I and x_j^u are the optimal demands of the informed and uninformed managers for the risky asset. Given the benchmark specified as in equation (3), the compensation defined by equation (1) can be written as

$$F_1^i \quad = \quad I + aW_0 + \left[(a+b)x_i - b\left(\int_0^{\lambda} x_j^I dj + \int_0^{1-\lambda} x_j^u dj\right)\right](d-P), \tag{4}$$

where x_i denotes the demand of either an informed or an uninformed manager i.

From equation (4), I can show that the compensation of any manager is linked to the demands of all other managers, which increases the uncertainty of managers' payoff because managers have different information. Thus, managers need to adjust their demands to hedge the new uncertainty. Lemma 1 shows the results.

Lemma 1. The optimal demands of both the informed and the uninformed man-agers are as follows:

[5] The terminal fund size can be expressed as $W_{1,i} = W_0 + x_i(d-P)$, and I further denote $W_{1,j}^I$ and $W_{1,j}^u$ as the fund sizes for informed and uninformed managers, respectively. By inputting $W_{1,i}$ into the
5 expression $\overline{W} = \int_w W_{1,j}^I dj + \int_{1-w} W_{1,j}^u dj$, I have equation (3).

$$x_i^I = \frac{a}{a+b}\underbrace{\frac{\mathrm{E}(d\,|\,s_i,P)-P}{a\tau\sigma^2_{d|s_i,P}}}_{\text{Demand without relative performance}} + \frac{b}{a+b}\underbrace{\left\{\int_0^{1-\lambda} x_j^u dj + \frac{\mathrm{cov}\left[d-P,(d-P)\int_0^{\lambda} x_j^I dj\,|\,s_i,P\right]}{\sigma^2_{d|s_i,P}}\right\}}_{\text{Hedging component}}, \tag{5}$$

$$x_u = \frac{a}{a+b\lambda}\underbrace{\frac{\mathrm{E}(d\,|\,P)-P}{a\tau\sigma^2_{d|P}}}_{\text{Demand without relative performance}} + \frac{b}{a+b\lambda}\underbrace{\frac{\mathrm{cov}\left[d-P,(d-P)\int_0^{\lambda} x_j^I dj\,|\,P\right]}{\sigma^2_{d|P}}}_{\text{Hedging component}}. \tag{6}$$

Proof. The proof is in the Appendix.

Lemma 1 shows the intermediate steps of the optimal demands from which the hedging behavior of each type of manager is observed. The optimal demands for both types of managers are the weighted averages of the demand without relative performance[6] and the hedging component. With partial equilibrium, the hedging component is the conditional covariance, and the sign of the covariance determines the trading aggressiveness of the managers. To understand the hedging component, I use the informed demand as an example, and similar logic can be applied to the uninformed demand.

I examine the conditional variance of the compensation of informed manager i. If I calculate the conditional variance of the compensation (equation (4)), I can obtain

$$\mathrm{var}(F_1^i\,|\,s_i,P) = \mathrm{var}\left\{\underbrace{\left[(a+b)x_i^I - b\int_0^{1-\lambda} x_j^u dj\right](d-P)}_{\mathrm{RV}_1} - \underbrace{b\int_0^{\lambda} x_j^I dj(d-P)}_{\mathrm{RV}_2}\,|\,s_i,P\right\}. \tag{7}$$

Conditional on s_i and P, expression (7) can be observed as the conditional variance of the sum of two random variables, RV_1 and RV_2. For RV_1, only d is random because x_i^I and x_j^u are in the information set of the informed manager i. For RV_2, however, $\int_0^{\lambda} x_j^I dj(d-P)$ is random. Thus, what matters is the covariance between the asset payoff and the average performance of the informed managers.[7] If manager i perceives both the asset payoff and the average performance of all informed managers to be very good (bad), it is optimal for the manager to increase his or her demand. With negative conditional covariance, however, manager i decreases his or her demand because the manager does not want to deviate too much from the average. In other words, with mean–variance utility, the conditional variance $\mathrm{var}(F_1^i\,|\,s_i,P)$ induces managers to reduce the tracking errors. To fully understand managers' hedging behavior, I need to derive the optimal trading strategies and equilibrium, which are shown in the next subsection.

A. Optimal Trading Strategies and Equilibrium

To solve the equilibrium, I need to derive the explicit forms of the demands for

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6 [6]I call $[\mathrm{E}(d\,|\,s_i,P)-P]/a\tau\sigma^2_{d|s_i,P}$ the demand without relative performance because it takes the same form as the demand if there is no relative performance (NRP) (by setting $b=0$). However, $\sigma^2_{d|s_i,P}$ in equation (5) is also affected by b in the equilibrium.

7 [7]Note that $\mathrm{cov}[d,(d-P)\int_0^{\lambda} x_j^I dj\,|\,s_i,P] = \mathrm{cov}[d,\lambda W_0 + (d-P)\int_0^{\lambda} x_j^I dj\,|\,s_i,P]$, and $\lambda W_0 + (d-P)\int_0^{\lambda} x_j^I dj$ is the average performance of all informed managers.

both types of managers, which are shown in Lemma 2.
Lemma 2. The explicit expressions of the optimal demands are

$$x_i^I = \frac{\mathrm{E}(d \mid s_i, P) - P}{[a + b(1-\lambda)]\tau\sigma^2_{d \mid s_i, P}} + \frac{b(1-\lambda)x_u}{a + b(1-\lambda)} + W \times \frac{b\lambda[2\mathrm{E}(d \mid s_i, P) + \sigma_\varepsilon\sqrt{\rho}\mathrm{E}(Y \mid s_i, P) - P - s_i]}{a + b(1-\lambda)}, \quad (8)$$

$$x_u = K\frac{\mathrm{E}(d \mid P) - P}{\tau\sigma^2_{d \mid P}} \quad (9)$$

where

$$K = \frac{a + b(1-\lambda)}{a(a+b)} + \frac{b\lambda}{a(a+b)}\frac{2\sigma^2_{d \mid P} - \sigma^2_{d \mid s_i, P}}{\sigma^2_{d \mid s_i, P}} + W \times \frac{2b^2\lambda^2(\sigma^2_{d \mid P} - \sigma^2_{d \mid s_i, P})\tau}{a(a+b)} \quad \text{and}$$

$$W = \frac{\beta_s}{[a + b(1 - 2\lambda\beta_s - \lambda\sigma_\varepsilon\sqrt{\rho}\beta_s^Y)]\tau\sigma^2_{d \mid s_i, P}}.$$ [8]

Proof. The proof is in the Appendix.

The demand of informed managers comprises three terms. The first two are the demand affected by the average demand of uninformed managers, and the third is the hedging demand for the average demand of all informed managers. If I set ρ =1 (as in Section IV.B), indicating that all informed managers have the same signal, the last term disappears. When all informed managers have the same signal, the average informed demand is a function of the signal and thus is in the informed manager's information set. For this reason, informed managers do not need to hedge the average demand of informed managers, so the third term disappears when ρ =1. If I set λ=1 (as in Section IV.C), the first two terms become $[\mathrm{E}(d \mid s_i, P) - P]/a\tau\sigma^2_{d \mid s_i, P}$, so the effect of uninformed managers disappears. I interpret the term $\mathrm{E}(Y \mid s_i, P)$ in the hedging demand as a learning effect because informed managers can learn something about the average demand of the informed managers[9] when $\rho \neq 0$ ($\neq 1$ as well). Note that Y is independent of the risky asset payoff d, so it only provides information about the average performance.

The uninformed demand is a constant K multiplied by [E(d | P)− P]/ $[\tau\sigma^2_{d \mid P}]$. Therefore, the consequence of uninformed hedging is only that trading aggressiveness changes, and the degree is measured by K . If all informed managers have the same signal (when ρ =1, as shown in Section IV.B), $W2b^2\lambda^2(\sigma^2_{d \mid P} - \sigma^2_{d \mid s_i, P})\tau/a(a+b)$ disappears. Thus, $W2b^2\lambda^2(\sigma^2_{d \mid P} - \sigma^2_{d \mid s_i, P})\tau/a(a+b)$ measures how uninformed managers react to informed managers' hedging behavior for the average demand of informed managers. As shown in the Appendix, W >0, so informed managers' hedging behavior leads uninformed managers to trade more aggressively.

In equilibrium, the conjectured price has the following form:

$$P = A + B\left[\left(\int_0^\lambda s_i di - \lambda\bar{d}\right) - Cu\right], \quad (10)$$

8 [8] For informed managers, the expressions of the conditional expectations $\mathrm{E}(Y \mid s_i, P)$ and $\mathrm{E}(d \mid s_i, P)$ are shown by equations (A-2) and (A-3) in the Appendix. For uninformed managers, $\mathrm{E}(d \mid P) = \bar{d} + \beta_p^u(P - A)$. Moreover, β_s, β_s^Y, $\sigma^2_{d \mid s_i, P}$, and $\sigma^2_{d \mid P}$ are quantities from the projection theorem, which can also be found in the Appendix.

9 [9] As shown in the Appendix and the next subsection, if I conjecture the demand of informed manager i as $x_i^I = Gs_i + HP + F$ for three constants G, H, and F, I have $\int_0^\lambda x_j^I dj = \lambda[G(d + \sigma_\varepsilon\sqrt{\rho}Y) + HP + F]$. Thus, $\mathrm{E}(Y \mid s_i, P)$ helps manager i learn $\int_0^\lambda x_j^I dj$.

where A, B, and C are three constants determined in equilibrium. In the Appendix, I show that A and B are functions of C and that B is a positive constant if C is positive. Moreover, I derive a fifth-degree polynomial in C (defined by equation (A-11) in the Appendix). If I define the polynomial as F(C), the equilibrium exists if and only if a positive C exists for F(C)=0. The existence of the equilibrium can be proven; however, I cannot show the uniqueness of the equilibrium. Instead, I characterize the conditions for the uniqueness of the equilibrium. The following proposition summarizes the results:

Proposition 1. Given the optimal demands in Lemma 2 and the conjectured price in equation (10), an equilibrium defined in Definition 1 exists. Define Q =[a +b(1−2λ)]σ_d^2+[a +b(1−λρ)] σ_ε^2, and the equilibrium is unique if

$$Q \quad < \quad \frac{\lambda}{\sigma_\varepsilon \sigma_u}\sqrt{\frac{5}{2}\frac{(1-\rho)(\sigma_\varepsilon^2+\sigma_d^2)(\sigma_d^2+\rho\sigma_\varepsilon^2)}{\tau}}, \tag{11}$$

or

$$\frac{\lambda^2(1-\rho)(\sigma_d^2+\rho\sigma_\varepsilon^2)^2}{(a+b)(\sigma_d^2+\rho\sigma_\varepsilon^2)Q+\rho Q^2} \quad < \quad 2\sigma_u^2\sigma_\varepsilon^2\tau^2. \tag{12}$$

Proof. The proof is in the Appendix.

Proposition 1 shows that the equilibrium exists and is unique when certain conditions are satisfied. When either inequality (11) or (12) holds, the fifth-degree polynomial has a unique positive root of C. Accordingly, the sufficient condition for inequality (11) is

$$\sigma_\varepsilon \sigma_u (a+b) \quad < \quad \frac{\sigma_d^2 \lambda}{\sigma_d^2+\sigma_\varepsilon^2}\left[\sqrt{\frac{5}{2}\frac{(1-\rho)}{\tau}}\right], \tag{13}$$

and the sufficient condition for inequality (12) is

$$\sigma_\varepsilon \sigma_u (a+b) \quad > \quad \left[\frac{1}{5}-(1-\rho)\rho\right]\lambda\sqrt{\frac{5}{2}\frac{(1-\rho)}{\tau}}. \tag{14}$$

Given inequalities (13) and (14), I identify some simple sufficient conditions for the uniqueness of the equilibrium. Note that when (1−ρ)ρ >1/5, inequality (14) always holds, implying that for sufficiently high correlation, the equilibrium is unique. When (1−ρ)ρ <1/5, the equilibrium is unique if $\sigma_d^2/(\sigma_d^2+\sigma_\varepsilon^2) > ⅕$, thus implying that the signal should be sufficiently precise.

