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What do the Prices of UK Inflation-linked Securities Say on Inflation Expectations, Risk Premia and Liquidity Risks?*

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Abstract

The difference between yields on nominal and inflation-linked government bonds or inflation swap rates are important indicators of the outlook for inflation and are monitored regularly by central banks, including the United Kingdom's Monetary Policy Committee (MPC). However, in the United Kingdom, inflation- linked instruments reference RPI inflation, whereas the MPC's target is CPI inflation of 2%. In this paper we extract market expectations for UK CPI inflation with the help of UK RPI-linked gilt prices, which is a novelty in the literature. To better extract useful information about expectations for CPI inflation, we develop a no-arbitrage term structure model and decompose the forward inflation curve into: measures of CPI inflation expectations; the expected wedge between RPI and CPI inflation; estimates of inflation risk premia and estimates of liquidity risk premia. We show that long-horizon expectations of CPI inflation fell in the 1990 s, after the introduction of inflation targeting and the creation of the MPC, and have since remained fairly stable at around 2%.

JEL Classification: C40; E31; E43; E52;G12

Keywords: Affine arbitrage-free dynamic term structure model, Breakeven inflation, Inflation expectations, Risk premia, Funding liquidity, Survey expectations

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1. Introduction

The yield spread between conventional and inflation-linked government bonds, commonly called the 'break-even inflation' (BEI) rate, has long been used as an indicator of market-based inflation expectations. And more recently, as a result of the development of the market for inflation derivatives, inflation swap rates have become an alternative indicator. These measures have been increasingly used in central bank publications, market commentaries and research. [1] For example, the BEI rates from bonds and swaps are monitored regularly by the UK Monetary Policy Committee (MPC), alongside other measures of inflation expectations such as those based on surveys.

However, BEI rates are imperfect indicators of expected inflation. They also reflect risk premia that compensate investors for inflation risk, along with potential liquidity risk. Moreover, in the UK, both inflation-linked bonds (known as "gilts" in the UK) and inflation swaps are linked to RPI inflation, whereas the MPC tar- gets CPI inflation. As we discuss below, the UK markets for RPI- linked gilts and RPI swaps are substantially deep and reasonably liquid. Yet, at present, a market for CPI-linked UK gilts does not exist. Similarly, the UK over-the-counter (OTC) CPI inflation swap market is highly illiquid with scarce transactions and limited price transparency. [2]Therefore, to derive market expectations of future CPI inflation rates we must decompose UK BEI rates, since these may contain three additional non-trivial components: an inflation risk premium to compensate for uncertainty about future inflation; a liquidity risk premium; and the spread between RPI inflation, to which market instruments are indexed, and CPI inflation, which is the measure that the MPC's inflation target refers to.

In this paper, we develop an affine term structure model of BEI rates, which allows us to better extract essential information from both gilt and inflation swap measures. The model decomposes market-implied BEI rates into measures of inflation expectations and risk premia using a no-arbitrage framework. The two main contributions of the paper are: (1) estimating expectations of the wedge between RPI and CPI inflation rates and so delivering a term structure of expectations for CPI inflation; and (2) modelling liquidity premia in gilt BEI rates by making use of inflation swap rates. We discuss each of these in more detail below.

First, given that the vast majority of sterling-denominated inflation-linked instruments reference the RPI rather than CPI measure of UK inflation, whereas the MPC's target is based on CPI inflation, we seek to model not only expectations for RPI inflation but also for CPI inflation. In practice, the difference between RPI and CPI inflation reflects a range of factors, such as different components being included in the calculation, different weights applied to the basket of goods and formula effects (geometric averages for RPI versus arithmetic averages for CPI). By jointly modelling RPI and CPI inflation we generate estimates for the future 'wedge' between RPI and CPI inflation that is priced into RPI BEI rates. Estimates of the expected 'wedge' between RPI and CPI inflation help policy makers and investors to extract not only expectations of RPI, but also CPI expectations from prices of UK financial instruments. This analysis and its findings are valuable for policy makers as well as being innovative in the literature, since to our knowledge the paper is the first study to look at this subject. Available UK studies look only at markets expectations of RPI inflation. While the inflation index issue does not exist for other markets: for example, both US and euro-area monetary policy inflation targets are expressed in terms of an annual rate of inflation based on CPI, the same index to which also all market inflation-linked instruments refer.

Second, we explicitly model the liquidity risk premium embedded in gilt BEI rates, which is driven by the relative illiquidity of index-linked (or inflation-linked [3]) gilts compared to conventional gilts. While there are several papers studying UK inflation risk premia (like Guimaraes (2014), or Joyce et al., (2010)), to our knowledge this is the first work with an explicit focus on liquidity premia in UK index-linked gilts. Previous evidence (mostly for the US market) suggests that the liquidity premium in Treasury inflation-protected securities (TIPS) yields can be substantial and vary over time. Nonetheless, there are large differences in the liquidity premium estimates available in the literature. Christensen and Gillan (2012) estimated a TIPS liquidity premium of the order of 30 to 40 basis points on average, ranging between 2 and 123 bps for a 10-year yield. Pflueger and Viceira (2013) estimated this liquidity premium to be about 70 bps for TIPS and 25 basis points for 10-year UK inflation linked gilts, with estimates generally being positive 4 but declining over time. D' Amico, Kim, and Wei (2014) model liquidity as a latent factor in no-arbitrage term structure models of nominal and TIPS yields and estimate an average 50 basis points liquidity premium for TIPS; applying their models to the UK data, they find that liquidity premia in index-linked gilt yields were fairly low (and smaller than liquidity premia in TIPS) prior the crisis, but they spiked to nearly 250 basis points at the height of the crisis.

In line with the literature, we assume that liquidity premia are present in gilt BEI rates but that liquidity premia are negligible for inflation swap BEI rates. Using the spread between gilt BEI and inflation swap BEI rates, we can therefore gain insights into liquidity conditions in bond markets. BEI rates adjusted for liquidity and inflation risk premia offer policy makers and investors measures of market expectations for RPI inflation over a range of future horizons. Without the adjustment, BEI rates could be substantially different from underlying market expectations, for example by up to 200 basis points based on our analysis.

It is common to analyse the BEI term structure using joint affine term structure models of nominal and real interest rates. The novelty of our paper lies in the fact we model the term structure of UK BEI rates directly. This approach simplifies the model and allows us to model both gilt and inflation swap BEI rates jointly with relative ease. It also avoids dealing with issues of there being a potential zero lower bound (ZLB) for UK nominal yields, which is not accounted for in the affine term structure model framework. While real and breakeven rates can turn deeply negative, nominal rates are more likely to be constrained on the downside. To model real, nominal and breakeven rates jointly, we would need some kind of shadow rate model in order to account properly for non-negativity in nominal rates. However, a shadow rate model applied to nominal and real yields jointly would add an extra complexity to an already highly parametrized non-linear model. Modelling breakeven rates directly keeps the model parsimonious and should not bias the estimates of inflation expectations and risk premia.

In addition, our model makes use of survey data covering professional forecasts of inflation, which help to identify the dynamics of the pricing factors and provide a reliable way to obtain robust decompositions (as shown in Joyce et al., (2010), Kim and Orphanides (2012), and Guimaraes (2014) among others).

Our results show that, after the introduction of inflation targeting in 1992 and the creation of the MPC in 1997, the significant falls in BEI rates reflected a fall in both inflation expectations and a fall in the inflation risk premium, suggesting that investors placed confidence in the new monetary policy framework. Both CPI and RPI inflation expectations have been reasonably stable at medium and long horizons since the introduction of the MPC in 1997.

The results also suggest that the negative sign of the risk premium in gilt BEI rates during the recent crisis was, to a large ex- tent, due to negative liquidity premia. These reflect periods of illiquidity in the market for index-linked gilts. Our estimates of inflation risk premia, which are required by investors to compensate them for uncertainty about future inflation, have been modestly positive during most of the sample. In addition, we show that the expected wedge between CPI and RPI inflation is quite volatile at short horizons but more stable at longer horizons, converging to around 66 basis points.

The rest of the paper is structured as follows: In Section 2 we motivate the analysis highlighting its relevance and interest for an audience of investors and policy makers; Section 3 explains the model setup and specifications; Section 4 describes the data and preliminary analysis; Section 5 discusses the empirical results; Section 6 presents the sensitivity analysis of the model; and, finally, Section 7 concludes the paper.

2. Motivation

In this section we discuss why the analysis carried out in the paper should be relevant and useful for investors and policy makers.

At the heart of our analysis is a Gaussian dynamic affine term structure model (GDATSM), applied jointly to inflation-linked gilts and swaps. Estimating a term structure model is essential if we want to extract market expectations, as there may be substantial inflation risk premia within BEI rates. It is also crucial if we want to understand the behaviour of liquidity premia in index-linked markets across time and maturities, which is important for various types of investors, e.g. for institutions like pension funds and insurances and other regular investors in the index-linked bonds; also for those accepting index-linked bonds as collateral.

2.1 Estimating the wedge between CPI and RPI

The Monetary Policy Committee (MPC) at the Bank of England has an explicit mandate to target CPI inflation of 2%. Economic agents' expectations about future CPI inflation play an important role in the MPC achieving its aim and delivering price stability. According to economic theory, inflation expectations have an effect on lots of economic choices, such as those concerning investments, long term acquisitions, mortgages, savings, or wage negotiations. And by shaping decisions in the real economy, inflation expectations impact realized inflation (the 'self-fulfilling prophecy' of inflation expectations). As a result, a rise in inflation expectations in the short-term runs the risk of increased inflationary pressures in the medium term. The MPC therefore monitors a range of indicators of inflation expectations to assess whether inflation expectations are anchored close to the inflation target.

In the UK, inflation-protected securities are linked to RPI inflation, while the MPC has a CPI inflation target. Therefore, when extracting policy relevant information from the gilts, it is essential to adjust for the wedge between CPI and RPI.

In this paper, we use a complex method, based on a GDATSM, to estimate the implied future wedge between the two inflation indices. There are various alternative ways to estimate future RPI- CPI wedge, but each of them has certain drawbacks, e.g.

- Long-run ('asymptotic') estimates - this method is silent about the RPI-CPI wedge and corresponding investor expectations at shorter horizons, which are crucial for assessing CPI expectations over the policy horizon.
- Projections by VAR-based models - this method is sensitive to a VAR specification, e.g. variable selection, coefficient restrictions, number of lags etc. Also wedge forecasts generated by VARs are not necessary consistent with investors'

expectations about the wedge.

- Market intelligence/surveys. These are infrequently conducted (relative to the frequency of information available from financial markets) and cover a subset of market participants.

Moreover, none of these methods rely on the rich and timely information provided by market prices. Only no-arbitrage models of market yields allow us to infer consistent market expectations across time and maturities. Such estimates of the market expected 'wedge' between RPI and CPI inflation help policy makers and investors to extract not only expectations of RPI, but also CPI expectations from the vast majority of sterling-denominated inflation- linked instruments referenced to RPI.

Finally, an important question is whether RPI-linked BEI rates are relevant for extracting forecasts of CPI inflation. We preliminarily examine to what extent BEI rates are good predictors for CPI inflation in a simple regression framework. Table 1 shows the result of this analysis. First, BEI rates from swaps have some predictive power for future CPI inflation 1-year ahead. Second, the whole BEI curve contains predictive information, and not only at maturities matching forecasting horizons.

Table 1
Estimated coefficient loadings β from one year ahead inflation predictive regressions $CPI_{t+12} = \alpha + \beta BEI_{H,t}$.

CPI forecasts, h = 12	1-year BEI, $BEI_{12,\,t}$	2-year BEI, $BEI_{24,\,t}$	3-year BEI, $BEI_{36,\,t}$
β	0.17 (1.91)	0.26 (2.09)	0.30 (1.97)

Note: The table shows the loading estimates β from the one-year ahead predictive regressions $CPI_{t+12} = \alpha + \beta BEI_{H,t}$ using various maturity breakeven inflation rates on the right-hand side ($BEI_{H,\,t}$ for H = 12,24,36.). The regressions are estimated at monthly frequency, with coefficient t-statistics given in parenthesis.

2.2 Estimating the liquidity premium

The second main question of the paper is to understand the importance of, and to account for, liquidity issues when analysing BEI rates from index-linked bonds. This question is of paramount importance not only for investors in index-linked bonds, but also for those responsible for the issuance and management of such bonds. BEI rates adjusted for liquidity and inflation risk premia allow policy makers and investors to infer market expectations of RPI inflation for the full range of future horizons. Without the adjustment, BEI rates can be substantially different from underlying inflation expectations, e.g. by up to 200 basis points as our subsequent analysis shows.

To analyse the liquidity premium, we have to rely on more than one source of inflation breakeven rates, using both inflation swaps and index-linked bonds. Analysing jointly inflation swap rates and BEI rates from gilts also gives us much richer information set.

One of the key assumptions in the model setup is that the difference between bond and swap BEI rates presents a liquidity premium. Below we rationalize this assumption in two steps: 1) index-linked gilts are assumed to be less liquid than nominal, which justifies a liquidity premium in gilt BEI rates; 2) inflation swap rates are assumed to be less contaminated by liquidity premia.

Gilt BEI rates are calculated from the difference between nominal and real yields. However BEI rates can be influenced by market liquidity conditions given that index-linked gilts are usually less liquid than nominal Treasury bonds. Previous studies have found fairly large bid-ask spreads and liquidity premia at certain times in index-linked government bond markets (see Bauer, 2015; Christensen and Gillan, 2012; and Fleckenstein et al., 2014). D' Amico, Kim, and Wei (2014) show that their model-implied index- linked gilt liquidity premium estimates can be linked to such observable measures of index-linked gilt liquidity as the difference between index-linked and conventional gilt asset swap spreads and the difference between the 10-year inflation swap rate and the 10- year gilt BEI rate.

For the UK, we find that nominal bonds generally have a much higher turnover and narrower bid-offer spreads than inflation linked gilts. And while their volumes have increased steadily over time since 1999, inflation-linked gilts are still relatively less actively traded than nominal gilts (Fig. 1). For recent data, trading in index-linked gilts only represents around 15% of total nominal and inflation-linked gilt turnover (up from less than 5% in 2004). Given the much larger turnover in nominal gilts, our conclusion is that we can realistically assume that nominal bonds are much more liquid than index-linked bonds in the UK market.

Moreover, while the turnover data captures only actual trading conditions, liquidity premia capture a combination of a current ease of trading and the risk that liquidity may deteriorate in the future. An alternative measure of (il)liquidity (the so-called Noise measure) is gaining momentum in the literature. This measure captures the average absolute yield curve fitting errors (For example, Fontaine and Garcia (2012) and Hu et al., (2013) use such measures for nominal bonds, while Ghrishchenko and Huang (2013) and D'Amico et al. (2014) apply it for TIPS.). Here we calculated this measure as the average absolute fitting errors from the Svensson yield curve and is likely to represent both current and forward looking liquidity risks. The fitting errors are large, and liquidity low, when funding constraints are particularly severe, preventing investors from arbitraging away differences across bond values. We have calculated the fitting errors in the UK nominal and real curves and created the 'equivalent' UK Noise indices. The measures, reported in Fig. 2, largely corroborate the considerations above; liquidity conditions in the UK gilt market are fine during normal times (like during 2004–2007), but abruptly deteriorate during turbulent times. And, crucially, according to this measure, UK nominal bonds appear persistently and substantially more liquid than UK index-linked gilts, especially in the post 2008 period, supporting our assumption.

We also assume here that liquidity premia in inflation swap BEI rates are likely to be smaller than those in gilt BEI rates. To support this assumption, it is worth recalling that the literature differentiates between the two notions of asset liquidity: funding liquidity, which relates to the ability of market participants to obtain funding, and market liquidity, which relates to the ease with which an asset can be traded (see, for example, Brunnermeier and Pedersen, 2009, Drehmann and Nikolaou, 2013).

First, we maintain that swap contracts contain minimal funding liquidity costs since they do not require large upfront payments, as is the case for bond investments. Hence, leveraged investors face lower capital constraints when gaining exposure to inflation-linked cash flows using inflation swaps compared to using index-linked bonds. As a result, inflation swap BEI rates may be less affected by market liquidity conditions than gilt BEI rates, since illiquid markets may be associated with higher funding costs and capital constraints.

Unfortunately, it is hard to measure market liquidity premia in inflation swaps due to a lack of quality data and transparency. We were not able to find UK inflation swap turnover data from publicly available sources. And it is also unsatisfactory to use estimated measures of turnover (e.g. from trade count data from Clarus 7 in the last 6 months and the average trade size from the swap depository https://rtdata.dtcc.com, which only capture swap transaction under Swap Execution Facility) as the majority of inflation swap transactions could be in the OTC market and so such estimates would only reflect a small proportion of total inflation swaps transactions.

While finding reliable turnover data on the UK inflation swap market liquidity may not be achievable, it could be helpful to look at the evidence based on data from the US inflation swap market. The UK and US inflation swap markets have a lot in common. They have both tended to trade in dealer-based OTC markets and have relatively few trades comparing to inflation-linked gilts (e.g. daily notional trading volume of US inflation swaps in 2010 was estimated to average $65 million, while in the TIPS market an estimated $5.0 billion per day was traded 8). Therefore, insights from US studies should be also valid for the UK case.

Using novel transaction data from the rapidly growing U.S. inflation swap market, Fleming and Sporn (2013) show that, de- spite its OTC nature and low level of trading activity, the inflation swap market is reasonably liquid and transparent, i.e. transaction prices are generally very close to widely available end-of-day quoted prices and realized bid-ask spreads are modest (only around 3 basis points). Consistent with their findings, Fleckenstein et al. (2014) attribute the spread between inflation swap rates and TIPS-implied breakeven rates to mispricing in the TIPS market and consider inflation swaps as a cleaner read of true inflation compensation.

Finally, we can also estimate a “Noise” (il)liquidity measure for the UK curves based on inflation swap rates. Supporting our assumption, the estimates 9 suggest that while before the financial crisis, the (il)liquidity measures were very small and comparable in the two markets, liquidity conditions in the index-linked bond market deteriorated substantially and stayed worse than those in the inflation swap market after the 2008 crisis. We therefore assume in our model that inflation swaps in the UK are very liquid compared to index-linked gilts. This assumption is similar to what is typically assumed in the credit risk literature where bonds carry a liquidity premium relative to credit default swaps.

In general, we admit that there can be an additional liquidity premia in inflation swaps. However, unlike in the bond markets, where nominal bonds are more liquid than real, and hence gilt implied BEIs would be biased downwards during liquidity crunches, it is unclear which direction swap rates should be driven by a liquidity premium. Therefore we abstract from it in this paper.

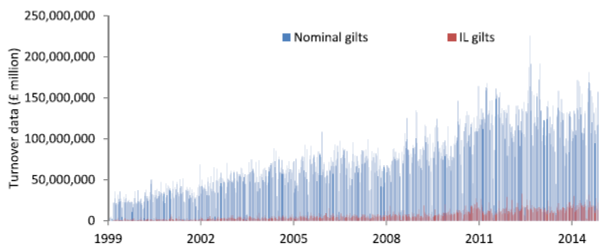

Fig. 1. UK Gilt turnover data.
Note: The figure shows the turnover volumes for the UK nominal bonds and inflation-linked gilts. Source: UK DMO.

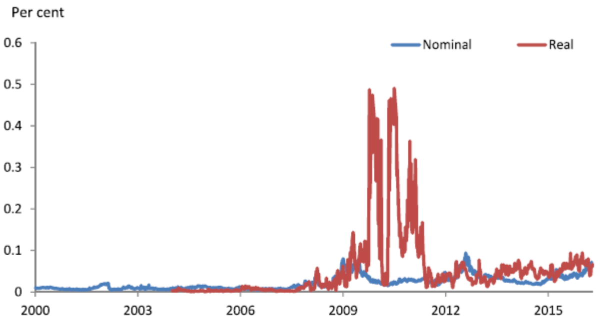

Fig. 2. Noise (il)liquidity measures from UK real and nominal yield curves.
Note: The measures are designed as the average absolute fitting errors from the corresponding Svensson yield curves (as in Fontaine and Garcia (2012), Hu et al., (2013) for nominal bonds, and in Ghrishchenko and Huang (2013) and D'Amico et al. (2014) for TIPS.). To allow comparison between them, both of the measures are calculated relative to mean yield values. The fitting errors are large, and liquidity low, when funding constraints are particularly severe, preventing investors from arbitraging away differences across bond values.

3. Affine term structure models of breakeven inflation rates

In this section we establish the term structure of breakeven inflation rates. In doing this we follow the standard affine models of nominal and real yield curves, where interest rates are assumed to be affine functions of risk factors. However, rather than the common approach of using joint affine term structure models of nominal and real interest rates, we model the term structure of UK BEI rates directly. This approach simplifies the model greatly and al- lows us to model gilt and inflation swap BEI rates jointly with relative ease.

In particular, the modelling approach adopted in this paper is based on a GDATSM for nominal and real rates. The main assumptions of such models are that all rates are driven by a set of pricing factors; short-term rates are affine functions of these factors; the factor dynamics are described by a VAR under the risk-adjusted pricing measure (Q) and under the physical ('real-world') pricing measure (P).

We start from defining short-term breakeven inflation rates as the difference between the nominal and real short rates, which can be modelled then as an affine function of factors. Subsequently, longer BEI rates can all be derived from the ratio of the nominal and real bond prices and be expressed as a function of corresponding risk factors.

We therefore show how we can decompose BEI rates into expected inflation rates and risk premia, and then how the expected inflation rates can be broken down into CPI inflation expectations and expectations of the wedge between RPI and CPI inflation. In turn, we decompose risk premia into inflation and liquidity components.

In one specification of our model we also try including professional survey expectations for both indices, which may help to identify model parameters that would otherwise be very imprecisely estimated and help to anchor the dynamics of the pricing factors.

Finally, we summarize the models for the RPI and CPI breakeven rates according to a state-space system, which can be estimated by standard econometric techniques. To do so we use a model normalization proposed by Joslin et al., (2011) (JSZ henceforth). This allows for model identification and also significantly simplifies its estimation.

3.1 Modelling CPI and RPI inflation rates

As is standard in affine term structure models, we assume that the one-period nominal risk-free interest rate at time t (i_t) is an affine function of a $M \times 1$ vector of unobserved risk factors, f_t :

$i_t = \delta_f f_t$,

where δ_f is a 1×M vector of constant factor loadings. The time t one-period risk-free real rate expressed in terms of CPI inflation, r_t^{CPI}, is also driven by the same factors, f_t:

$$r_t^{CPI} = \delta_f^{CPI}\mathbf{f}_t,$$

where δ_f^{CPI} is a M×1 vector of scalars.

Hence, by the Fisher equation, the short-term (i.e. one-period) CPI breakeven inflation rate at time t is given as

$$\pi_{t,1}^{CPI} = i_t - r_t^{CPI} = \left(\delta_f - \delta_f^{CPI}\right)\mathbf{f}_t \equiv \mathbf{J}\mathbf{f}_t \tag{1}$$

In what follows we stick to JSZ and normalise the factor vector in the above equation so that the factor loading coefficients for the short-term breakeven inflation rates are units, i.e.

$$\mathbf{J} = [\underbrace{1, 1 \ldots 1}_{M}]'$$

We also follow Guimaraes (2014) to assume, for the sake of simplicity and parsimony, that the one-period ahead expected CPI inflation rate at time t ($\pi_t^{e,CPI}$) is deterministic and equals the short-term breakeven inflation rate:

$$\pi_t^{e,CPI} = \pi_{t,1}^{CPI} = \mathbf{J}\mathbf{f}_t$$

The one-period RPI-linked risk-free real short rate at time t, r_t^{RPI}, is assumed to be driven by the risk factors f_t and an inflation wedge factor q_t:

$$r_t^{RPI} = \boldsymbol{\delta}_f^{RPI}\mathbf{f}_t + q_t = \left(\boldsymbol{\delta}_f^{CPI} + \boldsymbol{\theta}_f\right)\mathbf{f}_t + q_t, \tag{2}$$

where δ_f^{RPI} is a 1 by M vector of factor loadings, and θ_f = $\underbrace{[\theta_1, \ldots \theta_M]'}_{M}$ 1 by M vector, which represents the difference between δ_f^{RPI} and δ_f^{CPI}. The short-term RPI breakeven inflation rate($\pi_{t,1}^{RPI}$), which equals the expected one period RPI inflation rate at time t($\pi_t^{e,RPI}$), is modelled as the difference between the nominal and real short rate in a similar manner as the CPI breakeven inflation rate:

$$\begin{aligned}\pi_{t,1}^{RPI} = \pi_t^{e,RPI} &= i_t - r_t^{RPI} = \boldsymbol{\delta}_f\mathbf{f}_t - \left(\boldsymbol{\delta}_f^{CPI} + \boldsymbol{\theta}_f\right)\mathbf{f}_t - q_t \\ &= \mathbf{J}\mathbf{f}_t - \boldsymbol{\theta}_f\mathbf{f}_t - q_t\end{aligned} \tag{3}$$

As discussed above, the liquidity risk premia components in some RPI-linked instruments can be significant and need to be modelled. We do this by assuming that the short-term real rate used to price RPI-linked bonds ($r_t^{b,RPI}$) is adjusted by a liquidity, so we have:

$$r_t^{bRPI} = r_t^{RPI} + l_t = \boldsymbol{\delta}_f^{RPI}\mathbf{f}_t + q_t + l_t$$

where l_t denotes the one-period liquidity premium. The short-term RPI-linked gilt implied breakeven inflation rate is thus given as

$$\begin{aligned}\pi_{t,1}^{bRPI} = i_t - r_t^{bRPI} &= i_t - r_t^{RPI} - l_t = \pi_{t,1}^{RPI} - l_t \\ &= \mathbf{J}\mathbf{f}_t - \boldsymbol{\theta}_f\mathbf{f}_t - q_t - l_t\end{aligned} \tag{4}$$

We stack the latent factors f_t , the RPI-CPI wedge factor q_t and the liquidity spread factor l_t to get the state vector $\mathbf{x}_t = [\mathbf{f}_t', q_t, l_t]'$. As it is standard for GDATS models, we assume the state vector x_t follows a first-order Gaussian VAR under the risk-neutral pricing measure Q:

$$\begin{aligned}&\mathbf{x}_{t+1} = \boldsymbol{\kappa}^{\mathbb{Q}} + \boldsymbol{\Phi}^{\mathbb{Q}}\mathbf{x}_t + \boldsymbol{\Sigma}\varepsilon_{t+1}^{\mathbb{Q}} \\ &\boldsymbol{\varepsilon}_{t+1}^{\mathbb{Q}} \sim \mathcal{N}(0, \mathbf{I}),\end{aligned} \tag{5}$$

where we use the JSZ normalization and impose the following assumption on the drift of the Q-dynamics: κ^Q = $[\kappa_\infty^{\mathbb{Q}}, \underbrace{0 \ldots 0}_{M-1}, \kappa_q^{\mathbb{Q}}, \kappa_l^{\mathbb{Q}}]'$ is a K by 1 vector, where K=M+2; Φ^Q = $diag[\xi_1, \ldots, \xi_K]$, is a K by K matrix; the volatility matrix Σ is a K by K lower triangular matrix; the long-term mean of the CPI inflation rate under Q is given as $\kappa_\infty^{\mathbb{Q}}/(1-\xi_1)$.