B. Price Informativeness

In the literature, some analyses of relative performance contracts are based on passive benchmarks (e.g., Admati and Pfleiderer (1997)). For such contracts, managers tilt their portfolios toward the passive benchmark, which decreases the informativeness of prices. In my article, however, the benchmark is stochastic, and managers must rely on their private signals when they hedge the risk from the benchmark. Managers' hedging behavior may increase the weight they place on private signals, leading to more informative prices. For the analysis, consistent with the literature, I define the price informativeness as

$$I \quad = \quad \frac{1}{\sigma_{d|P}^2} \quad = \quad \frac{1}{\sigma_d^2}+\frac{\lambda^2}{\lambda^2\rho\sigma_\varepsilon^2+C^2\sigma_u^2}, \tag{15}$$

and I derive the results about price informativeness in the following proposition: Proposition 2. In equilibrium, the price is more informative in the presence of relative performance if the conditions

$$(2\lambda-1)\sigma_d^2-(1-\lambda\rho)\sigma_\varepsilon^2 \quad > \quad 0, \tag{16}$$

and

$$\sqrt{\frac{(1-\rho)(\sigma_d^2+\rho\sigma_\varepsilon^2)}{[(2\lambda-1)\sigma_d^2-(1-\lambda\rho)\sigma_\varepsilon^2]}} < \frac{a\tau\sigma_u\sigma_\varepsilon\{\lambda\rho^2\sigma_\varepsilon^2+\sigma_d^2[1+(2\lambda-1)\rho]\}}{(\sigma_d^2+\rho\sigma_\varepsilon^2)\lambda} \tag{17}$$

hold, and it is less informative otherwise.

Proof. The proof is in the Appendix.

Proposition 2 shows that prices can be either more or less informative in the presence of relative performance. Note that for a more informative price, I need the conditions in inequality (16) to hold, which at least means that $\lambda>1/2$. Thus, if relative performance causes the price to be more informative, a sufficient number of informed managers should exist in the market. Otherwise, the price is less informative. Moreover, with a sufficient number of informed managers (inequality (16) holds), the price is more informative when inequality (17) holds, and less informative otherwise. This finding of a less informative price is consistent with the literature; however, I show that benchmarking a portfolio can lead to a more informative price.

To understand the results, I assume that informed manager i submits an optimal demand with the form $x_i^I = Gs_i + HP + F$, in which G, H, and F are three constants that are unrelated to s_i and P. By some calculation, I can show $\int_0^\lambda x_j^I dj = \lambda[G(d+\sigma_\varepsilon\sqrt{\rho}Y)+HP+F]$. Through some manipulation, I can calculate the conditional covariance as

$$\begin{aligned}\operatorname{cov}[d,(d-P)\int_0^\lambda x_j^I dj \mid s_i, P] \\ = \lambda G[2\mathrm{E}(d\mid s_i,P)+\sigma_\varepsilon\sqrt{\rho}\mathrm{E}(Y\mid s_i,P)-P]\sigma^2_{d\mid s_i,P} \\ +\lambda(HP+K)\sigma^2_{d\mid s_i,P}.\end{aligned} \tag{18}$$

Note that informed managers essentially use the signal s_i and the price P to forecast two measures to determine the covariance. First, they forecast the expected return, which is $\mathrm{E}(d-P\mid s_i,P)$. Second, they forecast an uncertain part of the benchmark, which is $\mathrm{E}(d\mid s_i,P)+\sigma_\varepsilon\sqrt{\rho}\mathrm{E}(Y\mid s_i,P)$. In addition to the standard mean variance problem, managers use their signals to forecast the covariance.

From the projection theorem, I show that

$$\mathrm{E}(d\mid s_i,P) = \bar{d}+\beta_s(s_i-\bar{d})+\beta_p(P-A),$$

and

$$\mathrm{E}(Y\mid s_i,P) = \beta_s^Y(s_i-\bar{d})+\beta_p^Y(P-A).$$

Thus, I sum all of the coefficients of si and set the sum equal to G. Then, I can derive

$$G = \frac{\beta_s}{\tau\sigma^2_{d\mid s_i,P}[a+b(1-2\lambda\beta_s-\sigma_\varepsilon\lambda\sqrt{\rho}\beta_s^Y)]},$$

which is the weight that each informed manager places on his or her signal.[10] If I consider a passive benchmark such as that of Admati and Pfleiderer (1997), it is easy to show that the weight on the signal is $G=\beta_s/(\tau\sigma^2_{d\mid s_i,P}(a+b))$. Thus, each manager places less weight on the private signal if I compare it with the baseline case with b =0, which decreases the informativeness of the price. However, in the case with a stochastic benchmark, additional terms $b(1-2\lambda\beta_s-\sigma_\varepsilon\lambda\sqrt{\rho}\beta_s^Y)$ appear in the denominator because managers √ use their information to forecast the conditional covariance. If $1-2\lambda\beta_s-\sigma_\varepsilon\lambda\sqrt{\rho}\beta_s^Y<0$, each manager places more weight on the private signal. Because all of the information comes from informed managers, I

[10] Note that G =1/C, so G is always positive for C >0.

conjecture that the price is more informative if each manager places more weight on the private signal. I denote I_{NRP} as the price informativeness in the economy without relative performance for comparison purposes. Applying the projection theorem to $1-2\lambda\beta_s-\sigma_\varepsilon\lambda\sqrt{\rho}\beta_s^Y$, I can conjecture as follows:

$$\text{If} \quad \lambda^2(1-\rho)\sigma_\varepsilon^2(\sigma_d^2+\rho\sigma_\varepsilon^2) + C^2[(1-2\lambda)\sigma_d^2+(1-\lambda\rho)\sigma_\varepsilon^2]\sigma_u^2 \quad < \quad 0, \quad I > I_{\mathrm{NRP}}, \tag{19}$$

$$\text{If} \quad \lambda^2(1-\rho)\sigma_\varepsilon^2(\sigma_d^2+\rho\sigma_\varepsilon^2) + C^2[(1-2\lambda)\sigma_d^2+(1-\lambda\rho)\sigma_\varepsilon^2]\sigma_u^2 \quad > \quad 0, \quad I < I_{\mathrm{NRP}}.^{11} \tag{20}$$

Intuitively, a relative performance contract induces a manager to reduce the tracking errors between his or her performance and the benchmark. However, the benchmark is the average performance of all informed managers, which is uncertain. Thus, each manager needs to predict the conditional covariance based on his or her private signal and the price. Consequently, the effect of hedging depends on which piece of information, the manager's private signal or the price, is the best predictor. When the price is a more precise predictor of the average performance than the manager's private signal, managers place more weight on the price and less weight on the signal. As a result, the price aggregates less information and be-comes less informative. By contrast, when the manager's private signal is a more precise predictor than the price, managers place less weight on the signal. In this case, the price aggregates more information and becomes more informative.

My article thus differs from the previous literature by showing that prices can actually be more informative. For this reason, I more closely examine the case with a more informative price. Going back to the condition in inequality (17), it is more likely to be true when $a\tau\sigma_u$ is large. A large value of aτ essentially means that the managers are very risk averse and that they trade very conservatively by using private signals. In this case, the price aggregates less information and is likely to be a bad predictor of the covariance. Thus, managers are more likely to weigh their signals more heavily. A large value of σ_u means the price is very noisy and that it therefore is likely a poor predictor of the covariance. Thus, managers are more likely to weigh their signals more heavily.

Moreover, when ρ =1, the condition in inequality (17) never holds. Because this condition arises when informed managers observe a common signal, I can conclude that the increased informativeness of the price is caused by informed managers' behavior aimed at hedging the additional uncertainty.[12]12 When ρ =0, the condition in inequality (17) crucially depends on the relative values of σ_d^2 and σ_ε^2, which determine the informativeness of the signal. For a general ρ (0<ρ <1), its effect on the condition in inequality (17) is not monotonic. On the one hand, informed managers can better forecast the average performance based on higher ρ, which can make the condition in inequality (17) hold more easily. On the other hand, higher ρ decreases the weight that each manager places on the signal directly,[13] which has the opposite effect on the condition in inequality (17). When relative performance concerns exist, d I /dρ is not necessarily negative because a higher ρ can cause managers to better forecast the average performance.

[11] Those conjectures are rigorously confirmed in Proposition 3.

[12] Note that when ρ =1, the benchmark is not stochastic with respect to the informed managers. I discuss this situation in Section IV.B.

[13]When $b=0$, each manager places the weight $\beta_s/(\tau\sigma^2_{d|s_i,P})$ on the signal. From the projection
13 theorem, $\beta_s/(\tau\sigma^2_{d|s_i,P})=C^2\sigma_u^2/(a\tau[\lambda^2(1+\rho)\rho\sigma_\varepsilon^4+C^2\sigma_\varepsilon^2\sigma_u^2])$. Thus, the weight decreases with ρ.

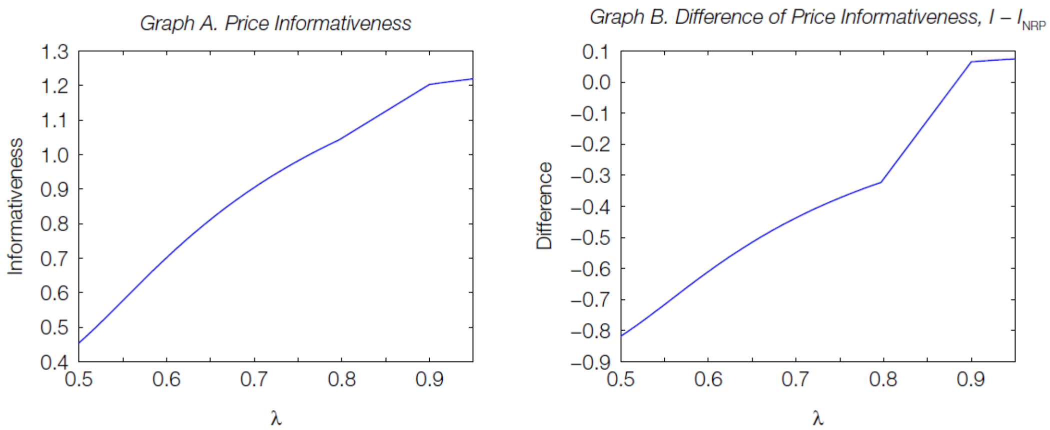
The effect of λ on the condition in inequality (17) is also not monotonic. Increasing λ causes the price to aggregate more information and to become more informative, which induces informed managers to weigh the signal less heavily. On the one hand, the condition in inequality (17) is easier to satisfy if the price is more informative. On the other hand, this condition is more difficult to satisfy if informed managers place less weight on their signal. Thus, the overall effect depends on the interaction between the two effects. For the purpose of illustration, I examine a numerical example.

Figure 1 shows the numerical example. Graph A shows how the price in-formativeness I changes with the fraction of informed managers, and Graph B shows the difference in price informativeness between the cases with and without relative performance (I − I_{NRP}). For Graph B, when the difference is negative, it indicates that relative performance leads the price to be less informative; when the difference is positive, it indicates a more informative price with the presence of relative performance.

FIGURE 1

Price Informativeness

Graphs A and B in Figure 1 present the price informativeness I in equilibrium. Graph A plots I as a function of λ. Graph B plots the difference in price informativeness between the cases with and without relative performance ($I - I_{\text{NRP}}$). Parameter values are as follows: $a=0.01$, $b=1$, $\sigma_d=2$, $\sigma_u=1$, $\sigma_\varepsilon=1$, $\tau=5$. I choose the values of λ and ρ such that the values in the Graph B can be both positive ($I > I_{\text{NRP}}$) and negative ($I < I_{\text{NRP}}$). When the values of the difference in Graph B are negative, the parameter values are $\lambda \in [0.5, 0.8]$ and $\rho=0.7$. When the values of the difference in Graph B are positive, the parameter values are $\lambda \in [0.9, 0.95]$ and $\rho=0.9$.



C. Comparative Statics

In this subsection, I generate comparative statics regarding the price informativeness for the different model parameters. For the purpose of comparison, I first present the comparative statics results in a benchmark economy without relative performance (b =0). The following lemma shows the comparative statics on the price informativeness I_{NRP} for the benchmark economy without relative performance:

Lemma 3. For the economy without relative performance, I show that $dI_{\text{NRP}}/d\lambda > 0$, $dI_{\text{NRP}}/d\sigma_\varepsilon^2 < 0$, $dI_{\text{NRP}}/d\sigma_u^2 < 0$, and $dI_{\text{NRP}}/d\rho < 0$.

Proof. The proof is in the Appendix.

For the comparative statics analysis in the benchmark economy, I consider how the price informativeness changes with respect to λ, σ_ε^2, σ_u^2 and ρ. Intuitively, more informative managers and more precise signals lead the price to be more informative, such that the price informativeness increases with λ and decreases with σ_ε^2. The price informativeness decreases with ρ because a higher ρ decreases the weight that each manager places on his or her signal. σ_u^2 has two effects on price informativeness. First, a higher σ_u^2 causes the price to be noisier, which causes managers to place more weight on the signal. Second, although the price aggregates more information, a higher σ_u^2 causes the price to be noisy. However, because the second effect is dominant, the price informativeness decreases with σ_u^2.

In addition to the parameters analyzed in Lemma 3, I also conduct a comparative statics analysis on the contract parameters a and b for the economy with relative performance. The following proposition summarizes the main results:

Proposition 3. For the equilibrium in Proposition 1, $dI/d\lambda > 0$, $dI/d\sigma_\varepsilon^2 < 0$, $dI/d\sigma_u^2 < 0$, $dI/da < 0$, $dI/db > 0$ if the condition in inequality (19) is true, and d I /db <0 if the condition in inequality (20) is true.

Proof. The proof is in the Appendix.

Compared with the benchmark economy, I have the same qualitative comparative statics results forλ, σ_ε^2, and σ_u^2 change the manner in which each manager forecasts the average performance; however, in general, a higher λ, a lower σ_ε^2, and a lower σ_u^2 cause the price to be more informative. Furthermore, although I do not present any comparative statics results for ρ in the proposition, the Appendix shows that the price informativeness can increase or decrease with ρ, which differs from the benchmark economy. The difference arises from the weight that each manager places on his or her private signal. When b =0, a higher ρ increases $\sigma_{d|s_i,P}^2$, which causes managers to place less weight on their signals. Given the concerns regarding relative performance, a higher ρ can cause managers to better forecast average performance, which causes each manager to place greater weight on his or her signal. As a result, d I /dρ is not necessarily negative.

I also analyze the effect that contract parameters a and b have on price informativeness. The sign of d I /db depends on the conditions in inequalities (19) and (20), which verifies my conjecture from the previous subsection. In the model, parameter a is always associated with the risk-aversion parameter τ , such that in-creasing a effectively induces managers to be more risk averse and to trade less aggressively on their information. As a result, the price informativeness decreases with a.

4. Special Information Structures

To thoroughly understand the model, I consider three special but familiar information structures in this section. First, I consider a baseline model with symmetric information, in which all managers receive a common signal (or identical signals). I regard this model as the baseline case. Second, I solve a model with asymmetric information, in which only some managers observe a common signal, and some do not. Third, I consider a model in which all managers receive “equally accurate” but independent signals.

A. The Baseline Case: The Model with Symmetric Information

This subsection presents the baseline model with symmetric information. In contrast to the general model, λ=1 and ρ =1 in this case. Solving the model, I find that each manager’s optimal demand is $x_i = [\mathrm{E}(d \mid s) - P]/(a\tau\sigma_{d|s}^2)$,[14] which has nothing to do with the relative performance parameter b. Given the same in-formation set, all managers submit the same demand and thus achieve the same performance, so this economy is the same as the benchmark economy without relative performance.

This part of the analysis is very similar to that in Kapur and Timmermann (2005); however, the setting is simpler. In addition to symmetric information, the

[14] The model is almost the same as the standard mean–variance problem and is easy to solve.

compensation in my model is linear. Note that in Kaniel and Kondor (2013), the incentive contract matters even when all managers have the same information. However, the incentive contract in their article is convex, which is motivated by fund flows. The compensation in my article is motivated by the "fulcrum" com-pensation and is hence linear.

B. Asymmetric Information

This subsection analyzes the special case when ρ =1, indicating that all in-formed managers observe a common signal (or identical signals). Therefore, the information set is F_I ={s}.[15] Uninformed managers observe the price and have the information set F_u ={P}. Because there is only one signal, the price performs the role of transmitting information.[16]

1. Optimal Trading Strategies and Equilibrium

The equilibrium definition is the same as Definition 1 but with different information sets. The following corollary outlines the results:

Corollary 1. The optimal demands of both informed and uninformed managers are as follows:

$$x_I = \frac{a}{a+b(1-\lambda)} \underbrace{\frac{\mathrm{E}(d\,|\,s)-P}{a\tau\sigma^2_{d|s}}}_{\text{Demand without relative performance}} + \frac{b(1-\lambda)}{a+b(1-\lambda)} x_u, \tag{21}$$

$$x_u = K_1 \frac{\mathrm{E}(d\,|\,P)-P}{\tau\sigma^2_{d|P}}, \tag{22}$$

where $K_1 = [a+b(1-\lambda)]/[a(a+b)] + [b\lambda/a(a+b)][(2\sigma^2_{d|P} - \sigma^2_{d|s})/\sigma^2_{d|s}]$. Given the optimal demands, there exists a noisy REE with the equilibrium price

$$P = A + B[(s-\bar{d}) - Cu],^{17} \tag{23}$$

for three constants A, B, and C.

Proof. The proof is in the Appendix.

In this economy, informed managers have no additional uncertainty in their compensation, but their demand is affected by the existence of relative performance. Consequently, the informed demand is a weighted average of the demand without relative performance and the uninformed demand. Informed managers place some weight on the uninformed demand and less weight on the signal. That is, the informed demand depends less on private information without relative performance than with relative performance. Note that even without any additional uncertainty, relative performance causes informed managers to trade less aggressively on their signals. This result holds as long as there are some uninformed managers.

Uninformed managers need to hedge the uncertainty generated by informed managers' performance, and this hedging behavior induces them to change their level of trading aggressiveness. It is easy to see that K1 >[[a +b(1−λ)]/a(a +b)]+[bλ/a(a +b)]=1/a. Because $(1/a)[\mathrm{E}(d\,|\,P)-P]/(\tau\sigma^2_{d|P})$ is the demand without relative performance, the uninformed managers trade more aggressively. Similar to the results in Lemma 1, the uninformed man-agers forecast the average performance based on the price, and they submit a hedging demand, which takes the form of the conditional

[15] Informed managers can also observe the price. However, the price is merely the sum of the noise and the signal and is therefore redundant to the signal.

[16] The mechanism is the same as that in the Grossman and Stiglitz (1980) model.

[17] With a little abuse of notations, I use the same parameters A, B, and C for all of the information structures.

covariance $\text{cov}[d,(d-P)\lambda x_I \mid P]$. From equation (21), the uncertain part for uninformed managers is the conditional expectation E(d |s), so I am effectively calculating $\text{cov}\{d,(d-P)[[\text{E}(d \mid s)-P]/\sigma^2_{d|s}] \mid P\}$, which can be simplified as $[(2\sigma^2_{d|P}-\sigma^2_{d|s})/\sigma^2_{d|s}][\text{E}(d \mid P)-P]$. Thus, I obtain the more aggressive coefficient K_1 for the optimal demand.

Intuitively, when the price increases, uninformed managers interpret this increase as good news and speculate that informed managers will buy some assets. However, they also know that informed managers have better information than they do, such that, on average, uninformed managers lose in terms of relative performance if the good news is actually realized. To minimize their losses, uninformed managers buy more assets than they would without relative performance. Similar logic applies when the price decreases.

When informed managers observe a good signal, they submit a buying order. When noise traders[18] submit a buying order or a small selling order (smaller than that of informed managers), uninformed managers have to sell the asset to clear the market. In this case, uninformed managers trade against information. For this reason, uninformed managers trade "correctly" with the information only when noise traders submit a large selling order. Suppose that uninformed managers submit a buying order when the signal is good. Informed managers trade less aggressively on their information in the presence of relative performance, which means that their buying order is smaller. Consequently, uninformed man-agers have to submit a larger buying order to clear the market, indicating that they trade more aggressively on the information extracted from the price.

Although informative trading is amplified by relative performance, uninformed managers cannot learn much because the price contains less information. Therefore, the Walrasian effect still dominates the information effect, engendering a more price-elastic demand. Moreover, as the uninformed managers trade more aggressively, the informed managers provide less informative trading signals.

2. Price Informativeness

To understand the effects on the price, I analyze the price informativeness by conducting a comparative statics analysis. I first study how different parameters change with respect to the relative performance parameter, b. Corollary 2 summarizes the results as follows:

Corollary 2. For the case with asymmetric information where ρ =1, I show that $\partial B/\partial b<0$, $\partial C/\partial b>0$, and $\partial K_1/\partial b>0$.

Proof. The proof is in the Appendix.

Inequality $\partial B/\partial b<0$ shows that as the importance of relative performance increases, the price becomes less responsive to information. Inequality $\partial C/\partial b>0$ shows that as the importance of relative performance increases, the significance of the effect of noise shocks on the price relative to information increases. Moreover, inequality $\partial K_1/\partial b>0$ shows that when relative performance is more important, uninformed managers trade more aggressively.

The model without relative performance solves the problem when b =0. The information structure in this section is the same as that in Grossman and Stiglitz (1980), in which managers maximize their utility over aW_1. Corollary 1 shows how B and C change with respect to b. If I denote B_{NRP} and C_{NRP} as the price parameters when there is no relative performance, I can immediately obtain B < B_{NRP}, C >C_{NRP}. B measures the responsiveness of the price to information, and C measures the

[18] Here, I interpret the negative noise supply as the demand of noise traders.

relative effect of information and noise shocks on the price. Thus, the price is less sensitive to managers' private signal and more affected by noise in an economy with relative performance than in the benchmark economy without relative performance. This result is observed because relative performance causes informed managers to trade less aggressively on their private information.

From the projection theorem, I can obtain the informativeness of the price

$$I = \frac{1}{\sigma^2_{d|P}} = \frac{\sigma_d^2+\sigma_\varepsilon^2+C^2\sigma_u^2}{\sigma_d^2(\sigma_\varepsilon^2+C^2\sigma_u^2)}, \tag{24}$$

which has a negative relationship with C. For this reason, the informativeness of the price is lower with relative performance than without relative performance. From the inequality ∂C/∂b >0, I show that the price informativeness decreases with b.

C. Differential Information

This section examines the differential information model,[19] in which all man-agers are equally informed with independent signals. In contrast to the full model, λ=1 and ρ =0 in this section. Because there are no uninformed managers, the informed demand is not affected by uninformed managers. Because ρ =0, a fund manager cannot learn anything from the signals of other managers. Therefore, the existence of relative performance affects the hedging behavior of only informed managers.

1. Optimal Trading Strategies and Equilibrium

The expression of the optimal demand in terms of the conditional covariance is almost the same as equation (5) except λ=1, and there exists a noisy REE with the equilibrium price

$$P = A+B[(d-\overline{d})-Cu], \tag{25}$$

where A, B, and C are three constants. In the Appendix, I show that A and B are functions of C, and C is the positive root of the cubic equation. Using a procedure similar to that in Section III, I summarize the results in the following corollary:

Corollary 3. The optimal demand is

$$x_i = \underbrace{\frac{\mathrm{E}(d\,|\,s_i,P)-P}{a\tau\sigma^2_{d|s_i,P}}}_{\text{Demand without relative performance}} + \underbrace{\frac{b\beta_s}{a[a+b(1-2\beta_s)]}\,\frac{2\mathrm{E}(d\,|\,s_i,P)-P-s_i}{\tau\sigma^2_{d|s_i,P}}}_{\text{Hedging demand}}. \tag{26}$$

Given the optimal demand in equation (26), the equilibrium exists. Define $Q_1 = a(\sigma_\varepsilon^2+\sigma_d^2)+b(\sigma_\varepsilon^2-\sigma_d^2)$, and the equilibrium is unique if $Q_1 < (\sigma_d/(\sigma_u\sigma_\varepsilon\tau))\sqrt{3(\sigma_\varepsilon^2+\sigma_d^2)}$ or $Q_1 > \sigma_d^2/[3(a+b)\tau^2\sigma_\varepsilon^2\sigma_u^2]$.

Proof. The proof is in the Appendix.

The optimal demand is the sum of the demand without relative performance and the hedging demand. Comparing this equation with equation (8) in Lemma 2, I show that there is no effect from hedging by uninformed managers. Therefore, the first term is simply the demand without relative performance, and the second term is the hedging demand for the average performance of all informed managers.

Note that I decompose the optimal demand in equation (26) into the "demand without relative performance" and the "hedging demand." Using the projection theorem, I show that the weight on the signal for the "demand without relative

[19] In the literature, this type of model is used to study how prices aggregate information (e.g., Grossman (1976), Diamond and Verrecchia (1981)).

performance" is $\beta_s/(a\tau\sigma^2_{d|s_i,P})$ and is $b\beta_s(2\beta_s-1)/(a\tau\sigma^2_{d|s_i,P}[a+b(1-2\beta_s)])$ for the hedging demand. From the expression of β_s and $\sigma^2_{d|s_i,P}$, I show that $\beta_s/(a\tau\sigma^2_{d|s_i,P})=1/(a\tau\sigma^2_\varepsilon)$, which is unrelated to relative performance. Thus, the effect on the price informativeness arises solely from the hedging demand. To be specific, it depends on $(2\beta_s-1)/(a+b(1-2\beta_s))$. I can show that $a+b(1-2\beta_s)$ is greater than 0, so whether hedging demand increases or decreases, the price informativeness depends on whether $\beta_s > ½$.