Given that the short term CPI and RPI breakeven inflation rates are correspondingly equal to one-period ahead expected CPI($\pi_t^{e,CPI}$) and RPI($\pi_t^{e,RPI}$) inflation rates, we can write them as affine functions of x_t:

CPI short breakeven: $\pi_{t,1}^{CPI} = \pi_t^{e,CPI} = i_t - r_t^{CPI} = \mathbf{Jf}_t = \boldsymbol{\delta}\mathbf{x}_t$ (6)

RPI short breakeven:
$$\pi_{t,1}^{RPI} = \pi_t^{e,RPI} = i_t - r_t^{*,RPI} = \mathbf{Jf}_t - \boldsymbol{\theta}_f \mathbf{f}_t - q_t = \bar{\boldsymbol{\delta}}\mathbf{x}_t \quad (7)$$

where $\delta=[\mathbf{J},0,0]$ and $\bar{\delta} = [\mathbf{J} - \boldsymbol{\theta}_f, -1, 0]$.

Similarly, we can rewrite Eq. (4) for the gilt implied short-term breakeven inflation rate as an affine function of x_t :

$$\pi_{t,1}^{bRPI} = \mathbf{Jf}_t - \boldsymbol{\theta}_f \mathbf{f}_t - q_t - l_t = \bar{\boldsymbol{\delta}}^b \mathbf{x}_t$$

where: $\bar{\delta}^b = [\mathbf{J} - \boldsymbol{\theta}_f, -1, -1]$.

Therefore the expected one period ahead RPI-CPI wedge(w_t) is

$$w_t = \pi_t^{e,RPI} - \pi_t^{e,CPI} = -\boldsymbol{\theta}_f \mathbf{f}_t - q_t = \left(\bar{\delta} - \delta\right)\mathbf{x}_t$$

Also the short-term liquidity premium(l_t) is given as

$$l_t = \pi_{t,1}^{RPI} - \pi_{t,1}^{bRPI} = (\bar{\boldsymbol{\delta}} - \bar{\boldsymbol{\delta}}^b)\mathbf{x}_t$$

3.2 Term structure of breakeven inflation rates

We let P t, n denote the price of an n-period nominal zero-coupon conventional bond at time t, and let $P_{t,n}$* denote the price of an n-period synthetic real index-linked bond at time t. Under the assumption of no-arbitrage, we have the following bond pricing equations under the risk-neutral (or risk-adjusted) pricing measure Q:

$$P_{t,n} = E_t^{\mathbb{Q}}[\exp(-i_t)P_{t+1,n-1}]$$

$$P_{t,n}^* = E_t^{\mathbb{Q}}\left[\exp(-r_t)P_{t+1,n-1}^*\right],$$

where r_t is a selected real rate.

The ratio of the nominal and real bond prices of the same maturity can be expressed as the ratio of their expected prices in one period adjusted for the short-term breakeven inflation rate (see the Appendix A1 for more details):

$$\frac{P_{t,n}}{P_{t,n}^*} = \frac{\exp(-i_t)E_t^{\mathbb{Q}}[P_{t+1,n-1}]}{\exp(-r_t)E_t^{\mathbb{Q}}\left[P_{t+1,n-1}^*\right]} \approx \exp[-\pi_{t,1}]E_t^{\mathbb{Q}}\left(\frac{P_{t+1,n-1}}{P_{t+1,n-1}^*}\right) \quad (8)$$

Given (8), the affine nature of the short-term breakeven inflation rate, and the VAR(1) dynamics of x_t under the pricing measure Q, we can show that the ratio of the n-period nominal and real bond prices is an exponentially affine function of x_t :

$$\frac{P_{t,n}}{P_{t,n}^*} = \exp(\tilde{a}_n + \tilde{\mathbf{b}}_n \mathbf{x}_t),$$

where the scalar $\tilde{a}_n$ and 1×K scalar vector $\tilde{b}_n$ can be solved with the recursive equations:

$$\tilde{a}_n = \tilde{a}_{n-1} + \tilde{\mathbf{b}}_{n-1}\kappa^{\mathbb{Q}} + 0.5\tilde{\mathbf{b}}_{n-1}\boldsymbol{\Sigma}\Sigma'\tilde{b}_{n-1}' + \tilde{a}_1 \quad (9)$$

$$\tilde{\mathbf{b}}_n = \tilde{\mathbf{b}}_{n-1}\boldsymbol{\Phi}^{\mathbb{Q}} + \tilde{\mathbf{b}}_1 \quad (10)$$

At its maturity, the price of a nominal bond is given as £1 while the real price of an index-linked bond equals 1 unit of goods. Consequently, the ratio of the nominal and real bond prices is equal to one at maturity, which gives the boundary conditions for the recursion equations above: $\tilde{a}_0 = 0$ and $\tilde{b}_0 = -0$.

Therefore the n-period synthetic CPI BEI rate($\pi_{t,n}^{CPI}$), RPI inflation swap based BEI rate($\pi_{t,n}^{RPI}$) and RPI linked gilt BEI rate($\pi_{t,n}^{bRPI}$) can all be derived with the following general equation:

$$\pi_{t,n}^{j} = -\frac{1}{n}\ln\frac{P_{t,n}}{P_{t,n}^{*,j}} = -\frac{1}{n}\left(\tilde{a}_n^j + \tilde{\mathbf{b}}_n^j \mathbf{x}_t\right),$$

where j in $\pi_{t,n}^{j}$ represents one of the three different types of inflation rates as mentioned above, and $P_{t,n}^{*,j}$ denotes the real price of an n-period corresponding index-linked bond at time t. Also the scalar $\tilde{a}_n^j$ and the 1 by K scalar vectors $\tilde{b}_n^j$ can be derived recursively as shown in Eqs. (9) and (10), where we just replace the previous initial conditions ($\tilde{a}_1$, $\tilde{b}_1$) with the new ones for different breakeven rates:

$$\pi_{t,n}^{CPI}: \ \tilde{a}_1^{CPI} = 0, \tilde{\mathbf{b}}_1^{CPI} = -\boldsymbol{\delta};$$

$$\pi_{t,n}^{RPI}: \ \tilde{a}_1^{RPI} = 0, \tilde{\mathbf{b}}_1^{RPI} = -\bar{\boldsymbol{\delta}};$$

$$\pi_{t,n}^{bRPI}: \tilde{a}_1^{bRPI} = 0, \tilde{\mathbf{b}}_1^{bRPI} = -\bar{\boldsymbol{\delta}}^{\boldsymbol{b}}.$$

Given the unobserved nature of the factors, normalization restrictions on the parameters are required to identify the model. As explained earlier, we follow the identification scheme proposed by Joslin et al. (2011) to normalise the short rate and the drift of the Q-dynamics, which allows for more efficient estimation of the parameters under both the real-world (P) and the risk-neutral (Q) measures (see Guimarães, 2014). We also follow JSZ by carrying out the transformation to get the new portfolio factors, which can be principal components of a panel of time series data. In the original JSZ paper, the real world dynamics of the portfolio factors can be estimated with OLS independently of the risk neutral dynamics estimation. Although this will not be the case for the Kalman Filter estimation that we perform in this paper (see section 2.5 for more discussion of the estimation), we use the observed portfolio factors to find good starting values for the latent portfolio factors in the Kalman Filter estimation.

Let z_t denote the latent portfolio factors, which are constructed to match the first K principal components of RPI gilt BEI rates, short term CPI and the RPI breakeven inflation rates. We assume the portfolio factor vector z_t follows a VAR(1) process under both real-world P and risk-neutral Q measures. (This is the case for essentially affine models, where market prices of risk, which transform the factor dynamics under the actual probability measure P into the risk-neutral dynamics, are assumed to be affine functions of the factors). The latent factors x_t (as defined in Section 2.1) can be recovered from z_t via the inverse transformation. The linear transformation from the original latent factors x_t to the portfolio factors z_t is given as below:

$$\mathbf{z}_t = \mathbf{a}_g + \mathbf{B}_g \mathbf{x}_t$$

where $a_g = G \cdot a$, $B_g = G \cdot B$, G is the loading matrix used to construct the principal component vector z_t , and a, B are defined in Appendix A2.

The real-world dynamics of z_t is given by the following transition equation:

$$\mathbf{z}_{t+1} = \boldsymbol{\kappa}^z + \boldsymbol{\Phi}^z \mathbf{z}_t + \boldsymbol{\Sigma}^z \boldsymbol{\varepsilon}_{t+1}^z$$
$$\boldsymbol{\varepsilon}_{t+1}^z \sim \mathcal{N}(0, \mathbf{I})$$

where κ^z is a K by 1 vector, ϕ^z is a K by K matrix; Σ^z is a K by K lower triangular matrix.

As shown in Appendix A2, the inflation swap breakeven inflation rate ($\pi_{t,n}^{RPI}$) and index-linked bond breakeven inflation rate($\pi_{t,n}^{bRPI}$) are affine functions in terms of the new state vector z_t plus measurement errors:

$$\pi_{t,n}^{RPI} = -\frac{1}{n}\left(a_n^{RPI} + \mathbf{b}_n{}^{RPI}\mathbf{z}_t\right) + e_{t,n} \tag{11}$$

$$\pi_{t,n}^{bRPI} = -\frac{1}{n}\left(a_n^{bRPI} + \mathbf{b}_n^{bRPI}\mathbf{z}_t\right) + e_{t,n}^b \tag{12}$$

where a_n^j and b_n^j ($j \in$ {CPI, RPI, bRPI}) are the scalar and the $1 \times K$ vector loadings for various types of inflation breakeven rates and we assume the error terms $e_{t,n}$ and $e_{t,n}^b$ both follow independent normal distribution $N(0, \omega)$ with the same volatility.

3.3 Inflation projections

Kim and Orphanides (2012) suggest that the typically short time series available for estimating dynamic term structure models lead to problems identifying the P dynamics of the factors and suggest the use of survey data to help with the identification. In our case, we use survey data to provide more information on expected inflation in the future. As discussed in previous sections (see Eqs. (6), (7), (11)), the expected one-period CPI and RPI inflation rates are given by

$$\pi_t^{e,CPI} = -a_1^{CPI} - \mathbf{b}_1^{CPI}\mathbf{z}_t;$$

$$\pi_t^{e,RPI} = -a_1^{RPI} - \mathbf{b}_1^{RPI}\mathbf{z}_t.$$

Expected CPI and RPI inflations between n and n+1 periods ahead at time t under P are respectively given by

$$\text{CPI}: \quad \pi_{t,t+n}^{e,CPI} = E_t\left(\pi_{t+n}^{e,CPI}\right) = -a_1^{CPI} - \mathbf{b}_1^{CPI}E_t(\mathbf{z}_{t+n}) = -a_n^{e,CPI} - \mathbf{b}_n^{e,CPI}\mathbf{z}_t$$

$$\text{RPI}: \quad \pi_{t,t+n}^{e,RPI} = E_t\left(\pi_{t+n}^{e,RPI}\right) = -a_1^{RPI} - \mathbf{b}_1^{RPI}E_t(\mathbf{z}_{t+n}) = -a_n^{e,RPI} - \mathbf{b}_n^{e,RPI}\mathbf{z}_t$$

where

$$a_n^{e,CPI} = a_1^{CPI} + \mathbf{b}_1^{CPI}(\mathbf{I} - \mathbf{\Phi}^z)^{-1}\left(\mathbf{I} - (\mathbf{\Phi}^z)^n\right)\boldsymbol{\kappa}^z$$

$$\mathbf{b}_n^{e,CPI} = \mathbf{b}_1^{CPI}(\mathbf{\Phi}^z)^n$$

$$a_n^{e,RPI} = a_1^{RPI} + \mathbf{b}_1^{RPI}(\mathbf{I} - \mathbf{\Phi}^z)^{-1}\left(\mathbf{I} - (\mathbf{\Phi}^z)^n\right)\boldsymbol{\kappa}^z$$

$$\mathbf{b}_n^{e,RPI} = \mathbf{b}_1^{RPI}(\mathbf{\Phi}^z)^n$$

Given that the surveys refer to annual inflation expectations rather than monthly inflation, we derive the model-implied annual inflation as

$$\pi_{t,t+n}^{a,e,CPI} = \sum_{i=1}^{12}\pi_{t,t+n+i}^{e,CPI} + e_{t,n}^{a,CPI} = \sum_{i=1}^{12}\left(-a_{n+i}^{e,CPI} - \mathbf{b}_{n+i}^{e,CPI}\mathbf{z}_t\right) + e_{t,n}^{a,CPI}$$

$$\pi_{t,t+n}^{a,e,RPI} = \sum_{i=1}^{12}\pi_{t,t+n+i}^{e,RPI} + e_{t,n}^{a,RPI} = \sum_{i=1}^{12}\left(-a_{n+i}^{e,RPI} - \mathbf{b}_{n+i}^{e,RPI}\mathbf{z}_t\right) + e_{t,n}^{a,RPI}$$

where the survey forecasts over n horizon are measured with a normally and independently distributed error term $e_{t,n}^{a,CPI} \sim N(0, \omega^{a,CPI})$ and $e_{t,n}^{a,RPI} \sim N(0, \omega^{a,RPI})$.

3.4 Breakeven inflation decomposition

The fitted BEI rates can be decomposed into two components: expectations for future inflation and a risk premium. For the inflation swap BEI($\hat{\pi}_{t,n}^{RPI}$) , we assume that the risk premium consists of only an inflation risk premium, given our assumption that the liquidity premium embedded in inflation swap rates is generally very small and difficult to identify. But for the fitted gilt BEI rate($\hat{\pi}_{t,n}^{RPI}$), the risk premium includes both an inflation risk premium and a liquidity premium. The expectations component and the inflation risk premium components should be the same for both inflation swap and gilt BEI rates.

The decompositions for fitted values of inflation swap (IS) and gilt BEI rates are given below

$$\text{IS BEI}: \hat{\pi}_{t,n}^{RPI} = exp_{t,n} + tp_{t,n}^{is} = exp_{t,n} + rp_{t,n}$$

$$\text{Gilt BEI}: \hat{\pi}_{t,n}^{bRPI} = exp_{t,n} + tp_{t,n}^{gilt} = exp_{t,n} + rp_{t,n} + lp_{t,n}^{gilt}$$

where we have:

Expected inflation: $exp_{t,n} = \frac{1}{n}\sum_{i=1}^{n} \pi_{t,t+i}^{e,RPI}$
Inflation risk premium: $rp_{t,\,n}$
Bond liquidity premium: $lp_{t,n}^{gilt} = \hat{\pi}_{t,n}^{b,RPI} - \hat{\pi}_{t,n}^{RPI}$
risk premium for inflation swap BEI: $tp_{t,n}^{is} = rp_{t,n}$
risk premium for gilt BEI: $tp_{t,n}^{gilt} = rp_{t,n} + lp_{t,n}^{gilt}$

One of the key assumptions made in this paper is that the difference between bond and swap BEI rates represents a liquidity premium. This is mainly because swaps and bonds have different characteristics, among which the most important is that swaps do not require large upfront payments as would be required for bond investments. Hence leveraged investors would face lower capital constraints to gain exposures to inflation-linked cashflows using inflation swaps compared to using index-linked bonds. These constraints will generally affect the ability to arbitrage between conventional and inflation linked bonds and it will tend to be priced as a charge e.g. a liquidity premium that makes the bond yield higher and hence the BEI rate lower. This characteristic is often referred to as 'shadow cost of capital' (see Garleanu and Pedersen (2011)). Some other reasons for swaps being more liquid than bonds include the more flexible nature of cash flows in swaps, which means that it is less likely for a swap to become "special" in the way that government bonds might. In addition, it can be difficult and costly to short physical bonds at some times. But, it is generally as easy to sell inflation protection as it is to buy protection in inflation swap markets.

3.5 State-space system and Kalman filter

We can summarise the above models for the RPI bond breakeven rates, RPI and CPI breakeven rates according to the following state-space system below.

$$\mathbf{z}_{t+1} = \kappa^{z} + \mathbf{\Phi}^{z}\mathbf{z}_t + \mathbf{w}_{t+1}, \mathbf{w}_{t+1} \sim \mathcal{N}\left(0, \mathbf{\Sigma}^{z}\mathbf{\Sigma}^{z\prime}\right) \tag{13}$$

$$\mathbf{y}_t = \mathbf{a} + \mathbf{B}\mathbf{z}_t + \mathbf{e}_t,\ \ \mathbf{e}_t \sim \mathcal{N}\left(0, \Omega\Omega'\right) \tag{14}$$

where

$$\mathbf{y}_t = \begin{pmatrix} \boldsymbol{\pi}_t^{bRPI} \\ \boldsymbol{\pi}_t^{RPI} \\ \boldsymbol{\pi}_t^{a,e,CPI} \\ \boldsymbol{\pi}_t^{a,e,RPI} \\ \pi_{t,1}^{CPI} \\ \pi_{t,1}^{RPI} \\ \pi_{t,1}^{bRPI} \end{pmatrix}, \quad \mathbf{a} = \begin{pmatrix} -\mathbf{a}^{bRPI} \\ -\mathbf{a}^{RPI} \\ -\mathbf{a}^{e,CPI} \\ -\mathbf{a}^{e,RPI} \\ -a_1^{CPI} \\ -a_1^{RPI} \\ -a_1^{bRPI} \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -\mathbf{B}^{bRPI} \\ -\mathbf{B}^{RPI} \\ -\mathbf{B}^{e,CPI} \\ -\mathbf{B}^{e,RPI} \\ -\mathbf{b}_1^{CPI} \\ -\mathbf{b}_1^{RPI} \\ -\mathbf{b}_1^{bRPI} \end{pmatrix}$$

and

$$\mathbf{a}^{bRPI} = \left(\frac{1}{n_1}a_{n_1}^{bRPI}, \frac{1}{n_2}a_{n_2}^{bRPI}, \ldots \frac{1}{n_N}a_{n_N}^{bRPI}\right)'$$

$$\mathbf{a}^{RPI} = \left(\frac{1}{n_1}a_{n_1}^{RPI}, \frac{1}{n_2}a_{n_2}^{RPI}, \ldots \frac{1}{n_N}a_{n_N}^{RPI}\right)'$$

$$\mathbf{a}^{e,CPI} = \left(\sum_{i=1}^{12} a_{n_1+i}^{e,CPI}, \sum_{i=1}^{12} a_{n_2+i}^{e,CPI}, \ldots \sum_{i=1}^{12} a_{n_N+i}^{e,CPI}\right)'$$

$$\mathbf{a}^{e,RPI} = \left(\sum_{i=1}^{12} a^{e,RPI}_{n_1+i}, \sum_{i=1}^{12} a^{e,RPI}_{n_2+i}, \ldots \sum_{i=1}^{12} a^{e,RPI}_{n_N+i} \right)'$$

$$\mathbf{B}^{bRPI} = \begin{pmatrix} \frac{1}{n_1}\mathbf{b}^{bRPI}_{n_1} \\ \frac{1}{n_2}\mathbf{b}^{bRPI}_{n_2} \\ \cdots \\ \frac{1}{n_N}\mathbf{b}^{bRPI}_{n_N} \end{pmatrix}, \quad \mathbf{B}^{RPI} = \begin{pmatrix} \frac{1}{n_1}\mathbf{b}^{RPI}_{n_1} \\ \frac{1}{n_2}\mathbf{b}^{RPI}_{n_2} \\ \cdots \\ \frac{1}{n_N}\mathbf{b}^{RPI}_{n_N} \end{pmatrix},$$

$$\mathbf{B}^{e,CPI} = \begin{pmatrix} \mathbf{b}^{e,CPI}_{n_1} \\ \mathbf{b}^{e,CPI}_{n_2} \\ \cdots \\ \mathbf{b}^{e,CPI}_{n_N} \end{pmatrix}, \quad \mathbf{B}^{e,RPI} = \begin{pmatrix} \mathbf{b}^{e,RPI}_{n_1} \\ \mathbf{b}^{e,RPI}_{n_2} \\ \cdots \\ \mathbf{b}^{e,RPI}_{n_N} \end{pmatrix}$$

$$\mathbf{\Omega} = \mathrm{diag}\Bigg(\underbrace{\omega, \ldots, \omega}_{for\ \pi^{bRPI}_t, \pi^{RPI}_t}, \underbrace{\omega^{a,CPI} .. \omega^{a,CPI}}_{for\ \pi^{a,e,CPI}_t}, \underbrace{\omega^{a,RPI}, \ldots, \omega^{a,RPI}}_{for\ \pi^{a,e,RPI}_t}, \underbrace{\omega, \omega, \omega}_{for\ \pi^{CPI}_{t,1} /\ \pi^{RPI}_{t,1}\ / \pi^{bRPI}_{t,1}} \Bigg)$$

The state Eq.(14) shows the real-world dynamics of the state vector, z_t. The measurement Eq. (14) gives the mapping between the observed variables and the state vector, where the observed variables include: RPI bond breakeven $\pi_{t,n}^{b,RPI}$; RPI inflation swap rates $\pi_{t,n}^{RPI}$; annual CPI survey expectations $\pi_{t,t+n}^{a,e,CPI}$; RPI survey expectations $\pi_{t,t+n}^{a,e,RPI}$. We also add one-month breakeven inflation rate($\pi^{CPI}_{t,1}$, $\pi^{RPI}_{t,1}$ and $\pi^{bRPI}_{t,1}$ for CPI, inflation swap and gilt RPI respectively) to pin down the short end of the breakeven rate curve.

We estimate the complete model(13) and (14) using maximum log-likelihood estimation, where the Kalman Filter is used to filter the factors. The log-likelihood function to be maximised is given as below:

$$\log \mathcal{L}(\mathbf{\Theta}; \mathbf{y}_{t=1,\ldots,T}) = \sum_{t=1}^{T} \log f(\mathbf{y}_t | \mathbf{y}_{t-1}, \mathbf{\Theta})$$

where $\mathbf{\Theta}$ is the parameter set that include all the parameters to be estimated, i.e. $\mathbf{\Theta} = \{\boldsymbol{\kappa}^{\mathbb{Q}}, \ \mathbf{\Phi}^{\mathbb{Q}}, \boldsymbol{\theta}_f, \boldsymbol{\kappa}^{z}, \mathbf{\Phi}^{\mathbf{z}}, \mathbf{\Sigma}^{\mathbf{z}}, \mathbf{\Omega}\}$. The parameters $\boldsymbol{\kappa}^{\mathbb{Q}}$, $\mathbf{\Phi}^{\mathbb{Q}}$ are defined in Eq. (5), $\boldsymbol{\kappa}^{z}$, $\mathbf{\Phi}^{\mathbf{z}}$, $\mathbf{\Sigma}^{\mathbf{z}}$ in (13), $\boldsymbol{\theta}_f$ in Eq. (2), and $\mathbf{\Omega}$ in Eq. (14).

4. Data and preliminary analysis

Our sample period spans October 1992 to December 2013 with data observations at a monthly frequency. The main reason for starting the sample period from October 1992 is to match a major change in the monetary policy framework in the UK, which adopted inflation targeting in October 1992, and hence to avoid a possible structural break in the data.

Gilt BEI rates(Fig.3.A) are computed as the difference between continuously compounded nominal and real spot rates (i.e. yields on zero-coupon bonds), which are estimated using the Variable Roughness Penalty (VRP) model by Anderson and Sleath (2001) and published by the Bank of England. For bond BEI rates, we use 3-, 4-, 5-, 7- and 10-year maturities from October 1992 to December 2013, sampled at monthly frequency on the 21st day of the month in line with the CPI and RPI data releases.

Inflation swap implied BEI rates 14 (Fig.3.B) are also obtained for the same bond maturities. Unfortunately, inflation swaps are only available from 2004. Our estimation methodology, which is based on the Kalman Filter, can cope with the missing data problem given that the estimation of state vectors will not be seriously affected by the missing data issue. In the Kalman Filter, the observable variables (the BEI rates and other inflation data) are used to improve the first-round estimate of the state vectors, rather than working as a direct input for calculating the state vectors.

The exclusion of maturities shorter than three years is due to the lack of good quality data at the short end of the BEI curves. According to Anderson and Sleath (2001), constraints are applied to the VRP model to guarantee stability at the short end of the curve by omitting index-linked bonds with short maturities or bonds that are unsuitable due to the small number available in the specific curve segment. This creates gaps in the time series of real spot rates, and hence of BEI rates, at shorter maturities.

In order to address the issue of a lack of short maturity data, we also include proxies for one-month CPI and RPI breakeven rates (i.e. $\pi_{t,1}$ and $\bar{\pi}_{t,1}$) in the model (see data plot in Fig.3.C). These are approximated by regressing the month-on-month CPI and RPI inflation on the lagged year-on-year CPI and RPI inflation rates (Fig.3.D). The UK CPI and RPI price index data we use are non- seasonally adjusted, and published monthly by the Office for National Statistics (ONS). The realised rate of year-on-year inflation is calculated as the annual log change of the price index. We use lagged year-on-year inflation rates as explanatory variables instead of lagged month-on-month inflation rates to avoid seasonality exhibited in month-on-month inflation time series, which is highly undesirable. Therefore the one-month CPI and RPI inflation breakeven rates are approximated by a linear function of lagged year-on-year inflation rates in our paper.

Our proxy for the one-month RPI breakeven rate is useful for identifying the short end of inflation swap BEI curves. For the gilt BEI curve, we need to adjust for the liquidity premium, as dis- cussed in the previous sections. A proxy for the one-month bond breakeven rate is derived as the one-month RPI breakeven rate adjusted for a short-term liquidity spread, estimated by regressing bond-swap breakeven spreads (i.e. liquidity premium) on the corresponding maturities at each period. This assumes the term structure of liquidity premia follows a straight line and the value of the short term liquidity spread can be inferred by extending this line to the one-month maturity. We apply this short term liquidity spread adjustment to the one-month RPI breakeven rate for the period after 2004. We cannot do the same adjustment for the period before 2004 due to the lack of inflation swap data. Therefore, we approximate the one-month gilt BEI inflation rate using the one-month RPI inflation rate for the period before 2004. This is a simplistic assumption, but may not be unreasonable given that inflation was fairly stable over the period in question.

We supplement the dataset with survey data for CPI and RPI inflation expectations 1 to 10 years ahead from Consensus Economics (Fig.3E-F). They are available semi-annually from April 2004 for CPI and from April 1990 for RPI. In particular, we use the mean estimates of individual panelists' inflation expectations for CPI and RPI from Consensus forecasts publications. And although one issue with the survey forecasts is that they tended to underestimate future realized inflation over the given sample, nonetheless they provide a better forecasting performance than the benchmark comparator Random Walk, as can be seen from Table 2 showing mean square errors produced by CPI surveys relative to those produced by a random walk.

Table 2
Relative mean squared errors of survey CPI forecasts with respect to random walk.

Forecast horizon:	1 year	2 years	3 years	4 years	5 years
RMSE	0.67	0.75	1.31	0.62	0.90

Note: Value less than one indicates superior forecasts by the surveys. The surveys are presented by the mean estimates of individual panellists' inflation expectations for CPI and RPI from Consensus Survey forecasts publications. Sample: 2005–2014.

Another issue with the RPI survey data is that the forecast is actually given for the RPIX inflation (RPI inflation excluding mortgage interest payments). Following Joyce et al., (2010) who noted that at medium to long horizons the RPI/RPIX wedge is likely to be small, we do not take into account the difference between the two indices.