The equilibrium is unique when either $Q_1<(\sigma_d/\sigma_u\sigma_\varepsilon\tau)\sqrt{3(\sigma^2_\varepsilon+\sigma^2_d)}$ or $Q_1>\sigma^2_d/(3(a+b)\tau^2\sigma^2_\varepsilon\sigma^2_u)$ holds. Similar to the analysis of the full model, the simple sufficient condition is identified as $\sigma^2_d/(\sigma^2_\varepsilon+\sigma^2_d)>(a+b)/(9(a-b))$.[20] Accordingly, the equilibrium is unique when b >a, implying that relative performance is somewhat important. When b <a, the equilibrium is unique when the signal is sufficiently precise.

2. Price Informativeness

As in the analysis in Section III, the optimal demand takes the form $x^I_i= Gs_i+HP+F$, and I can derive

$$G = \frac{\beta_s}{\tau\sigma^2_{d|s_i,P}[a+b(1-2\beta_s)]}.$$

Compared with the baseline case with b =0, in this case, each manager places more weight on his or her private signal when $1-2\beta_s<0$, which is a special case of inequality (19), and β_s is a function of C. By implicitly differentiating the cubic equation (equation (A-13) in the Appendix) with respect to C, I can derive the following corollary:

Corollary 4. In equilibrium, the price is more informative in the presence of relative performance if

$$\sigma^2_d > \sigma^2_\varepsilon \quad \text{and} \quad a\tau\sigma_u\frac{\sigma_\varepsilon}{\sigma_d}\sqrt{\sigma^2_d-\sigma^2_\varepsilon} > 1,$$

and is less informative otherwise.

If I set λ=1 and ρ =0, inequality (17) in Proposition 2 becomes

$$a\tau\sigma_u\frac{\sigma_\varepsilon}{\sigma_d}\sqrt{\sigma^2_d-\sigma^2_\varepsilon} > 1$$

in Corollary 4. Thus, the price informativeness can be analyzed in a simplified information structure. Note that relative performance causes the price to be more informative only if the signal is not too noisy $(\sigma^2_d>\sigma^2_\varepsilon)$. With such a signal, the condition $a\tau\sigma_u(\sigma_\varepsilon/\sigma_d)\sqrt{\sigma^2_d-\sigma^2_\varepsilon}>1$ is equivalent to $((1/\sigma^2_\varepsilon)-(1/\sigma^2_d))\sigma^2_uC^2>1$. Thus, the condition is likely to be true when $\sigma^2_uC^2$ is large or when $(1/\sigma^2_\varepsilon)-(1/\sigma^2_d)$ is large. $\sigma^2_uC^2$ measures the noisiness of the price; hence, a large $\sigma^2_uC^2$ indicates that the price is not very precise. When $(1/\sigma^2_\varepsilon)-(1/\sigma^2_d)$ is large, the signal is relatively good. Thus, the condition indicates that each manager places more weight on the signal when the price is less precise or when the signal is relatively good. As a consequence, the price aggregates more information and therefore provides more information.

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[20] It is determined that the sufficient condition for $Q_1<(\sigma_d/\sigma_u\sigma_\varepsilon\tau)\sqrt{3(\sigma^2_\varepsilon+\sigma^2_d)}$ is $\sigma_u\sigma_\varepsilon<(\sqrt{3}/\tau(a+b))\sqrt{\sigma^2_d/(\sigma^2_\varepsilon+\sigma^2_d)}$, and the sufficient condition for $Q_1>\sigma^2_d/(3(a+b)\tau^2\sigma^2_\varepsilon\sigma^2_u)$ is $\sigma_\varepsilon\sigma_u>$
20 $(1/\tau)\sqrt{1/3(a^2-b^2)}$. Thus, the simple sufficient condition can then be derived.

The intuition of the term $a\tau\sigma_u$ is similar to the full model; however, I now need to analyze $(\sigma_\varepsilon/\sigma_d)\sqrt{\sigma_d^2-\sigma_\varepsilon^2}$. If $\sqrt{\sigma_d^2-\sigma_\varepsilon^2}$ is very large, the signal is very precise. Then, each manager weighs his or her signal more heavily, so the price aggregates more information and becomes more precise. However, as the precision of the price increases, the price may become a more precise predictor than the managers' signals, and the managers m weigh the price more heavily instead. For this reason, a large value of $(\sigma_\varepsilon/\sigma_d)\sqrt{\sigma_d^2-\sigma_\varepsilon^2}$ indicates that the signal should be relatively precise but not "too good."

I also calculate the parameter B for the equilibrium price, which measures the sensitivity of the price to the aggregated information. B is a function of C, and it is easy to show that d B/dC <0. If I denote B_{NRP} as the parameter B in the economy without relative performance, I can easily obtain the following results:

- If $a\tau\sigma_u(\sigma_\varepsilon/\sigma_d)\sqrt{\sigma_d^2-\sigma_\varepsilon^2}>1$, $B>B_{\text{NRP}}$.
- If $a\tau\sigma_u(\sigma_\varepsilon/\sigma_d)\sqrt{\sigma_d^2-\sigma_\varepsilon^2}<1$ or $\sigma_d^2<\sigma_\varepsilon^2$, $B<B_{\text{NRP}}$.

Thus, when relative performance causes managers to weigh their signals more heavily, the price is more sensitive to aggregate information and vice versa. Moreover, $d\sigma^2_{d|s_i,P}/C>0$, so $\sigma^2_{d|s_i,P}$ can be analyzed directly by following the analysis of C.

D. Public and Private Signals

This subsection extends the framework of Section IV.C by allowing managers to observe two signals, $s_i=d+\varepsilon_i$ and $s_c=d+\varepsilon_c$, where s_c is the public signal that is observable to all managers, and s_i is the private signal, which is observable to manager i only. $\varepsilon_i\sim\mathrm{N}(0,\sigma_\varepsilon^2)$; and $\varepsilon_c\sim\mathrm{N}(0,\sigma_c^2)$, ε_i and ε_c are independent.

In Section IV.C, the public information is represented by the price, which is endogenously determined. The price plays two roles: It i) signals the average performance of all managers and ii) aggregates information. Therefore, the effect of relative performance on public information is unclear. To separate the two roles, I introduce an additional exogenous public signal and study the effect of relative performance on the public signal. The following corollary presents the optimal demand and the price:

Corollary 5. The optimal demand is

$$x_i = \frac{1}{a}\frac{\mathrm{E}(d\,|\,s_i,s_c,P)-P}{\tau\sigma^2_{d|s_i,s_c,P}} + \frac{b\beta_s}{a[a+b(1-2\beta_s)]}\frac{2\mathrm{E}(d\,|\,s_i,s_c,P)-P-s_i}{\tau\sigma^2_{d|s_i,s_c,P}}. \quad (27)$$

The equilibrium price has the following form:

$$P = A+B[(d-\bar{d})+D(s_c-\bar{d})-Cu], \quad (28)$$

where A, B, and C have the same expressions as in Corollary 3, and D =(1+ $(b/a))(\beta_c/\beta_s)$.[21]

Proof. Given this expression of the optimal demand, the market-clearing condition $\int_0^1 x_i=S+u$ becomes an affine function of d, $(s_c-\bar{d})$ and u. By plugging the expression of price into the previous equation, I can find the parameters A, B, C, and D. A, B, and C have the same forms as in the case of differential information. Setting the coefficient of $(s_c-\bar{d})$ to 0, I have $D=-\beta_c(a+b)/$

[21] β_c and β_s are parameters from the projection theorem.

$((a+b)\beta_p B-[a+b(1-\beta_s)]B)$. Because $B=(a\beta_s+(a+b)\beta_p B)/[a+b(1-\beta_s)]$, I have $D=(1+b/a)(\beta_c/\beta_s)$.

The price continues to perform the role of aggregating information, and the effects of hedging are the same as those in Section IV.C. There is a new parameter, D, which measures the relative effects of the public information and the aggregated private information on the price. If I consider a special case where $\sigma_c^2=\sigma_\varepsilon^2$, I have $\beta_c/\beta_s=1$.[22] Thus, from the optimal demand in equation (27), it is easy to see that relative performance increases the sensitivity of the price to the public signal, and when b/a increases, the price becomes more responsive to public information.

Using the projection theorem, $\mathrm{E}(d\,|\,s_i,s_c,P)=\bar{d}+\beta_s(s_i-\bar{d})+\beta_c(s_c-\bar{d})+\beta_p(P-A)$, and the optimal demand in equation (27), I show that each manager weighs the public signal more than the private signal. Thus, on the aggregate level, the price is more sensitive to public information than to private information. Intuitively, placing more weight on the private signal makes the price aggregate more information, enabling other managers to learn more private information from the price. Without relative performance, the public signal and the aggregated private signals have the same effect on the price (when $\sigma_c^2=\sigma_\varepsilon^2$). With relative performance, however, the price is more responsive to the public signal than to the private signal.

Generally, if the public signal does not play the role of aggregating information, relative performance causes managers to place more weight on the public signal and less weight on the private signals. However, if the public signal (price) aggregates information, placing less weight on the private signals decreases the information aggregation. For this reason, if the information content of the public signal (price) is endogenous in the economy, relative performance does not necessarily cause managers to place more weight on the public signal.

E. Discussion of Equilibrium

Although this article focuses on price informativeness, it is nonetheless interesting to examine the implications of relative performance on other equilibrium quantities, such as the expected return and price volatility. In the literature, when studies consider relative performance with a passive benchmark or an index (e.g., Basak and Pavlova (2013), Cuoco and Kaniel (2011)), a common result is that managers tend to increase the demand of assets within the benchmark and thus increase asset prices and volatilities. Kapur and Timmermann (2005) find that relative performance may lower the equity premium. In my model, the most important information structures are asymmetric information in Section IV.B and differential information in Section IV.C. Therefore, I discuss the effects on the equilibrium for each information structure. To analyze the equilibrium, I first define the expected returns and the asset volatility as $\mathrm{E}(r)=\bar{d}-\mathrm{E}(P)$ and $\sigma_P^2=\mathrm{var}(P)$. For ease of comparison, I use the case without relative performance (b =0) as the baseline case for each information structure.

Section IV.B analyzes the case in the presence of the uninformed managers, and the price takes the form in equation (23), so I have $\mathrm{E}(r)=\bar{d}-A$ and $\sigma_P^2=B^2[(\sigma_d^2+\sigma_\varepsilon^2)+C^2\sigma_u^2]$. In this case, the expected return is likely to be lower than that in the baseline case when σ_ε^2 and σ_u^2 are high. On the one hand, the informed managers trade less informatively in the presence of relative performance

[22] I consider this special case because it has the special property that $\beta_c/\beta_s=1$. With this property, I do not have to calculate the entire equilibrium. Otherwise, β_c and β_s are functions of the equilibrium price parameters that need to be solved. Moreover, with this assumption, managers place the same weight on the private and public signals, which serves as a natural benchmark.

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concerns, which diminishes the informativeness of the price. For uninformed managers, a less informative price carries more uncertainty, so they will accept a lower price or require a higher risk premium. On the other hand, uninformed managers trade more aggressively, which increases the price. If I examine the expression of A, the first effect is illustrated primarily by a higher $\sigma^2_{d|P}$ in comparison with the baseline case, and the second effect is illustrated primarily by a higher K_1. Although the way in which the expected return changes depends on the interaction of the two effects, the second effect likely dominates the first when σ^2_{ε} and σ^2_{u} are high. Relative performance has two effects on volatility: The first arises from the signal, and the second arises from the noise. Because the price is less sensitive to the signal, the volatility that arises from the signal is lower. Effectively, the effect of informed managers in this economy is lower in the presence of relative performance; thus, the effect of the noise on the price is stronger, and the volatility from the noise is larger. When σ^2_{ε} and σ^2_{u} are high, the first effect is likely to be strong, and the second effect is likely to be strong when σ^2_{u} is high.

From the results in Section IV.C, I have $\mathrm{E}(r)=\overline{d}-A$ and $\sigma^2_P=B^2\sigma^2_d+B^2C^2\sigma^2_u$, and the baseline case is the economy without uninformed managers (λ=1) and relative performance (b =0). The expected return is lower when dC/db <0 or when the price is more informative. On the one hand, a more informative price decreases the conditional volatility $\sigma^2_{d|s_i,P}$, which decreases the risk premium and increases the price. On the other hand, if I do not consider any information effect, I can show, from the optimal demand in equation (26), that the managers' hedging behavior effectively increases the total demand,[23] which also increases the price. Thus, the two effects together lead to a higher price and a lower expected return. When dC/db >0 or when the price is less informative, the two effects are opposite;[24] hence, the way in which expected return changes depends on which effect is dominant. The volatility also consists of two parts, arising from the aggregate signals and the noise supply. The effects of relative performance on the aggregate signals (B) and the noise supply (C) are opposite. When σ^2_{d} is high, the effect on aggregate signals is likely to be strong, whereas when σ^2_{u} is high, the effect on the noise supply is likely to be strong. The overall effect also depends on whether relative performance increases or decreases the informativeness of the price.