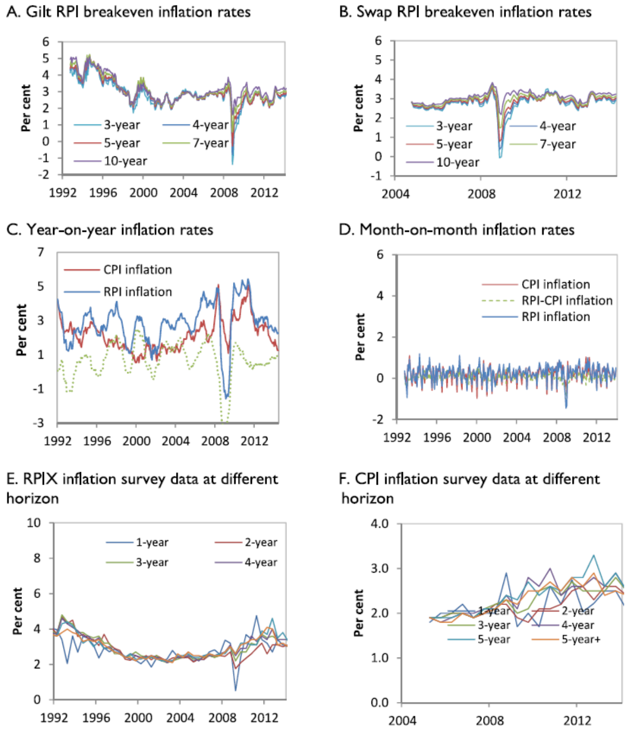

Fig. 3. Inflation breakeven rates, realised inflation and surveys.
Note: Bond RPI breakeven inflation rates are computed as the difference between continuously compounded nominal and real spot yields published by the Bank of England. RPI Swaps breakeven inflation rates are by the Bank of England. CPI and RPI inflation surveys are from Consensus Economics. CPI surveys and RPI swaps are only available after October 2004. Monthly inflation rates used in estimation have been annualised and obtained by regressing month-on-month inflation data corresponding to the actual date for price on year-on-year inflation data that have a 1-month lag and correspond to release date.

As our JSZ portfolio weights are chosen to be the same as the loadings used to construct the principal components, we carried out principal component analysis following standard practice in the term structure literature, to identify the number of factors required to explain the variance in BEI rates, CPI inflation and RPI inflation (Table 3). The analysis shows that 5 principal components are required to explain 99.79% of the data variance for gilt BEI, CPI and RPI inflation for the sample period from 1992 to 2013. We did a similar analysis for both gilt and inflation swap BEI rates, but excluding CPI and RPI inflation, with a data sample from 2004 to 2013. In this case we need at least 4 principal components to explain 99.94% of the variance. A portfolio of inflation swap BEI rates, and CPI and RPI inflation data from 2004 to 2014 would only require 3 factors. So, overall, we need at least 5 factors for our model in order to fit the gilt and inflation swap BEI rates as well as RPI and CPI inflation. This also shows that modelling breakeven rates directly instead of from a joint nominal and real curve estimation made our specifications more parsimonious given that we need at least 6 or 7 factors to explain the same proportion of variance of a portfolio of BEI rates, nominal rates and CPI and RPI inflation.

Table 3
Principal Components of a portfolio of BEI rates (3 to 10-year maturity), CPI and RPI inflation.

	Gilt BEI, CPI and RPI inflation (from 1992 - 2013)		IS BEI, CPI and RPI inflation (from 2004 - 2013)		IS and gilt BEI (from 2004 - 2013)	
Principal component	% variance explained	Cumulative %	% variance explained	Cumulative %	% variance explained	Cumulative %
1	83%	83.00%	76.00%	76.000%	76.30%	76.30%
2	10%	93.00%	13.2%	89.200%	13.50%	89.80%
3	4.2%	97.20%	10.2%	99.400%	7.91%	97.71%
4	2.5%	99.70%	0.54%	99.940%	1.99%	99.70%
5	0.09%	99.79%	0.02%	99.960%	0.30%	100.00%

Note: The samples consist of bond and swap inflation breakeven rates of maturities of 3, 4, 5 and 10 years.

5. Results

Based on the principal component analysis, our preferred model has 5 factors and it is estimated for the sample period between October 1992 and the end of December 2013. The model fits bond and swap BEI rates well at all maturities. For example, the fitting errors of 5 and 10 year BEI rates are less than 20 basis points (see Fig.4) in absolute terms. In Table 4 we report the estimated model parameters: $\Theta = \{\kappa^{\mathbb{Q}}, \Phi^{\mathbb{Q}}, \theta_f, \kappa^z, \Phi^z, \Sigma^z, \Omega\}$. We found that the diagonal parameters in $\Phi^{\mathbb{Q}}$ are all significant with the largest element very close to 1 (i.e. 0.990), showing the high persistency of the dynamics of factors under Q. We find that the largest eigenvalue for the matrix Φ^z is 0.988, which is also very high. This suggests that the factors are also highly persistent under P measure.

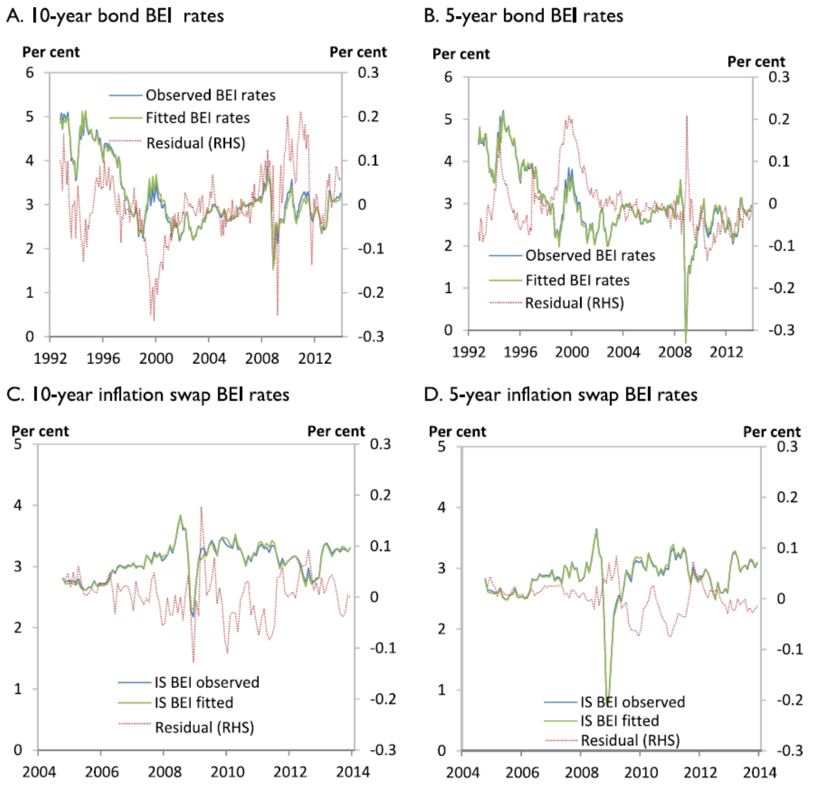

Fig. 4. Actual and fitted bond and swap spot breakeven rates at selected maturities.
Note: IS BEI stands for Inflation Swap Breakeven rates. The sample period of the preferred model is October 1992 and December 2013. Swap data are only available after May 2004. All data are by the Bank of England. The observed bond and inflation swap breakeven rates are plotted with reference to the left hand axis. Residuals are plotted with reference to the right hand axis in percentage points. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Table 4
Estimated parameters.

Parameter	Estimation	Parameter	Estimation	Parameter	Estimation
$\boldsymbol{\kappa}^{Q}$		$\boldsymbol{\Phi}^{z}$		$\boldsymbol{\Sigma}^{z}$	
κ^{Q}_{∞}	3.03E-05	$\Phi^{z}_{1,1}$	0.964621**	$\Sigma^{z}_{1,1}$	0.000422**
	(0.000245)		(0.010718)		(1.71E-05)
κ^{Q}_{q}	-4.16E-07	$\Phi^{z}_{2,1}$	-0.02783	$\Sigma^{z}_{2,1}$	-8.27E-05**
	(0.000312)		(0.060369)		(3.22E-05)
κ^{Q}_{l}	2.86E-10	$\Phi^{z}_{3,1}$	0.234792**	$\Sigma^{z}_{3,1}$	-1.62E-05**
	(2.52E-07)		(0.084033)		(6.97E-06)
$\boldsymbol{\Phi}^{Q}$		$\Phi^{z}_{4,1}$	-0.21543	$\Sigma^{z}_{4,1}$	4.66E-05**
			(0.281361)		(5.50E-06)
ξ_1	0.990392**	$\Phi^{z}_{5,1}$	0.001544	$\Sigma^{z}_{5,1}$	-4.66E-06
	(0.00494)		(0.884889)		(2.86E-06)
ξ_2	0.980790**	$\Phi^{z}_{1,2}$	0.029709**	$\Sigma^{z}_{2,2}$	0.000331**
	(0.008495)		(0.015132)		(2.67E-05)
ξ_3	0.878017**	$\Phi^{z}_{2,2}$	0.977231**	$\Sigma^{z}_{3,2}$	6.00E-05**
	(0.018632)		(0.068983)		(2.21E-05)
ξ_4	0.986378**	$\Phi^{z}_{3,2}$	-0.14111	$\Sigma^{z}_{4,2}$	3.46E-05**
	(0.063425)		(0.116641)		(9.94E-06)
ξ_5	0.988050**	$\Phi^{z}_{4,2}$	0.296531	$\Sigma^{z}_{5,2}$	-3.73E-05**
	(0.000783)		(0.308671)		(2.69E-06)
$\boldsymbol{\theta}^{*}_{f}$		$\Phi^{z}_{5,2}$	0.000354	$\Sigma^{z}_{3,3}$	8.15E-05
			(0.961827)		(1.56E-05)
θ^{*}_{f1}	0.055388	$\Phi^{z}_{1,3}$	-0.00461	$\Sigma^{z}_{4,3}$	-1.53E-06**
	(0.219931)		(0.004413)		(5.74E-06)
θ^{*}_{f2}	0.000297	$\Phi^{z}_{2,3}$	0.023694	$\Sigma^{z}_{5,3}$	-4.06E-06**
	(0.286948)		(0.020434)		(1.61E-06)
θ^{*}_{f3}	-0.000465	$\Phi^{z}_{3,3}$	0.920517**	$\Sigma^{z}_{4,4}$	5.96E-05
	(0.285091)		(0.038479)		(7.78E-06)
$\boldsymbol{\kappa}^{z}$		$\Phi^{z}_{4,3}$	-0.08653	$\Sigma^{z}_{5,4}$	-1.60E-05**
			(0.096722)		(1.58E-06)
κ^{z}_{1}	0.000150	$\Phi^{z}_{5,3}$	0.000432	$\Sigma^{z}_{5,5}$	5.17E-06
	(0.000106)		(0.299253)		(3.64E-06)
κ^{z}_{2}	1.95E-05	$\Phi^{z}_{1,4}$	-0.0051		
	(0.000129)		(0.00327)		
κ^{z}_{3}	-5.35E-05*	$\Phi^{z}_{2,4}$	-0.01177		
	(3.14E-05)		(0.019835)		
κ^{z}_{4}	5.75E-06	$\Phi^{z}_{3,4}$	0.035233		
	(3.30E-05)		(0.026455)		
κ^{z}_{5}	-1.88E-06	$\Phi^{z}_{4,4}$	0.888555**		
	(1.23E-05)		(0.086601)		
$\boldsymbol{\Omega}$		$\Phi^{z}_{5,4}$	0.000179		
			(0.292131)		
ω	5.54E-05**	$\Phi^{z}_{1,5}$	-6.99E-07		
	(5.60E-07)		(0.001412)		
$\omega^{a,\,CPI}$	0.000275**	$\Phi^{z}_{2,5}$	0.001904		
	(1.96E-05)		(0.005389)		
$\omega^{a,\,RPI}$	0.000322**	$\Phi^{z}_{3,5}$	1.95E-05		
	(1.52E-05)		(0.011348)		
		$\Phi^{z}_{4,5}$	0.0012		
			(0.030619)		
		$\Phi^{z}_{5,5}$	0.959584**		
			(0.08411)		

Note: The table shows the estimates of the five-variable state vector dynamics under Q and P distributions: $\mathbf{z}_{t+1} = \boldsymbol{\kappa}^{z} + \boldsymbol{\Phi}^{z}\mathbf{z}_t + \mathbf{w}_{t+1}, \mathbf{w}_{t+1} \sim \mathcal{N}(0, \boldsymbol{\Sigma}^{z}\boldsymbol{\Sigma}^{z\prime})$, together with the measurement error variance estimates from $\mathbf{y}_t = \mathbf{a} + \mathbf{B}\mathbf{z}_t + \mathbf{e}_t,\ \mathbf{e}_t \sim \mathcal{N}(0, \boldsymbol{\Omega}\Omega')$. Significance level: **5%; *10%. Numbers in parenthesis are standard deviations, which are calculated by using the outer product of the scores, as explained in Greene (2011) on his discussion of the BHHH estimator.[20] The largest eigenvalue for the $\boldsymbol{\Phi}^{z}$ matrix is 0.988.

We used the JSZ method, which specifies the dynamics of factors under P and Q rather than the prices of risk and stochastic discount factor (SDF) directly. Still, we can back out the market prices of risk from the P and Q dynamics, given that the factors under the P measure are distributed according to

$$z_{t+1} = \kappa^{z} + \Phi^{z} z_t + \Sigma^{z}\varepsilon^{z}_{t+1},$$

while they also follow Gaussian process under Q, but with different coefficients:

$$z_{t+1} = \kappa^{zQ} + \Phi^{zQ} z_t + \Sigma^{zQ}\varepsilon^{zQ}_{t+1}$$

The coefficients κ^{zQ}, Φ^{zQ} are determined from the two equations: $x_{t+1} = \kappa^{Q} + \Phi^{Q} x_t + \Sigma\varepsilon^{Q}_{t+1};\ z_t = a_g + B_g x_t$, so that $\kappa^{zQ} = B_g\kappa^{Q} + (I - B_g\Phi^{Q}B_g^{-1})\ a_g;\ \Phi^{zQ} = B_g\Phi^{Q}B_g^{-1}z_t.$

The market prices of risk, λ_t, which transform P into Q and determine SDFs, are assumed to be affine in factors: $\lambda_t = \lambda_0 + \lambda_1 z_t$. They can be recovered from the following equations:

$$\kappa^{z} = \kappa^{zQ} + \Sigma^{z}\lambda_0,\ \ \Phi^{z} = \Phi^{zQ} + \Sigma^{z}\lambda_1$$

In our approach, we model directly the breakeven rates and abstract from nominal and real interest rates and therefore cannot estimate respective SDFs. But we can get the idea of the expected BEI SDF at time t by analysing the prices of risk component. This component is also present in the nominal SDF. Fig.5 provides the plot of the time series of $-0.5\,\lambda'_t\,\lambda_t$, which is the focal component and a useful diagnostic of the validity of the SDF. The series shows that the risk pricing was biggest at the end of 2008, after the col- lapse of Lehman Brothers and at the height of the global financial crisis. On the contrary, the market discounting of risk stayed persistently at minimal, close to zero, values during the period of low volatility and low interest rates of 2003–2006.