5. Convex Compensation

In the context of the mutual fund industry, the compensation in equation (1) may have a different interpretation. I can conceptualize $aW_{1,i}$ as the fixed proportion of assets under management and $b(W_{1,i}-\overline{W})$ as the fund inflows or outflows depending on the fund performance. Therefore, even when the fee structure is flat (a fixed proportion of assets under management), relative performance concerns can still stem from fund flows. Moreover, empirical evidence shows that the flow–performance relationship is actually convex (Chevalier and Ellison (1997)). Thus, it would be interesting to analyze the implications of the convex contract under the REE framework.

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[23] [23]Note that $b\beta_s/(a[a+b(1-2\beta_s)])$ is positive, so the total demand is actually increased if I do not consider any information effect.

[24] When d B/db >(<) 0, dC/db <(>) 0.

To consider a convex contract, I follow a payoff structure similar to that of Cuoco and Kaniel (2011) and rewrite the compensation (1) as

$$\begin{aligned} F_{1,i} &= I + aW_{1,i} + b_1(W_{1,i} - \overline{W})\mathbf{1}_{\{\text{good market}\}} \\ &\quad + b_2(W_{1,i} - \overline{W})\mathbf{1}_{\{\text{bad market}\}}, \end{aligned} \tag{29}$$

where $b_1 > b_2$, and $\mathbf{1}_{\{\text{condition}\}}$ is the indicator function, which takes the value 1 if the condition is true. Thus, the compensation in equation (29) captures the idea that the fund inflows are very high in a good market but that the fund outflows are not so high in bad times.

Solving the full REE model with the convex payoff requires the inference of a truncated, multidimensional normal distribution, which makes the solution messy. Instead, I will consider the special information structure and conduct the numerical analysis based on some simplified assumptions. For comparison purposes, I set $b_1 = b$ and $b_2 = 0$. Thus, the convex contract that I am considering here is an option contract. When the market is good, the option part (or relative performance) of the contract pays off. When the market is bad, however, the option part is 0. Moreover, I consider an asymmetric-information setting (Grossman and Stiglitz (1980)) and analyze informed and uninformed managers' behavior. In other words, I consider an information structure where all informed managers observe a common signal ($\rho = 1$).

For this information structure, with a convex contract, the informed managers submit an optimal demand conditional on the private signal s. For simplicity, I consider $d - P > 0$ as a good market and $d - P < 0$ as a bad market. Thus, informed managers need to know whether $E(d \mid s) - P$ is greater than 0 based on their private signals. I solve the optimal demand in the following proposition:

Proposition 4. Consider a special information structure where $\rho = 1$. With a convex compensation, informed manager demand is

$$X_I = x_{I1}\mathbf{1}_{\{s>k\}} + x_{I2}\mathbf{1}_{\{s<k\}}, \tag{30}$$

where

$$\begin{aligned} x_{I1} &= \frac{[E(d \mid s) - P]}{[a + b(1-\lambda)]\tau\sigma^2_{d|s}} + \frac{b(1-\lambda)x_u}{[a + b(1-\lambda)]}, \\ x_{I2} &= \frac{[E(d \mid s) - P]}{a\tau\sigma^2_{d|s}}, \quad \text{and} \\ k &= \frac{(\sigma_d^2 + \sigma_\varepsilon^2)P}{\sigma_d^2} - \frac{\sigma_\varepsilon^2 \overline{d}}{\sigma_d^2}. \end{aligned}$$

Moreover, if I consider a fictitious economy with only informed managers, and the asset supply is $S^* = S + u - (1-\lambda)x_u$, there exists a unique equilibrium price

$$P^* = P_1\mathbf{1}_{\{s>k\}} + P_2\mathbf{1}_{\{s<k\}}, \tag{31}$$

where P1 and P2 are shown in the Appendix.

Proof. The proof is in the Appendix.

In this economy, informed managers submit different demands, which are conditional on the realizations of their signals. When the relative performance is above a certain threshold, the contract is effectively linear, and the demand x_{I1} is the same as that in Section III. When the relative performance is below the benchmark, it is 0, and the demand is x_{I2}, which is the same as that without relative performance.

From equation (30), I show that informed managers place weight on the demand of uninformed managers only when the signal is good. If the signal is bad, they focus on their private information. Thus, in the case with a convex contract, informed managers trade more informatively than in the case with a linear relative performance contract but less informatively than in the case without relative performance. Uninformed managers effectively make an inference from a truncated normal distribution. I follow the procedure of Yuan (2005) and assume a fictitious economy in which only informed managers trade. In this economy, the market-clearing condition is $\lambda[x_{I1}\mathbf{1}_{\{s>k\}}+x_{I2}\mathbf{1}_{\{s<k\}}]=S^{*}$, and then I can derive the fictitious equilibrium price in equation (31).

Note that P∗ and the true equilibrium price P are equivalent sufficient statistics for d in the Blackwell sense (Yuan (2005)). Thus, uninformed managers must infer d from P∗, and the conditional moments are

$$\begin{aligned}\mathrm{E}(d\mid P,x_u) &= \Pr(s>k)E_1+\Pr(s<k)E_2,\\ \mathrm{var}(d\mid P,x_u) &= \Pr(s>k)V_1+\Pr(s<k)V_2,\end{aligned}$$

where E_1, E_2, V_1, and V_2 are conditional moments of d from the truncated normal distribution, which are shown in the Appendix.

In addition to the standard inference problem presented by Yuan (2005), the uninformed managers are also subject to a convex contract. If E(d | P)− P is smaller than 0, there is no relative performance for the contract, and the problem is similar to that of Yuan (2005). However, if E(d | P)− P >0, the uninformed managers have relative performance concerns and must therefore adjust their demand. They need to extract information about informed managers' signal and determine when the informed managers' option contract is in the money. In other words, they need to infer when s >k. I further denote ES_1 and ES_2 as conditional means of the signal s from the truncated normal distribution, which are defined in the Appendix. Then, the inference condition for s >k is $\Pr(s>k)\mathrm{ES}_1+\Pr(s<k)\mathrm{ES}_2>k$.

Furthermore, I need to determine the demand of uninformed managers $x_u=x_u^{*}$, and optimal demand becomes a fixed-point problem, which is shown in the following proposition:

Proposition 5. The demand for uninformed managers is a unique fixed point of the following equation:

$$x_u = K_2\frac{\Pr(s>k)E_1+\Pr(s<k)E_2-P}{\tau[\Pr(s>k)V_1+\Pr(s<k)V_2]}, \tag{32}$$

where

$$K_2=\left\{\begin{array}{r}[k_1\mathbf{1}_{\{\Pr(s>k)\mathrm{ES}_1+\Pr(s<k)\mathrm{ES}_2<k\}}+k_2\mathbf{1}_{\{\Pr(s>k)\mathrm{ES}_1+\Pr(s<k)\mathrm{ES}_2>k\}}]\mathbf{1}_{\{\Pr(s>k)E_1+\Pr(s<k)E_2>P\}}\\ +\dfrac{1}{a}\mathbf{1}_{\{\Pr(s>k)E_1+\Pr(s<k)E_2<P\}}\end{array}\right\}.$$

k_1 and k_2 are two constants and are shown in the Appendix.

Proof. The proof is in the Appendix.

The result in the proposition is very similar to that of Yuan (2005), in which $K_2=1$. For this reason, uninformed managers change their trading aggressiveness in the presence of relative performance. Define $g(x_u)=K_2[\Pr(s>k)E_1+\Pr(s<k)E_2-P]/[\tau[\Pr(s>k)V_1+\Pr(s<k)V_2]]-x_u$, and x_u is a unique real root for $g(x_u)=0$. The equilibrium price can be obtained by the market-clearing condition. However, because x_u is not a linear form of the equilibrium price, multiple equilibria exist.[25] In general, the expressions of the solutions are complicated, and for expositional purposes, I conduct a numerical analysis for uninformed managers' demand and compare it with the solution from a linear contract.

[25] For more details, please see Yuan (2005).

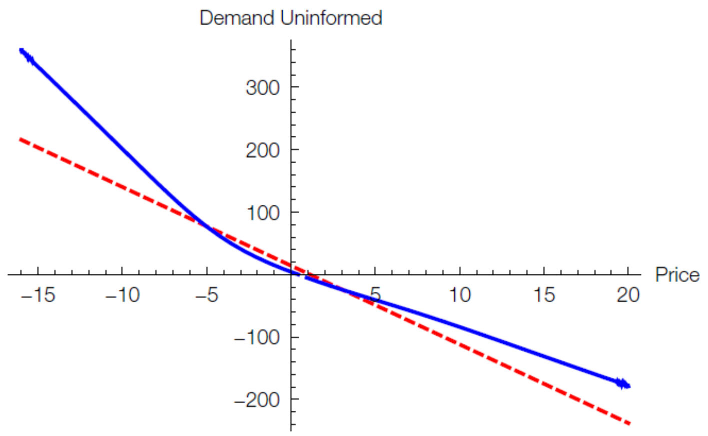
Figure 2 provides an example of uninformed managers' demand with a convex contract. I show that the uninformed managers' demand has a nonlinear relationship with the price, reflecting the "confusion" effect, consistent with Yuan (2005). When the price is very low, the option part is likely to pay off, inducing the managers to trade more aggressively under a convex contract than under a linear contract. By contrast, when the price is high, the option part is likely to pay 0, so uninformed managers trade less aggressively under a convex contract than under a linear contract.

A convex contract affects uninformed managers in two ways. First, when uninformed managers make an inference on the basis of the fictitious price, they may confuse whether the option part of informed managers pays off. I show the first effect from the conditional moments. Second, based on the realization of the price, uninformed managers need to determine when the options embedded in their contracts have a positive value. If the option value is positive, they have relative performance concerns, and they need to predict the performance of informed managers. However, the option compensation of informed managers may or may not be in the money. Thus, uninformed managers need to conjecture whether the option compensation of informed managers is in the money. In any circumstance, uninformed managers change their trading aggressiveness. Consequently, this effect is captured by the parameter K2, which is a combination of several indicator functions.

FIGURE 2

Uninformed Managers' Optimal Demands

Figure 2 presents how the optimal demands of the uninformed managers change with respect to the equilibrium price. The solid line corresponds to the optimal demand with a convex contract, and the dashed line corresponds to a linear contract. Parameter values are as follows: $a=0.01$, $b=b_1=2$, $S=2$, $\overline{d}=1$, $\sigma_d=2$, $\sigma_u=5$, $\sigma_\varepsilon=1$, $\lambda=0.7$, $\tau=3$.



6. Concluding Remarks

In this article, I develop a noisy REE for delegated portfolio management in the presence of relative performance incentives. I focus on the informative trading of fund managers and price informativeness when both informed and uninformed managers have an incentive to reduce tracking errors with respect to a certain benchmark. The benchmark is the average performance of all managers, which is endogenously given in the model. Thus, individual managers have concerns about their performance relative to their peer group.

In the presence of relative performance, the existence of uninformed managers causes informed managers to trade less informatively, diminishing the informativeness of the price. Uninformed managers trade more aggressively to hedge the additional uncertainty regarding the average performance of informed managers.

Moreover, each informed manager uses his or her private signal and the price to forecast the benchmark because it contains other managers' information. When an informed manager's private signal is a better predictor of the benchmark relative to the price, the manager weighs the price more heavily, causing the price to be more informative.

My article contributes to the literature by analyzing price informativeness within a delegated portfolio management framework when fund managers' compensation is linked to certain benchmarks. In the literature, fund managers normally tilt their portfolios toward the benchmark and thus trade less informatively. The result holds as long as the benchmark is exogenous. When the benchmark is endogenous and uncertain, the way in which each manager uses his or her information changes. When a manager uses his or her signal more than the price, the equilibrium price is more informative.