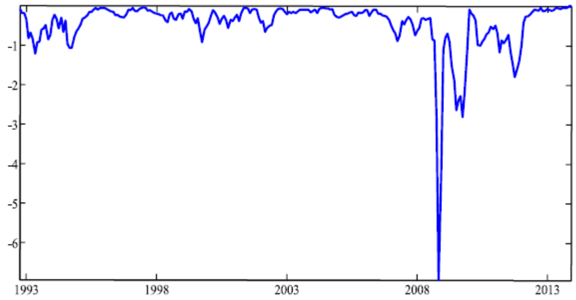

Fig. 5. $-0.5\lambda_t'\lambda_t$ component of the SDF.
Note: Figure provides the plot of the time series of $-0.5\lambda'_t\lambda_t$, which is the focal component and a useful diagnostic of the validity of the SDF. λ_t is a vector of market prices of risk.

5.1 Gilt and swap BEI rates decomposition

As Fig.6 (panels A-B) shows, both long and medium term (i.e.10 and 5-year respectively) gilt BEI rates fell significantly after the introduction of inflation targeting in 1992 and drifted downwards during the 1990 s. This is partially accounted for by a fall in inflation expectations. The fall in breakeven rates was also associated with a significant fall in inflation risk premia, suggesting investors had more confidence in the new monetary policy framework and/or were less uncertain about future inflation. These results are consistent with the earlier findings by Joyce et al. (2010) and Abrahams et al. (2015).

With the exception of Q3-2008, when both 5 and 10-year RPI inflation expectations peaked at around 3.5%, and the subsequent falls in inflation expectations during the height of the financial crisis, our measures of medium and long-term RPI inflation expectations are reasonably stable and average around 2.8% since 1998.

Estimates of the 10-year gilt BEI term premium(Fig.6.A) were generally positive and decreasing across the sample period, averaging around 1% until 1997, 20 basis points between 1997 and 2008 and minus 10 basis points thereafter, in line with the estimates by Abrahams et al. (2015). Fig.6.B shows that the 5-year gilt BEI term premium also exhibits a similar downward trend over the sample.

The decomposition of this term premium for the 10-year gilt BEI rate after 2004 (Fig.6.C) shows that the inflation risk premium was, on average, 15 basis points. The maximum level was reached in October 2009 at 75 basis points. It went down to −40 basis points in Q4-2011. Fig.6.D shows the 5-year risk premium decomposition, where the inflation risk premium is slightly negative (−6 basis points on average after the crisis) but the liquidity premium is much lower (−44 basis points on average after the crisis). This is rather different from the estimates discovered in previous studies, such as Guimaraes(2014), which found large negative inflation premia in the medium and long-term gilt BEI rates since the crisis. This may be because those estimates not only include the inflation risk premium, which is driven by uncertainty about future inflation risk, but also a liquidity premium. Therefore our model suggests that the negative sign of gilt BEI term premia since the crisis is more the result of market liquidity factors rather than a strongly negative inflation risk premium.

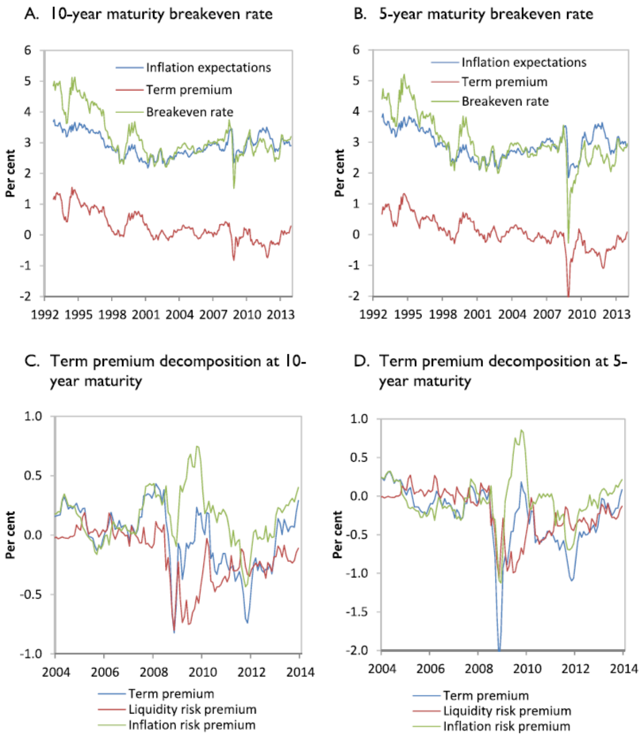

Fig. 6. Bond spot BEI rates decomposition - preferred model.
Note: Bond breakeven rates are observed. Term premium is the sum of liquidity and inflation premia. There is no swap data before 2004, therefore it is not possible to distinguish between inflation and liquidity premium before then. CPI bond and swap data are not available in UK, hence decomposition only refers to RPI inflation and RPI linked bond and swap data. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Our estimates of the liquidity premium explains a large part of the total risk premium in some periods, especially in a time of crisis, as in 2008, when it accounted for 98% of the total term premium and its absolute value was as high as 80 basis points for the 10-year gilt BEI rate. These estimates are in the range with those found earlier in the literature, e.g. between negligible estimates by Pflueger and Viceira (2013) and 200 bp by D' Amico, Kim, and Wei (2014). We believe that the relatively high liquidity premium estimate reflects a combination of funding constraints in the market for inflation-linked gilts in that period and exceptional movements driven by a flight-to-quality phenomenon towards conventional gilts and unwinding of derivatives positions by institutional investors following the failure of Lehman Brothers. The liquidity premium at the 10 years maturity otherwise averaged −30 basis points after 2009 and stabilised at around −20 basis points after 2012. The rise in risk premia after September 2012 was instead primarily driven by inflation risk premia rather than liquidity premia, as the latter remained fairly constant. We also found that the estimated liquidity factor turns out to be very similar to both the funding illiquidity proxy in Malkhozov et al. (2017) and the liquidity premium estimates in Pflueger and Viceira (2013). This lends further support for the robustness of the liquidity premium estimates.

As an additional plausibility check, we have calculated the liquidity measure from the fitting errors in the index-linked yield curve, the so-called Noise index. Fig.7 plots the liquidity factor estimated by our model alongside the Noise index from the UK real yield curves. Strikingly, despite being independently constructed by two distinct methodologies, the BEI Noise liquidity index (calculated from the cross-section of fitting errors of the UK real curves) and the BEI liquidity factor (based on the difference between the rates on inflation-linked bonds and inflation swaps and estimated by our GDATSM) are highly correlated (correlation index is around 0.8), suggesting the validity of our approach to estimate the liquidity factor in the UK index-linked gilts.

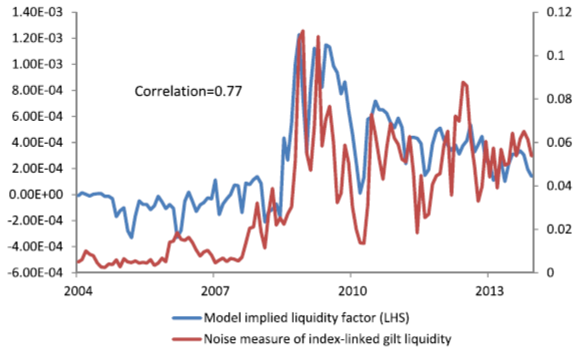

Fig. 7. Noise measure of index-linked gilt liquidity and the modelled liquidity factor.
Note: The BEI Noise liquidity index is calculated from the cross-section of fitting errors of the UK real curves; the BEI liquidity factor is based on the difference between the rates on inflation-linked bonds and inflation swaps and estimated by our GDATSM.

On average, the term structure of inflation risk premia is upward sloping (Fig.8), in line with the existing studies. Intuitively, this is because inflation uncertainty is likely to be increasing with the time horizon, although the slope of our estimates varies over time. We find that in 2000–2004 the term structure was flat (where 10-year inflation risk premium is about the same as that of 5-year) and after 2004 it became upward sloping again (with the 10-year inflation risk premium lying above that of 5-year). The term structure of liquidity premia (Fig.8) is flat and positive between 2004 and 2008, and downward sloping but negative thereafter.

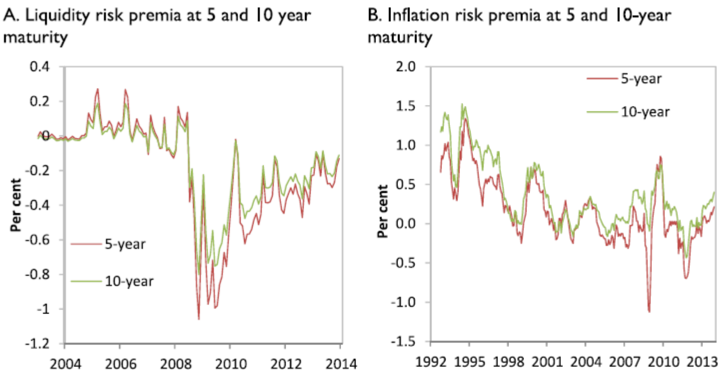

Fig. 8. Liquidity and inflation risk premia in bond BEI rates.

Inflation swap BEI rates (Fig.9) are generally less volatile than corresponding maturity bond BEI rates, which may be more significantly affected by liquidity conditions. This could imply that their movements are more driven by changes in inflation expectations and may corroborate views from the Bank's market contacts that swap BEI rates represents a more reliable indicator of inflation expectations, compared to bond BEI rates.

5.2 Estimated CPI expectations and RPI-CPI wedge

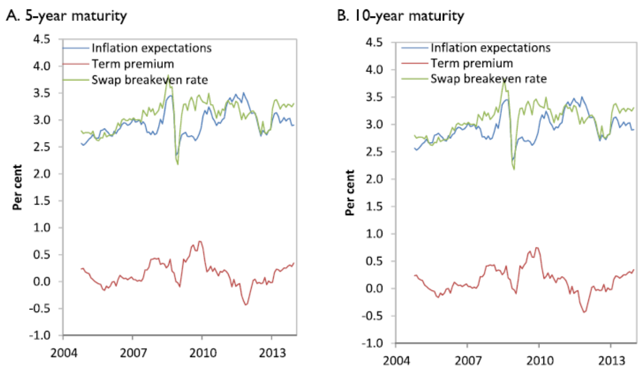

Fig. 9. Inflation swap spot BEI rates decomposition - the preferred model.
Note: Swap breakeven rates are observed. RPI inflation expectations and inflation risk premia in swap and bond breakeven rates are identical; term premium in swap breakeven rates is the same as inflation risk premium. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Our estimates of RPI and CPI inflation expectations for 2-, 3-, 5- and 10-year horizons are reported in Fig. 10. The key finding is that long-term (i.e. 10-year) RPI and CPI inflation expectations are very stable and well anchored, with the latter close to the MPC's 2% CPI inflation target. But the estimates of RPI and CPI expectations at shorter horizons are more volatile, with the former the most volatile (Fig.10. A).

Estimates of long-term CPI expectations average 2.3% over the whole sample. We also found that after the crisis long-term expectations for both CPI and RPI have been slightly more volatile than during the period before the crisis, but after the independence of the Bank of England (i.e. from 1997 to 2008).

Importantly, given that markets are forward looking, market expectations of CPI inflation rates are different from naïve estimates of inflation such as current inflation rates or average inflation over past 3 years. The two naïve measures generate considerably larger forecasting errors than the model implied expectations: mean squared forecasting errors (MSFE) are 1.80 and 3.10 versus 1.59 correspondingly. Indeed, as it can be seen from Fig.10.E, the measure based on average values over past 3 years is too smooth; and there are several episodes when current inflation rates and market expectations have been moving in opposite directions. Moreover, the CPI inflation rates have been rather volatile over the sample, while the model estimates of the market expectations of CPI rates at the end of the policy horizon (3-years) have been more stable, similar to inflation expectations measures based on surveys. For example, over the last decade SEF respondents expected inflation in 2–3 years to be within 20 basis points of 2%. This suggests that our model implied inflation expectations are better proxies for the market expected rates than the naïve forecasts of inflation.