My finding of a more informative price is new to the literature, and it provides new empirical implications. In the previous literature, the price informativeness is always reduced by benchmarking, so a more informative price would be regarded as an anomaly. However, my article shows that the price informativeness depends on the nature of the benchmark and that the price can be more informative with an endogenous benchmark. Moreover, my article also predicts the equilibrium behavior of different types of fund managers. For example, when relative performance concerns become stronger, the managers who trade increasingly more aggressively are likely to be uninformed managers. To separate the roles of the price in providing a public signal and in aggregating information, I introduce an exogenous public signal to the model. I show that in the presence of relative performance, managers place more weight on the public signal, so the price becomes more sensitive to public information. These results are also suitable for empirical tests.

Finally, for the purpose of comparison, I consider an option-like incentive contract for managers. In particular, I consider a special case with the Grossman and Stiglitz (1980) information structure and show that informed managers trade more aggressively on their signals under an option contract than under a linear contract. Moreover, uninformed managers trade less aggressively under an option contract than under a linear contract if they expect the option to be out of the money. By contrast, if they expect the option to be in the money, they trade more aggressively.

Appendix. Proofs and Figures

1. Proof of Lemma 1

Given the informed managers' information structure, the conditional mean and variance of the compensation are as follows:

$$\begin{aligned}
\mathrm{E}(F\,|\,s_i,P) &= I+aW_0+\left[(a+b)x_i-b\int_0^{1-\lambda}x_j^u dj\right][\mathrm{E}(d\,|\,s_i,P)-P] \\
&\quad -b\mathrm{E}\left[(d-P)\int_0^{\lambda}x_j^I dj\,|\,s_i,P\right], \\
\mathrm{var}(F\,|\,s_i,P) &= \left[(a+b)x_i-b\int_0^{1-\lambda}x_j^u dj\right]^2\mathrm{var}(d\,|\,s_i,P) \\
&\quad -2\left[(a+b)x_i-b\int_0^{1-\lambda}x_j^u dj\right]b\,\mathrm{cov}\left[(d-P),(d-P)\int_0^{\lambda}x_j^I dj\,|\,s_i,P\right] \\
&\quad +f(o),
\end{aligned}$$

where f (o) is the function that is unrelated to x_i. By the first-order condition (FOC), I can obtain equation (5) in Lemma 1. Similarly, I can derive equation (6) for uninformed managers.

2. Proof of Lemma 2

I assume the informed manager demand has the form $x_i^I = Gs_i + HP + K$ for three constants, G, H, and K . Then, $\int_0^\lambda x_j^I dj = \lambda(Gd + HP + K) + \int_0^\lambda \varepsilon_j dj$ and then the conditional covariance is

$$\text{(A-1)} \quad x_i^I = \frac{1}{a+b}\frac{\mathrm{E}(d \mid s_i, P) - P}{\tau\sigma^2_{d \mid s_i, P}} + \frac{b}{a+b}\int_0^{1-\lambda} x_j^u dj + \frac{\lambda b G[2\mathrm{E}(d \mid s_i, P) + \sigma_\varepsilon\sqrt{\rho}\mathrm{E}(Y \mid s_i, P) - P] + \lambda b(HP + K)}{a+b}.$$

By some manipulation, I show that $\mathrm{cov}(d, d^2 \mid s_i, P) = 2\mathrm{E}(d \mid s_i, P)\sigma^2_{d \mid s_i, P}$.[26] Because Y and d are independent, it can be shown that

$$\mathrm{cov}(d, dY \mid s_i, P) = \mathrm{E}(d^2 Y \mid s_i, P) - \mathrm{E}(d \mid s_i, P)\mathrm{E}(dY \mid s_i, P) = \mathrm{E}(Y \mid s_i, P)\sigma^2_{d \mid s_i, P}$$

Then I rewrite equation (5) in Lemma 1 as

$$\text{(A-2)} \quad \mathrm{E}(d \mid s_i, P) = \bar{d} + \beta_s(s_i - \bar{d}) + \beta_p(P - A).$$

$$\text{(A-3)} \quad \mathrm{E}(Y \mid s_i, P) = \beta_s^Y(s_i - \bar{d}) + \beta_p^Y(P - A).$$

Plugging the conditional expectations into equation (A-1) and matching each term with $x_i^I = Gs_i + HP + K$, I derive three equations with three unknowns. Solving the system of equations, I obtain

$$G = \frac{\beta_s}{\tau\sigma^2_{d \mid s_i, P}[a + b(1 - 2\lambda\beta_s - \sigma_\varepsilon\lambda\sqrt{\rho}\beta_s^Y)]},$$

$$H = \frac{\beta_p - 1}{[a + b(1-\lambda)]\tau\sigma^2_{d \mid s_i, P}} + \frac{b\lambda G(2\beta_p - 1 + \sigma_\varepsilon\sqrt{\rho}\beta_p^Y)}{a + b(1-\lambda)},$$

$$K = \frac{\bar{d} - \beta_s\bar{d} - \beta_p A}{[a + b(1-\lambda)]\tau\sigma^2_{d \mid s_i, P}} + \frac{b(1-\lambda)x_u}{a + b(1-\lambda)} + \frac{b\lambda G[2\bar{d} - 2\beta_s\bar{d} - 2\beta_p A - \sigma_\varepsilon\sqrt{\rho}(\beta_s^Y\bar{d} + \beta_p^Y A)]}{a + b(1-\lambda)}.$$

Rearranging the expression $x_i^I = Gs_i + HP + K$, I obtain expression (8) in Lemma 2.

For uninformed managers, the conditional covariance in expression (6) is

$$\mathrm{cov}\left[d, (d-P)\int_0^\lambda x_j^I dj \mid P\right] = \int_0^\lambda \mathrm{cov}[d, (d-P)x_j^I \mid P]dj,$$

so I need to use expression (8). By some calculation, I have following results:

$$\text{(A-4)} \quad \mathrm{cov}\{d, (d-P)[\mathrm{E}(d \mid s_i, P) - P] \mid P\} = [\mathrm{E}(d \mid P) - P](2\sigma^2_{d \mid P} - \sigma^2_{d \mid s_i, P}),$$

$$\text{(A-5)} \quad \mathrm{cov}[d, (d-P)\mathrm{E}(Y \mid s_i, P) \mid P] = \mathrm{E}(Y \mid P)\sigma^2_{d \mid P},$$

$$\text{(A-6)} \quad \mathrm{cov}\{d, (d-P)[2\mathrm{E}(d \mid s_i, P) - P] \mid P\} = 2\mathrm{E}(d \mid P)(2\sigma^2_{d \mid P} - \sigma^2_{d \mid s_i, P}) - P(3\sigma^2_{d \mid P} - 2\sigma^2_{d \mid s_i, P}),$$

$$\text{(A-7)} \quad \mathrm{cov}\{d, (d-P)s_i \mid P\} = [2\mathrm{E}(d \mid P) - P]\sigma^2_{d \mid P} + \sigma_\varepsilon\sqrt{\rho}\mathrm{E}(Y \mid P)\sigma^2_{d \mid P}.$$

By plugging expression (8) into the conditional covariance and combining equations (A-4),(A-5), (A-6), and (A-7), I derive equation (9) in Lemma 2.

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[26] I use the fact that if $X \sim \mathrm{N}(\mu, \sigma_X^2)$, $\mathrm{E}(X^3) - \mu\mathrm{E}(X^2) = 2\mu\mathrm{E}(X^2) - 2\mu^3$. Then $\mathrm{cov}(d, d^2 \mid s_i, P) = \mathrm{E}(d^3 \mid s_i, P) - \mathrm{E}(d \mid s_i, P)\mathrm{E}(d^2 \mid s_i, P) = 2\mathrm{E}(d \mid s_i, P)\mathrm{E}(d^2 \mid s_i, P) - \mathrm{E}(d \mid s_i, P)^3 = 2\mathrm{E}(d \mid s_i, P)\sigma^2_{d \mid s_i, P}$.

3. Proof of Proposition 1

The market-clearing condition $\int_0^{\lambda} x_j^I dj + (1-\lambda)x_u = S + u$ can be written as an affine function of $\int_0^{\lambda} s_i di - \lambda \overline{d}$ and u, so the constant term and all coefficients of random variables should be 0. When the constant term is 0, I derive A as

$$\text{(A-8)} \qquad A = \overline{d} - \frac{S}{\Pi_1\lambda + \Pi_2\lambda + \Pi_3 K},$$

where $\Pi_1 = 1/([a+b(1-\lambda)]\tau\sigma^2_{d|s_i,P})$, $\Pi_2 = b\beta_s\lambda/([a+b(1-\lambda)][a+b(1-2\lambda\beta_s - \lambda\sigma_\varepsilon\sqrt{\rho}\beta_s^Y)]\tau\sigma^2_{d|s_i,P})$, $\Pi_3 = (a+b)(1-\lambda)/([a+b(1-\lambda)]\tau\sigma^2_{d|P})$. A is the function of β_s, β_s^Y, $\sigma^2_{d|s_i,P}$, and $\sigma^2_{d|P}$. β_s and β_s^Y are from the projection theorem, equations (A-2) and (A-3), and $\sigma^2_{d|s_i,P}$ and $\sigma^2_{d|P}$ are conditional variances. When the coefficient of $(\int_0^{\lambda} s_i di - \lambda\overline{d})$ is 0, I solve B as

$$\text{(A-9)} \qquad B = \frac{\begin{bmatrix} \Pi_1(\beta_s + \lambda\beta_p B) \\ +\Pi_2(2\beta_s + 2\lambda\beta_p B + \sigma_\varepsilon\sqrt{\rho}(\beta_s^Y + \lambda\beta_p^Y B) - 1) \\ +\Pi_3 K\beta_p^u B \end{bmatrix}}{\Pi_1\lambda + \Pi_2\lambda + \Pi_3 K}.$$

B is the function of β_s, β_s^Y, $\sigma^2_{d|s_i,P}$, $\sigma^2_{d|P}$, $\beta_p B$, $\beta_p^Y B$, and $\beta_p^u B$, which are quantities from the projection theorem. When the coefficient of u is 0, I solve C as

$$\text{(A-10)} \qquad C = \frac{\tau\sigma^2_{d|s_i,P}}{\beta_s}[a + b(1 - 2\lambda\beta_s - \lambda\sigma_\varepsilon\sqrt{\rho}\beta_s^Y)].$$

In particular, β_s, β_s^Y, $\sigma^2_{d|s_i,P}$, $\sigma^2_{d|P}$, $\beta_p B$, $\beta_p^Y B$, and $\beta_p^u B$ are functions of parameter C, which indicates that A and B are functions of C. For this reason, the equilibrium is fully solved if C is determined.

By the projection theorem, I have $\beta_s = (C^2\sigma_d^2\sigma_u^2)/[\lambda^2(1-\rho)\sigma_\varepsilon^2(\sigma_d^2 + \rho\sigma_\varepsilon^2) + C^2(\sigma_\varepsilon^2 + \sigma_d^2)\sigma_u^2]$, $\beta_s^Y = C^2\sigma_u^2\sigma_\varepsilon\sqrt{\rho}/[\lambda^2(1-\rho)\sigma_\varepsilon^2(\sigma_d^2 + \rho\sigma_\varepsilon^2) + C^2(\sigma_\varepsilon^2 + \sigma_d^2)\sigma_u^2]$, and $\sigma^2_{d|s_i,P} = \sigma_d^2[\lambda^2(1-\rho)\rho\sigma_\varepsilon^4 + C^2\sigma_\varepsilon^2\sigma_u^2]/[\lambda^2(1-\rho)\sigma_\varepsilon^2(\sigma_d^2 + \rho\sigma_\varepsilon^2) + C^2(\sigma_\varepsilon^2 + \sigma_d^2)\sigma_u^2]$. Plugging the expressions of β_s, β_s^Y, and $\sigma^2_{d|s_i,P}$ into equation (A-10) and by some manipulation, I can derive

$$\text{(A-11)} \qquad N_1C^5 - N_2C^4 + N_3C^3 - N_4C^2 - N_5 = 0,$$

where

$$\begin{aligned}
N_1 &= (\sigma_\varepsilon^2 + \sigma_d^2)\sigma_u^4, \\
N_2 &= \tau\sigma_\varepsilon^2\sigma_u^4\{[a + b(1-2\lambda)]\sigma_d^2 + [a + b(1-\lambda\rho)]\sigma_\varepsilon^2\}, \\
N_3 &= \lambda^2(1-\rho)\sigma_\varepsilon^2(\sigma_d^2 + \rho\sigma_\varepsilon^2)\sigma_u^2, \\
N_4 &= \{(a+b)(\sigma_d^2 + \rho\sigma_\varepsilon^2) + [a + b(1-2\lambda)]\sigma_d^2\rho + [a + b(1-\lambda\rho)]\sigma_\varepsilon^2\rho\} \\
&\quad \times \sigma_u^2\lambda^2(1-\rho)\sigma_\varepsilon^4\tau, \\
N_5 &= (a+b)\tau\lambda^4(1-\rho)^2\rho\sigma_\varepsilon^6(\sigma_d^2 + \rho\sigma_\varepsilon^2).
\end{aligned}$$

Note that N_1, N_3, N_4, and N_5 are all positive. To determine the equilibrium, I set $F(C) = N_1C^5 - N_2C^4 + N_3C^3 - N_4C^2 - N_5$ and find that $F(\infty) = \infty$ and $F(0) < 0$. By the intermediate value theorem, there exists at least one solution for $F(C) = 0$. This proves the existence of the equilibrium. For uniqueness, I define $Q = [a + b(1-2\lambda)]\sigma_d^2 + [a + b(1-\lambda\rho)]\sigma_\varepsilon^2$ and identify the following sufficient conditions:

1. First, $F'(C) = 5N_1C^4 - 4N_2C^3 + 3N_3C^2 - 2N_4C$ and $F''(C) = 20N_1C^3 - 12N_2C^2 + 6N_3C - 2N_4$. It is observed that $F'(0) = 0$ and $F''(0) < 0$. Therefore, for positive C, if $F''(C)$ is an increasing function, $F'(C)$ is first negative and then positive. Consequently, $F(C) = 0$ has a unique positive root if $F'''(C) = 60N_1C^2 - 24N_2C + 6N_3 > 0$. $F'''(C)$ is a convex function and meets its minimum value at $C^* = N_2/5N_1$. Solving $F'''(C^*) > 0$, I find that $Q < \lambda\sqrt{5(1-\rho)(\sigma_\varepsilon^2 + \sigma_d^2)(\sigma_d^2 + \rho\sigma_\varepsilon^2)/2\tau}/(\sigma_\varepsilon\sigma_u)$.

2. When $Q>\lambda\sqrt{5(1-\rho)(\sigma_\varepsilon^2+\sigma_d^2)(\sigma_d^2+\rho\sigma_\varepsilon^2)/5\tau}/(\sigma_\varepsilon\sigma_u)$, it is possible for $F'''(C)<0$ and for $F''(C)$ to not be an increasing function for positive C. However, if $F''(C)$ is first negative and then positive, $F(C)=0$ still has a unique positive root. If the value of $F''(C)$ is negative when $F'''(C)=0$, $F''(C)$ is first negative and then positive for $C>0$. Defining C^* as the root for $F'''(C)=0$, I rewrite $F''(C^*)=-4N_2^2(C^*)^2+4N_3C^*-2N_4$, which achieves its global maximum at $C^*=N_3/2N_2$. Thus, the sufficient condition for $F''(C^*)<0$ is to show that $-4N_2^2(N_3/2N_2)^2+4N_3(N_3/2N_2)-2N_4<0$, which can be simplified as $\lambda^2(1-\rho)(\sigma_d^2+\rho\sigma_\varepsilon^2)^2/[(a+b)(\sigma_d^2+\rho\sigma_\varepsilon^2)Q+\rho Q^2]<2\sigma_u^2\sigma_\varepsilon^2\tau^2$.

Moreover, $\lambda^2(1-\rho)(\sigma_d^2+\rho\sigma_\varepsilon^2)^2/[(a+b)(\sigma_d^2+\rho\sigma_\varepsilon^2)Q+\rho Q^2]<2\sigma_u^2\sigma_\varepsilon^2\tau^2$ can be rewritten as

(A-12) $$2\sigma_u^2\sigma_\varepsilon^2\tau^2\rho Q^2+2\sigma_u^2\sigma_\varepsilon^2\tau^2(a+b)(\sigma_d^2+\rho\sigma_\varepsilon^2)Q-\lambda^2(1-\rho)(\sigma_d^2+\rho\sigma_\varepsilon^2)^2 \quad > \quad 0.$$

Note that the left-hand side of inequality (A-12) increases with Q, so if inequality (A-12) holds for $Q=\lambda\sqrt{5(1-\rho)(\sigma_\varepsilon^2+\sigma_d^2)(\sigma_d^2+\rho\sigma_\varepsilon^2)/5\tau}/(\sigma_\varepsilon\sigma_u)$, it holds for all $Q>\lambda\sqrt{5(1-\rho)(\sigma_\varepsilon^2+\sigma_d^2)(\sigma_d^2+\rho\sigma_\varepsilon^2)/5\tau}/(\sigma_\varepsilon\sigma_u)$. The sufficient condition for inequality (A-12) for $Q=\lambda\sqrt{5(1-\rho)(\sigma_\varepsilon^2+\sigma_d^2)(\sigma_d^2+\rho\sigma_\varepsilon^2)/5\tau}/(\sigma_\varepsilon\sigma_u)$ to hold is

$$\lambda\tau(\sigma_d^2+\rho\sigma_\varepsilon^2)^2\{5\lambda(1-\rho)^2\rho+\sigma_u\sigma_\varepsilon(a+b)\sqrt{10(1-\rho)\tau}-\lambda(1-\rho)\} \quad > \quad 0,$$

which can be simplified as inequality (14). This concludes the proof.

4. Proofs of Proposition 2, Proposition 3, and Lemma 3

Proof for dC/db: Differentiating F(C)=0 implicitly with respect to b, I obtain

$$\frac{dC}{db} = \frac{\begin{bmatrix}\lambda^4(1-\rho)^2\rho\sigma_\varepsilon^6(\sigma_d^2+\rho\sigma_\varepsilon^2)\tau+\sigma_u^4[(1-2\lambda)\sigma_d^2+(1-\lambda\rho)\sigma_\varepsilon^2]C^4\sigma_\varepsilon^2\tau \\ +\{(\sigma_d^2+\rho\sigma_\varepsilon^2)+[(1-2\lambda)\rho]\sigma_d^2+(1-\lambda\rho)\sigma_\varepsilon^2\rho\}\sigma_u^2\lambda^2(1-\rho)\sigma_\varepsilon^4C^2\tau\end{bmatrix}}{5N_1C^4-4N_2C^3+3N_3C^2-2N_4C}.$$

The denominator of dC/db is positive, so the sign of dC/db depends on the sign of the numerator. By some manipulation, I show that the sign of the numerator depends on

$$G(C) \quad = \quad \lambda^2(1-\rho)\sigma_\varepsilon^2(\sigma_d^2+\rho\sigma_\varepsilon^2)+[(1-2\lambda)\sigma_d^2+(1-\lambda\rho)\sigma_\varepsilon^2]\sigma_u^2C^2.$$

Thus, the condition for dC/db <(or>)0 is the same as inequality (19) (or inequality (20)).

Proof for Proposition 2. From the expression of $G(C)$, it is easy to see that when $(1-2\lambda)\sigma_d^2+(1-\lambda\rho)\sigma_\varepsilon^2>0$, the price is always less informative. Define C' such that $G(C')=0$, which means

$$C' \quad = \quad \frac{\lambda\sigma_\varepsilon}{\sigma_u}\sqrt{\frac{(1-\rho)(\sigma_d^2+\rho\sigma_\varepsilon^2)}{[(2\lambda-1)\sigma_d^2-(1-\lambda\rho)\sigma_\varepsilon^2]}}.$$

If $dC/db<0$, I need $C>C'$, which also means $F(C')<0$. If I plug C' in $F(C')<0$, I can derive inequality (17). Similarly, I can also derive the condition for $dC/db>0$.

Proof for dC/da: Differentiating F(C)=0 implicitly with respect to a, I have

$$\frac{dC}{da} = \frac{\begin{bmatrix}\sigma_\varepsilon^2\{\lambda^4(1-\rho)^2\rho\sigma_\varepsilon^4(\sigma_d^2+\rho\sigma_\varepsilon^2) \\ +C^2\lambda^2(1-\rho)\sigma_\varepsilon^2[(1+\rho)\sigma_d^2+2\rho\sigma_\varepsilon^2]\sigma_u^2 \\ +C^4(\sigma_d^2+\sigma_\varepsilon^2)\sigma_u^4\}\tau\end{bmatrix}}{5N_1C^4-4N_2C^3+3N_3C^2-2N_4C},$$

which is greater than 0. Given the expression of I , I can obtain the results in the proposition.

Proof for $dI/d\lambda$, $dI/d\rho$, $dI/d\sigma_u^2$, and $dI/d\sigma_\varepsilon^2$: Given the expression of price informativeness I, it is easy to derive $dI/d\lambda=2\lambda[C^2-C\lambda(dC/d\lambda)]\sigma_u^2/(\lambda^2\rho\sigma_\varepsilon^2+C^2\sigma_u^2)^2$, $dI/d\sigma_\varepsilon^2=-[\lambda^4\rho+2\lambda^2\sigma_u^2C(dC/d\sigma_\varepsilon^2)]/(\lambda^2\rho\sigma_\varepsilon^2+C^2\sigma_u^2)^2$, $dI/d\sigma_u^2=-[\lambda^2C^2+2\lambda^2\sigma_u^2C(dC/d\sigma_u^2)]/(\lambda^2\rho\sigma_\varepsilon^2+C^2\sigma_u^2)^2$, and $dI/d\rho=-[\lambda^4\sigma_\varepsilon^2+2\lambda^2\sigma_u^2C(dC/d\rho])/(\lambda^2\rho\sigma_\varepsilon^2+C^2\sigma_u^2)^2$. Differentiating $F(C)=0$ implicitly with respect to λ, I have

$$\frac{dC}{d\lambda} = \frac{\begin{bmatrix} 4(a+b)(1-\rho)^2\rho\sigma_\varepsilon^6(\sigma_d^2+\rho\sigma_\varepsilon^2)\tau\lambda^3 - 3bC^2(1-\rho)\rho\sigma_\varepsilon^4(2\sigma_d^2+\rho\sigma_\varepsilon^2)\sigma_u^2\tau\lambda^2 \\ -2C^2(1-\rho)\sigma_\varepsilon^2\sigma_u^2\{(\sigma_d^2+\rho\sigma_\varepsilon^2)C-(a+b)\sigma_\varepsilon^2[(1+\rho)\sigma_d^2+2\rho\sigma_\varepsilon^2]\tau\}\lambda \\ -b\sigma_\varepsilon^2(2\sigma_d^2+\rho\sigma_\varepsilon^2)\sigma_u^4\tau C^4 \end{bmatrix}}{5N_1C^4-4N_2C^3+3N_3C^2-2N_4C}.$$

Given the expression of $dI/d\lambda$, the sign of $dI/d\lambda$ depends on the sign of $C^2-\lambda dC/d\lambda$. By the fact that $F(C)=0$, I show that $C^2-\lambda dC/d\lambda>0$, so $dI/d\lambda>0$. Differentiating $F(C)=0$ implicitly with respect to σ_ε^2 and with the fact that $F(C)=0$, I obtain

$$\frac{dC}{d\sigma_\varepsilon^2} = \frac{\begin{bmatrix} (\sigma_d^2\sigma_u^4/\sigma_\varepsilon^2)C^5 \\ +\lambda^2(1-\rho)\sigma_\varepsilon^2\sigma_u^2\tau\{[a+b+(a+b-2b\lambda)\rho]\sigma_d^2+2\rho[2(a+b)-b\lambda\rho]\sigma_\varepsilon^2\}C^2 \\ +(a+b)\tau\lambda^4\rho\sigma_\varepsilon^4(1-\rho)^2(2\sigma_d^2+3\rho\sigma_\varepsilon^2) \end{bmatrix}}{5N_1C^4-4N_2C^3+3N_3C^2-2N_4C}.$$

It is easy to see that $dC/d\sigma_\varepsilon^2>0$, so I have $dI/d\sigma_\varepsilon^2<0$. Differentiating $F(C)=0$ implicitly with respect to σ_u^2, I obtain

$$\begin{aligned} \frac{dC}{d\sigma_u^2} &= \frac{-2N_1C^5+2N_2C^4-N_3C^3+N_4C^2}{(5N_1C^4-4N_2C^3+3N_3C^2-2N_4C)\sigma_u^2} \\ &= \frac{-N_1C^5+N_2C^4-N_5}{(5N_1C^4-4N_2C^3+3N_3C^2-2N_4C)\sigma_u^2}. \end{aligned}$$ [27]

Defining $f_1(C)=-N_1C^5+N_2C^4$, I have $f_1(C'')=0$ at $C''=N_2/N_1$ and $f_1(C)<0$ for $C>N_2/N_1$. It is easy to check $F(C'')<0$, so I know $C^*>C'$ for $F(C^*)=0$. Thus, I have $f_1(C^*)<0$ for $F(C^*)=0$ and can conclude that $dC/d\sigma_u^2<0$. Differentiating $F(C)=0$ implicitly with respect to ρ, I have

$$\frac{dC}{d\rho} = \frac{\begin{bmatrix} 4(a+b)\lambda^4\sigma_\varepsilon^8\tau\rho^3+3\lambda^3\sigma_\varepsilon^6[(a+b)\lambda(\sigma_d^2-2\sigma_\varepsilon^2)+bC^2\sigma_u^2]\tau\rho^2 \\ -2\lambda^2\sigma_\varepsilon^4\{(a+b)\lambda^2\sigma_\varepsilon^2(2\sigma_d^2-\sigma_\varepsilon^2)\tau \\ +C^2\sigma_u^2[(a+b-2b\lambda)\sigma_d^2+(2a+b(2+\lambda))\sigma_\varepsilon^2\tau]-\sigma_u^2C^3\}\rho \\ +\lambda\sigma_\varepsilon^2\{\lambda(\sigma_d^2-\sigma_\varepsilon^2\sigma_u^2)\sigma_u^2C^3+\tau(a+b)\lambda^3\sigma_d^2\sigma_\varepsilon^4 \\ +2\sigma_\varepsilon^2\tau\lambda((a+b)\sigma_\varepsilon^2-b\lambda\sigma_d^2)\sigma_u^2C^2-b\sigma_\varepsilon^2\sigma_u^4\tau C^4\} \end{bmatrix}}{5N_1C^4-4N_2C^3+3N_3C^2-2N_4C},$$

which could be either positive or negative.