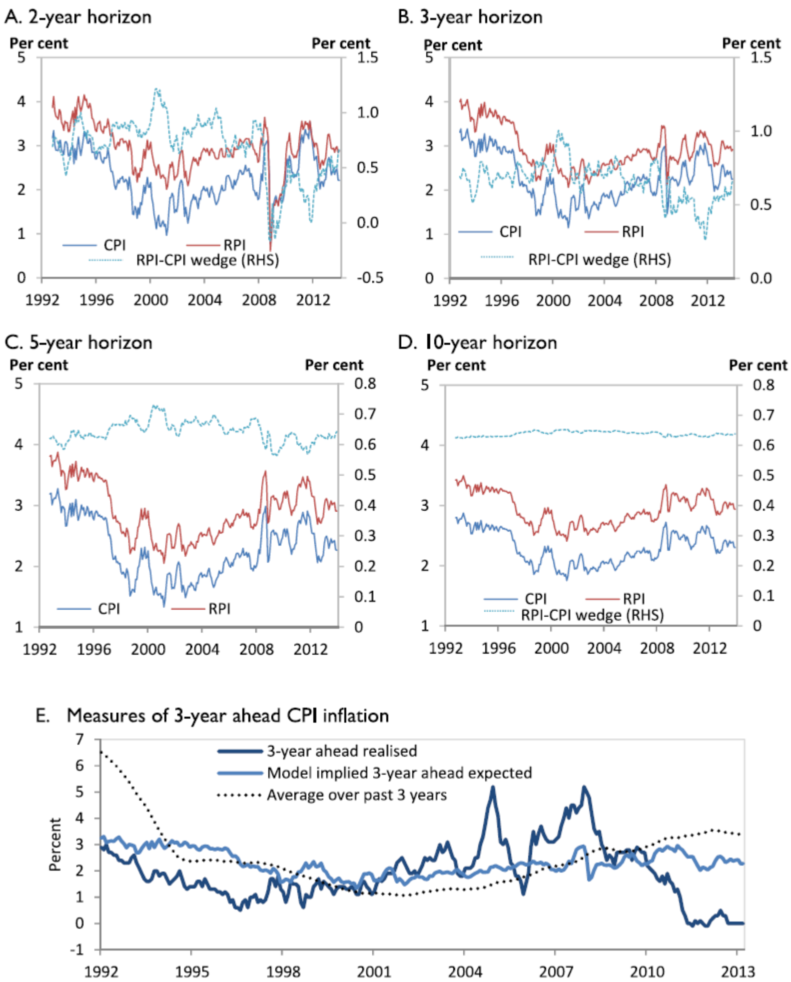

Fig. 10. Expected 1-year CPI and RPI inflation rates over various horizons.
Note: CPI and RPI inflation rates are plotted with reference to the left hand axis. The RPI-CPI inflation difference is plotted with reference to the right hand axis (RHS).

As regards estimated expectations for the RPI-CPI ‘wedge’ (i.e. the spread between RPI and CPI inflation), we can distinguish three periods with significantly different features (Fig.11). The first period is between 1992 and 1997 (see Fig.11.A), where more than 50% of the expected RPI-CPI wedge term structures are downward sloping. The second period is between 1998 and 2007 (Fig.11.B), where more than 75% of the expected RPI-CPI wedge term structures are downward sloping. The last period is between 2008 and 2013 (Fig.11.C), where more than 75% of the expected RPI-CPI wedge term structures are upward sloping. The contrast between the 2nd and 3rd periods is especially large. This suggests the market expected a higher than average RPI-CPI wedge at short horizons before the 2008 crisis but a lower wedge after the crisis. Therefore the short-term measure of RPI inflation expectations is a better proxy for CPI expectations after the crisis than it was before the crisis.

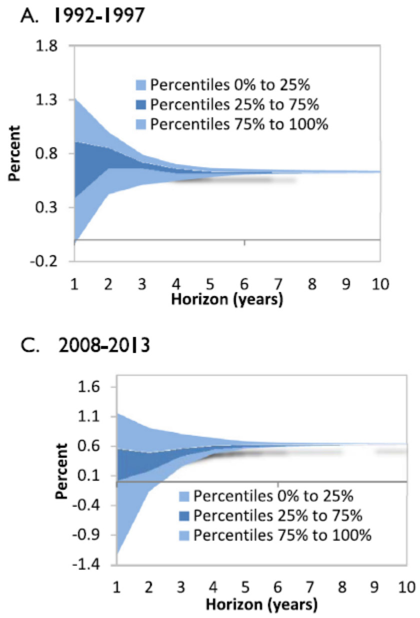

Fig. 11. Dispersion of expected annual RPI-CPI inflation wedge over a 10-year horizon over three different periods.

We also find that the dispersion of expectations for the RPI and CPI wedge is largest for shorter horizons. The expected wedge appears to mean-revert beyond 4 to 5 years. Fig. 12 further demonstrates this point by showing the term structure of the wedge dispersion, where the dispersion starts at a high level at the shortest maturity and then falls quickly to a very low level after 3 years.

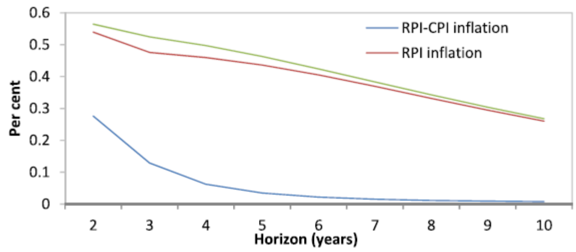

Fig. 12. Annual RPI-CPI wedge dispersion over different forecast horizons (i.e. Standard deviation of RPI-CPI inflation difference).
Note: Figure shows the term structure of the dispersion of expectations for the RPI and CPI wedge.

For longer horizons, the expected RPI/CPI wedge appears fairly stable at around 66 basis points. At face value this suggests that estimates of long-term RPI expectations can be transformed to estimates of long-term CPI expectations via a constant adjustment.

In other words, we could approximate long-term CPI inflation expectations by subtracting a constant wedge of 66 basis points from the measure of RPI inflation expectations. We cannot apply a similar constant adjustment to short-term RPI inflation measures, however, given that our estimates of the expected RPI-CPI wedge change significantly from month to month at shorter horizons (i.e. the 2–5 year horizon).

It is also worth noting that our estimates for the expected long- run RPI-CPI wedge are a little lower than some other estimates available in the literature. For example, the Bank of England's recent discussions with market participants suggest that they generally expect the wedge to average around 80–100 basis points in the long-term (see Domit and Roberts-Sklar, 2015). However, these market forecasts might have been adjusted up following methodological changes in 2010 by the ONS.

Indeed, historically RPI has tended to be higher than CPI, with a large part of the difference between the two indices explained by the formula effect arising due to the different averaging techniques applied to actual price quotes (CPI uses a geometric average whereas RPI uses an arithmetic average). Prior to 2010, the contribution from the formula effect had been relatively stable around 50 basis points. It then increased markedly, coinciding with a change in the ONS' clothing price collection practices. The change increased the dispersion of relative prices within ONS' sample and, because an arithmetic average is greater than the corresponding geometric average by an amount dependent on the dispersion, pushed RPI inflation up relative to CPI. As a result, the formula effect may have increased by 20 basis points or more and now is estimated to be above 0.7pp.

Inopportunely, given the short sample period since 2010, our model is unlikely to fully capture these changes yet. So we may slightly underestimate more recent expectations for the long run RPI/CPI wedge.

Finally, our model estimates can be also used to get 'clean' estimates of real rates, both CPI- and RPI-linked, directly from nominal yields. Fig.13 shows a time series plot of the observed 10-year nominal and inferred from them real yields. The real yields are derived by subtracting from the nominal yields our model-implied estimates of inflation risk premia and corresponding inflation expectations for CPI or RPI inflation rates. According to our estimates, the fall in nominal rates in the end of the 1990ies is mostly accounted for by the fall in inflation components, and to a lesser extent, by the fall in real rates. Instead, the post-crisis decline in nominal rates is almost entirely mirrored by the real rates. Nonetheless, here we cannot say if the real rates fell due to the fall in real term premia or expected risk-free real rates, as the information on the real rates is outside of our model framework.

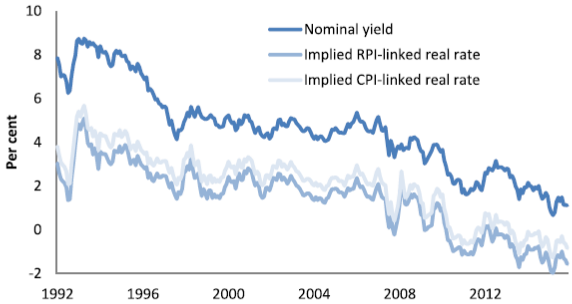

Fig. 13. 10-year real yields extracted from 10-year nominal bond yields.
Note: The figure displays 10-year spot yields on nominal bonds (source: Bank of England) and inferred from them 10-year real yields. The real yields are derived by subtracting from the nominal yields our model implied estimates of inflation risk premia and corresponding inflation expectations for RPI or CPI inflation rates. The real rates derived this way can still contain non-trivial real term premia, and hence they are not necessary equivalent to risk-free rates. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

6. Sensitivity analysis

In this section we test the robustness of our results to the choice of sample period, to the inclusion of survey information and short maturity data, and to the liquidity assumption. We carry out various sensitivity exercises and report the results in Figs 14–18.

6.1 Sensitivity to the sample period

We estimate the model across different periods (with the same end date but different starting dates) in order to check the model sensitivity to two possible structural breaks: (1) the introduction of inflation targeting in 1992, and (2) the creation of an independent MPC at the Bank of England in 1997. The longest sample period goes back to 1989 when CPI data are available for the first time. We find that the model is fairly robust to the choice of the sample period as the estimated BEI rates, term premia and expectations are all very similar to each other for different sample periods (Fig.14).

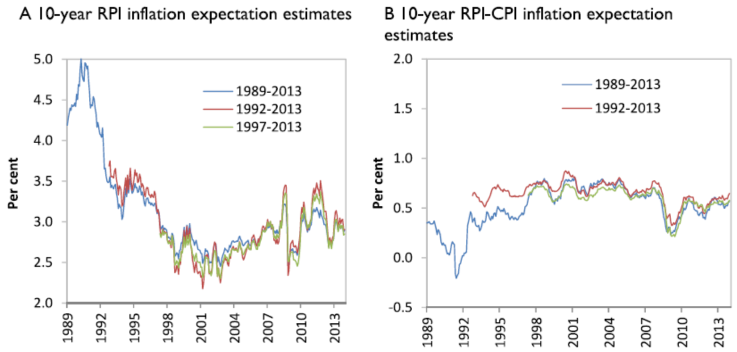

Fig. 14. Sensitivity analysis for different sample periods with the same end date by different start dates.
Note: The model is estimated across different periods (with the same end date but different starting dates). The longest sample period goes back to 1989 when CPI data are available for the first time.

We also estimated the model by gradually expanding the end date of the sample by 1 year from 2006, with the starting date in Oct 1992. We find that expanding the data sample stabilises both CPI and RPI inflation expectations in the post-1998 period (Fig. 15), as the range of projections narrows over time. This suggests that our preferred sample period (from 1992 to 2013) is long enough to guarantee the stable estimation of the decomposition.

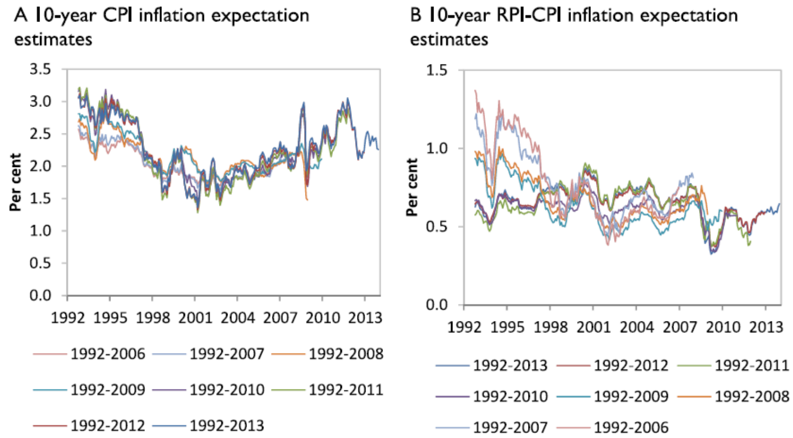

Fig. 15. Sensitivity analysis for different sample periods with the same start date by different end dates.
Note: The model is estimated by gradually expanding the end date of the sample by 1 year from 2006, with the starting date in Oct 1992.

6.2 Impact of inclusion of survey data

We re-estimated the preferred 5-factor model without survey data. The results (Fig.16) show that the estimates of CPI expectations become very sensitive to the sample selection if the model excludes survey data. This is also true for the estimates of RPI expectations. Therefore the inclusion of the survey data helps to improve the robustness of the estimation of inflation expectations to different sample choices.

6.3 Impact of liquidity assumption

To test the impact of the liquidity assumption, we re-estimated the model without the inflation swap BEI data so that the liquidity premium cannot be identified in the model. The principal component analysis suggests that 4 factors are enough for the reduced dataset (which includes gilt BEI rates but not inflation swap data).

We found that excluding inflation swap BEI data (but still including survey data) mainly affects the inflation risk premia estimation, which become negative after 2004 (Fig.16), in line with Guimaraes (2014). This is due to the fact that the new estimation of the inflation risk premium also includes the unidentified liquidity premium component. Therefore, the impact of the liquidity assumption affects mostly the estimation of the inflation risk premium rather than the expectation component.

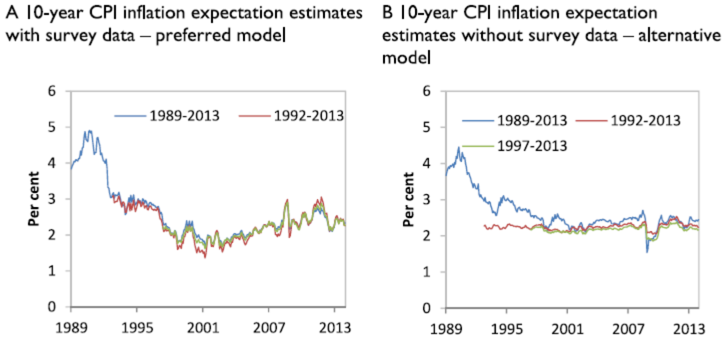

Fig. 16. Sensitivity analysis for the impact of survey data.
Note: The alternative model also includes 5 factors but does not include survey data.

We also estimated the joint model of real and nominal yields as in Guimaraes (2014) and compared the results with those from the four-factor specification of our model without inflation swaps. Figs 16C-D show that the resulting risk premia and inflation expectations estimates are very similar in two cases, suggesting that estimating inflation risk premia directly on BEI rates is consistent with a more computationally demanding estimation based on real and nominal rates.