Proof of Lemma 3. If $b=0$, $C=a\tau\sigma^2_{d|s_i,P}/\beta_s$. From the projection theorem, I have a cubic equation, $\sigma_u^2C^3-a\tau\sigma_\varepsilon^2\sigma_u^2C^2-a\tau\lambda^2(1-\rho)\rho\sigma_\varepsilon^4=0$, for C. This equation has a unique positive root. Following the procedure for the calculation of $dI/d\lambda$, $dI/d\rho$, $dI/d\sigma_u^2$, and $dI/d\sigma_\varepsilon^2$, I can easily derive the comparative statics results for I_{NRP}.

5. Proof of Corollary 1

The optimization problem is similar to the proof of Lemma 2, so I can derive the results of the optimal demands. Similar to the proof of Proposition 1, the price can be obtained by the market-clearing condition $\lambda x_I+(1-\lambda)x_u=S+u$. It is easy to show

[27] The second step comes from the fact that F(C)=0.

$$A = \bar{d} - \frac{S\tau\sigma^2_{d|s}[a+b(1-\lambda)]}{\left[\lambda+(1-\lambda)(a+b)K_1\left(\frac{\sigma^2_{d|s}}{\sigma^2_{d|P}}\right)\right]},$$

$$B = \frac{\left[\lambda\beta_s+(1-\lambda)(a+b)K_1\left(\frac{\sigma^2_{d|s}}{\sigma^2_{d|P}}\right)\beta_P B\right]}{\left[\lambda+(1-\lambda)(a+b)K_1\left(\frac{\sigma^2_{d|s}}{\sigma^2_{d|P}}\right)\right]}, \quad \text{and}$$

$$C = \tau\sigma_\varepsilon^2\frac{[a+b(1-\lambda)]}{\lambda},$$

where $\sigma^2_{d|P}$ and $\beta_P B$ are functions of C by the projection theorem.

6. Proof of Corollary 2

$\partial C/\partial b>0$ is obvious from the expression of C, and it is easy to see that $\partial\sigma^2_{d|P}/\partial b = [2C\sigma_u^2\sigma_d^2\sigma_d^2/(\sigma_d^2+\sigma_\varepsilon^2+C^2\sigma_u^2)^2](\partial C/\partial b)>0$. Combining those results, I can show that $\partial K_1/\partial b>0$. For simplicity, I differentiate $(a+b)K_1$ with respect to b instead of K_1. Given $(a+b)K_1=1+b(1-\lambda)/a+[b(1-\lambda)/a][(2\sigma^2_{d|P}-\sigma^2_{d|s})/\sigma^2_{d|s}]$, it is easy to show that $(a+b)\partial K_1/\partial b=[\lambda a/a(a+b)][((2\sigma^2_{d|P}-\sigma^2_{d|s})/\sigma^2_{d|s})-1]+(2b\lambda/(a\sigma^2_{d|s}))(\partial\sigma^2_{d|P}/\partial b)>0$. By the fact that $((2\sigma^2_{d|P}-\sigma^2_{d|s})/\sigma^2_{d|s})-1>0$, I have $\partial K_1/\partial b>0$. By some manipulation, the parameter B can be written as $B=[\lambda\beta_s(\sigma_\varepsilon^2+C^2\sigma_u^2)+(1-\lambda)\sigma^2_{d|s}(a+b)K_1]/[\lambda(\sigma_\varepsilon^2+C^2\sigma_u^2)+(1/\sigma_d^2)(1-\lambda)\sigma^2_{d|s}(a+b)K_1(\sigma_d^2+\sigma_\varepsilon^2+C^2\sigma_u^2)]$. Plugging the expressions of K_1 and C into B, and taking the derivative with respect to b, I find $\partial B/\partial b<0$.

7. Proof of Corollary 3

The optimization problem is similar to the proof of Lemma 2. Given the conjectured price, the market-clearing condition $\int_0^1 x_i=S+u$ becomes an affine function of d and u. Following the same procedure as the proof of Proposition 1, I have $A=\bar{d}-Sa\tau\sigma^2_{d|s_i,P}[a+b(1-2\beta_s)]/[a+b(1-\beta_s)]$, $B=[b\beta_s-(a+b)(\beta_s+\beta_P B)]/[b\beta_s-(a+b)]$, and $C=\tau\sigma^2_{d|s_i,P}[a+b(1-2\beta_s)]/\beta_s$. By the projection theorem, I have $\beta_s=\sigma_d^2C^2\sigma_u^2/[\sigma_d^2\sigma_\varepsilon^2+C^2(\sigma_\varepsilon^2+\sigma_d^2)\sigma_u^2]$ and $\sigma^2_{d|s_i,P}=\sigma_d^2\sigma_\varepsilon^2C^2\sigma_u^2/[\sigma_d^2\sigma_\varepsilon^2+(\sigma_d^2+\sigma_\varepsilon^2)C^2\sigma_u^2]$. Then C becomes the root of the following cubic equation:

$$a_1C^3-a_2C^2+a_3C-a_4 = 0, \quad \text{(A-13)}$$

where $a_1=(\sigma_\varepsilon^2+\sigma_d^2)\sigma_u^2$, $a_2=[a(\sigma_\varepsilon^2+\sigma_d^2)+b(\sigma_\varepsilon^2-\sigma_d^2)]\tau\sigma_\varepsilon^2\sigma_u^2$, $a_3=\sigma_d^2\sigma_\varepsilon^2$, and $a_4=(a+b)\sigma_d^2\sigma_\varepsilon^4\tau$. By defining $f(C)=a_1C^3-a_2C^2+a_3C-a_4$, I can show that $f(0)<0$ and $f(\infty)=\infty$. By the intermediate value theorem, there exists at least one solution for $f(C)=0$. For the uniqueness, I need to look at the property of $f'(C)$ and $f''(C)$, and the procedure is similar to the proof of Proposition 1.

8. Proofs of Propositions 4 and 5

Informed managers observe the signal, so the option pays off when $\mathrm{E}(d\,|\,s)>P$. Because $\mathrm{E}(d\,|\,s)=\bar{d}+\beta_s(s-\bar{d})$, informed managers effectively expect the option contract to be in the money when the signal is sufficiently good, or $s>k$.[28] When the option contract is in the money, the optimal demand is $x_{I1}=[\mathrm{E}(d\,|\,s)-P]/[a+b(1-\lambda)](\tau\sigma^2_{d|s})+[b(1-\lambda)x_u/(a+b(1-\lambda))]$. When $\mathrm{E}(d\,|\,s)<P$ or $s<k$, informed managers expect that the option contract is out of money. For this reason, the compensation is linear, so informed managers submit an optimal demand $x_{I2}=(\mathrm{E}(d\,|\,s)-P)/(a\tau\sigma^2_{d|s})$. This concludes the proof of Proposition 4.

[28]

I solve the price P^* by the market-clearing condition $\lambda x_{I1}\mathbf{1}_{\{s>k\}}+\lambda x_{I2}\mathbf{1}_{\{s_i<k\}}=S+u-(1-\lambda)x_u$. Depending on the realizations of the indicator functions and the expressions of x_{I1} and x_{I2}, I have the following prices:

$$
\begin{aligned}
P_1 &= \bar{d}+\frac{\sigma_d^2}{\sigma_d^2+\sigma_\varepsilon^2}(s-\bar{d})-\left[\frac{[a+b_1(1-\lambda)]S^*}{\lambda}-b_1(1-\lambda)x_u\right]\frac{\sigma_d^2\sigma_\varepsilon^2\tau}{\sigma_d^2+\sigma_\varepsilon^2},\\
P_2 &= \bar{d}+\frac{\sigma_d^2}{\sigma_d^2+\sigma_\varepsilon^2}(s-\bar{d})-\frac{a\tau}{\lambda}\frac{\sigma_d^2\sigma_\varepsilon^2}{\sigma_d^2+\sigma_\varepsilon^2}S^*.
\end{aligned}
$$

Uninformed managers need to make an inference about d from the price. Following Yuan (2005), I have following conditional moments:

$$
\begin{aligned}
\mathrm{E}(d\mid P,x_u) &= \Pr(s>k)E_1+\Pr(s<k)E_2,\\
\mathrm{var}(d\mid P,x_u) &= \Pr(s>k)V_1+\Pr(s<k)V_2,
\end{aligned}
$$

where $\Pr(s>k)=1-\Pr(s_i<k)=\Phi[(1/\sigma_d^2)(P-\bar{d})]$, and

$$
\begin{aligned}
E_1 &= \mathrm{E}(d\mid P=P_1,x_u=x_u^*,s<k), \quad & E_2 &= \mathrm{E}(d\mid P=P_2,x_u=x_u^*,s>k),\\
V_1 &= \mathrm{var}(d\mid P=P_1,x_u=x_u^*,s<k), \quad & V_2 &= \mathrm{var}(d\mid P=P_2,x_u=x_u^*,s>k).
\end{aligned}
$$

I can easily get the closed-form expressions following the formulas from Greene ((1990), pp. 707–708).

For the compensation of the uninformed managers, the option pays 0 if $\Pr(s>k)E_1+\Pr(s<k)E_2-P<0$. Because there is no relative performance, the optimal demand is $x_u=[\Pr(s>k)E_1+\Pr(s<k)E_2-P]/(a\tau[\Pr(s>k)V_1+\Pr(s<k)V_2])$.

When $\Pr(s>k)E_1+\Pr(s<k)E_2-P<0$, uninformed managers have concerns about relative performance because the option contract is expected to be in the money. However, the option contract of informed managers may or may not be in the money, so uninformed managers need to infer when the option contract of informed managers pays off. Define the conditional mean of the signal s as

$$
\mathrm{ES}_1 = \mathrm{E}(s\mid P=P_1,x_u=x_u^*,s<k), \quad \mathrm{ES}_2 = \mathrm{E}(s\mid P=P_2,x_u=x_u^*,s>k).
$$

When $\Pr(s>k)\mathrm{ES}_1+\Pr(s<k)\mathrm{ES}_2>k$, the demand for the informed managers is x_{I1}. The optimization problem of the uninformed managers is similar to that in Section IV.B, and I have $x_u=k_1[\Pr(s>k)E_1+\Pr(s<k)E_2-P]/[a\tau[\Pr(s>k)V_1+\Pr(s<k)V_2]]$, where $k_1=[a+b(1-\lambda)]/[a(a+b)]+[b\lambda/a(a+b)][(2\sigma_{d|P}^2-\sigma_{d|s_i,P}^2)/\sigma_{d|s_i,P}^2]$.

When $\Pr(s_i>k)\mathrm{ES}_1+\Pr(s_i<k)\mathrm{ES}_2<k$, the demand for the informed managers is x_{I2}. Following a procedure similar to that in Section IV.B, I have $x_u=k_2[\Pr(s>k)E_1+\Pr(s<k)E_2-P]/[a\tau[\Pr(s>k)V_1+\Pr(s<k)V_2]]$, where $k_2=[a+b\lambda(2\sigma_{d|P}^2-\sigma_{d|s}^2)/\sigma_{d|s}^2]/[a(a+b\lambda)]$.

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[28] By the projection theorem, I can show that $k=((\sigma_d^2+\sigma_\varepsilon^2)P/\sigma_d^2)-(\bar{d}\sigma_\varepsilon^2/\sigma_d^2)$.

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