It is worth noting that here we work with a relatively parsimonious model based on BEI rates only, as it requires fewer latent factors to fit the data and avoids estimating the overparametrized joint nominal-real model. The fact that without using swaps the model results are similar to those from Guimaraes (2014) confirms that the implications of BEI model are consistent with the standard GDATSMs that take both nominal and real yields as inputs. Instead, the difference between our benchmark results and those from the Guimaraes (2014) come from the use of inflation swaps data and liquidity factor.

6.4 Impact of exclusion of data on very short maturity rates

Although data on very short maturity rates on inflation linked bonds are patchy and often not reliable, we can still use shorter maturity data from inflation swaps as an input to the model estimation. We included such a variant of the model estimation in our sensitivity analysis. In particular, we added 1-year rates from inflation swaps into the set of the observed 3-, 4-, 5-, 7- and 10-year maturity data used for the estimation.

As a result, the main findings reported in the paper do not change. For example, as Fig.s 18A-C show, the estimates of liquidity premia and term premia at medium and long horizons are robust to the inclusion of a shorter 1-year maturity rate in the estimation. The estimates of expected inflation rates at various horizons are also robust. However, the estimates of premia at short maturities are more sensitive (Fig. 18D). In fact, when the input to the model estimation contains the information on very short maturities, resulting term premia estimates at very short maturities display substantially more time variation and amplitude and can dive deep in the negative territory (e.g. up to −400 basis points, as during the deflation scare of 2009). Even though such estimates are out of the range of what has been documented by previous studies, it is difficult to judge how plausible such negative values of the inflation risk premium are, as so far a consensus in the literature has not been reached on the magnitude and even the sign of the inflation risk premium.

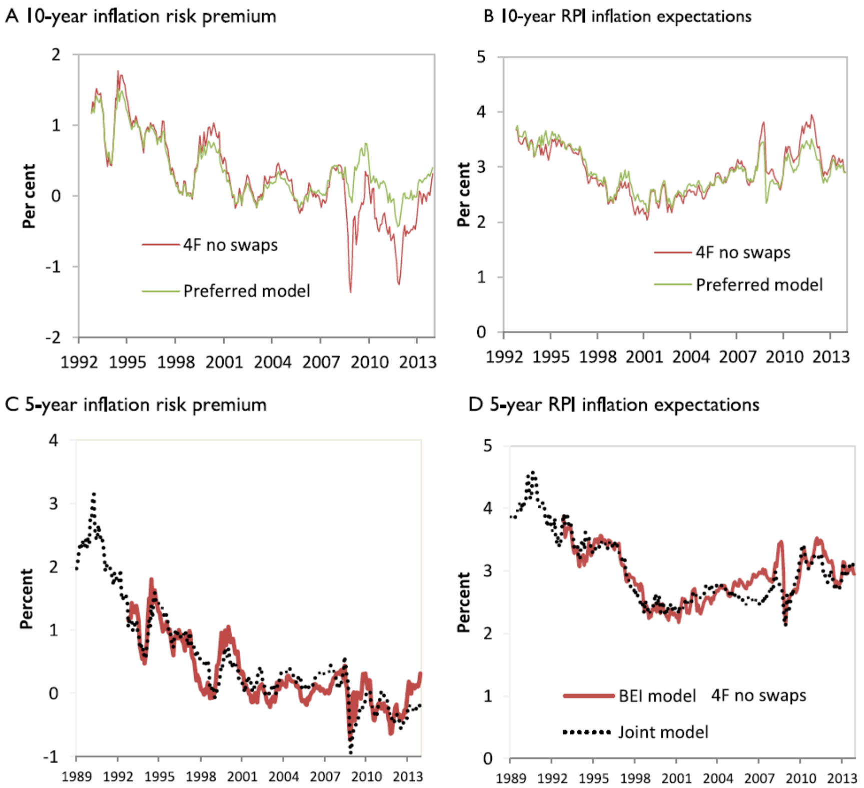

Fig. 17. Sensitivity analysis for the impact of liquidity assumption.
Note: To test the impact of the liquidity assumption, we re-estimated the model without the inclusion of inflation swap data where liquidity premium is not modelled. This is a 4 factors model. The joint model is based on real and nominal curves as in Guimaraes (2014), which is estimated by AACM method.

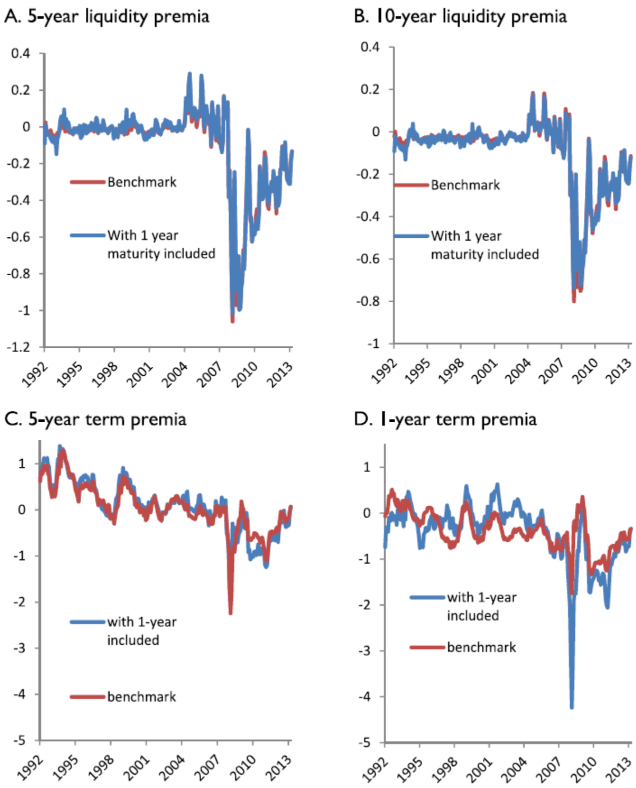

Fig. 18. Sensitivity analysis for the impact of inclusion of data on very short maturity rates.
Note: Here we use shorter maturity data from inflation swaps as an input to the model estimation in our sensitivity analysis. In particular, we added 1-year rates from inflation swaps into the set of the observed 3-, 4-, 5-, 7- and 10-year maturity data used for the estimation. Figures show the estimates of liquidity premia and term premia at short, medium and long horizons.

7. Conclusion

The breakeven inflation rates implied from traded financial instruments (in particular index-linked gilts and inflation swaps), should contain rich information on inflation expectations. However, UK BEI rates cannot be interpreted as market forecasts of future CPI inflation, which is the measure of inflation targeted by the UK MPC. This is because BEI rates in the UK refer to RPI rather than CPI inflation, and also because BEI rates include risk premia, which compensate for inflation risk and liquidity risk in some cases.

To address these limitations and extract more helpful information from BEI rates, we develop a no-arbitrage term structure model to decompose breakeven inflation rates into CPI inflation expectations, expectations for the 'wedge' between RPI and CPI inflation, and risk premia. We further decompose estimates of risk premia in gilt BEI rates into inflation risk and liquidity premia components.

There are a few novel features in our model. First, we model BEI rates directly, without modelling together nominal and real yields as many previous studies have done. Second, we model both bond and inflation swap BEI rates jointly, which allows us to identify the liquidity premium in gilt BEI rates. Third, we incorporate professional survey data on inflation forecasts into our model to improve the estimation of the real world dynamics. The plausibility tests carried out demonstrate the robustness of our model estimation to the sample choice and the liquidity assumption, and also show the importance of including survey data.

We find that our estimates for CPI and RPI inflation expectations have been reasonably stable at medium and long-term horizons since 1997. But long-term expectations for both CPI and RPI are slightly more volatile after the crisis compared to the decade just before the crisis (i.e. the period between 1997 and 2008).

The term structure of inflation risk premia is found to be upward sloping on average, consistent with inflation uncertainty increasing with the time horizon. Liquidity premia in gilt BEI rates are found to explain a large part of the total risk premium in gilt BEI rates during certain periods, especially in the crisis period after 2008. The results suggest that the negative sign of the risk premium in gilt BEI rates during these periods was, to a large extent, the result of negative liquidity premia, which we conclude were driven by periods of illiquidity in the market for index-linked gilts. This also suggests that inflation swap BEI rates may be a more reliable indicator of inflation expectations, compared to bond BEI rates.

Finally, our model implies that expectations for the wedge between CPI and RPI inflation are quite volatile for short horizons but very stable (converging to 66 basis points) at longer horizons. At face value this suggests that our estimates for long-term RPI expectations can be transformed to get a view on long-term CPI inflation expectations via a simple constant adjustment. We also note, however, that our estimates for the long-run RPI-CPI wedge are a little lower than some other forecasts. For example, the Bank of England's more recent discussions with market participants suggest that they generally expect the wedge will average around 80–100 basis points in the long-term. In some cases, such forecasts of the long-run wedge were adjusted upwards following methodological changes in 2010 by the ONS, which our model will not fully capture given the short sample period afterwards, so the model may underestimate recent expectations for the future RPI/CPI wedge.

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Appendix

A.1. Breakeven Rates Approximation

The appendix explains how we can derive the approximation relationship in (8). First we will show the following relationship holds for some constant c_n:

$$\frac{E_t^{\mathbb{Q}}(P_{t+1,n-1})}{E_t^{\mathbb{Q}}(P^*_{t+1,n-1})} = \exp(c_n) E_t^{\mathbb{Q}}\left(\frac{P_{t+1,n-1}}{P^*_{t+1,n-1}}\right) \tag{15}$$

Let's start by denoting $P_{t,n}/P^*_{t,n}$ as $P^{\pi}_{t,n}$, which gives the inflation breakeven rate as $\pi_{t,n} = -P^{\pi}_{t,n}/n$. We then take log on the left-hand-side of the above equation and obtain the following.

$$\begin{aligned}
\ln\left(\frac{E_t^{\mathbb{Q}}(P_{t+1,n-1})}{E_t^{\mathbb{Q}}(P^*_{t+1,n-1})}\right) &= E_t^{\mathbb{Q}}(\ln P_{t+1,n-1}) + 0.5V_t^{\mathbb{Q}}(\ln P_{t+1,n-1}) \\
&\quad - E_t^{\mathbb{Q}}\left(\ln P^*_{t+1,n-1}\right) - 0.5V_t^{\mathbb{Q}}\left(\ln P^*_{t+1,n-1}\right) \\
&= E_t^{\mathbb{Q}}\left(\ln P^*_{t+1,n-1} + \ln P^{\pi}_{t+1,n-1}\right) \\
&\quad + 0.5V_t^{\mathbb{Q}}\left(\ln P^*_{t+1,n-1} + \ln P^{\pi}_{t+1,n-1}\right) \\
&\quad - E_t^{\mathbb{Q}}\left(\ln P^*_{t+1,n-1}\right) - 0.5V_t^{\mathbb{Q}}\left(\ln P^*_{t+1,n-1}\right) \\
&\Rightarrow \\
\ln\left(\frac{E_t^{\mathbb{Q}}(P_{t+1,n-1})}{E_t^{\mathbb{Q}}(P^*_{t+1,n-1})}\right) &= E_t^{\mathbb{Q}}\left(\ln P^{\pi}_{t+1,n-1}\right) + 0.5V_t^{\mathbb{Q}}\left(\ln P^{\pi}_{t+1,n-1}\right) \\
&\quad + COV_t^{\mathbb{Q}}\left(\ln P^*_{t+1,n-1}, \ln P^{\pi}_{t+1,n-1}\right)
\end{aligned} \tag{16}$$

Given that we have $\ln(E_t^{\mathbb{Q}}(P^{\pi}_{t+1,n-1})) = E_t^{\mathbb{Q}}(\ln P^{\pi}_{t+1,n-1}) + 0.5V_t^{\mathbb{Q}}(\ln P^{\pi}_{t+1,n-1})$, the constant term c_n in Eq (15) thus equals the covariance term in the above equation

$$c_n = COV_t^{\mathbb{Q}}\left(\ln P^*_{t+1,n-1}, \ln P^{\pi}_{t+1,n-1}\right)$$

Second, we argue that the constant term c_n only plays an insignificant role and can be dropped in (15). Therefore the following approximation will hold:

$$\frac{P_{t,n}}{P^*_{t,n}} = \frac{\exp(-i_t)E_t^{\mathbb{Q}}[P_{t+1,n-1}]}{\exp(-r_t)E_t^{\mathbb{Q}}\left[P^*_{t+1,n-1}\right]} \approx \exp[-\pi_{t,1}]E_t^{\mathbb{Q}}\left(\frac{P_{t+1,n-1}}{P^*_{t+1,n-1}}\right)$$

We believe it is justifiable to assume $c_n \approx 0$ for the following reasons: (1) The covariance term c_n is negligibly small compared to the sum of the first two terms in Eq (16) for all maturities that we have used to fit the model (i.e. 3 years to 10 years). preliminary results show that one month realised covariance terms (calculated using daily breakeven and real yield data) between 3, 6, and 10 year $\ln P_{t,n}^{\pi}$ and $\ln P_{t,n}^{*}$ are either under or just above 0.01% of the sum of the expectation and the variance terms in Eq (16) although they are calculated under P rather than Q. (2) Our model is an inflation only model which does not include any nominal or real yield data. As a result, the covariance term will be estimated using extra nominal/real yield data if we are to include this term. This adds unnecessary complexity without bringing any real benefits. (3)The assumption on covariance term (c_n) will have no impact on any dynamic analysis. This is because the covariance term is a constant and will not change with time. Therefore, it has no impact on any dynamic analysis such as how expectation/term premium components change over time.

A.2. JSZ transformation

As discussed in the main text, z_t is constructed to match the first K principal components of RPI linked gilt implied breakeven rates, short term CPI and the RPI breakeven inflation rates. The linear transformation from the original latent factors x_t to the portfolio factors z_t is given as below:

$$\mathbf{z}_t = \mathbf{G}\cdot\begin{pmatrix}\boldsymbol{\pi}_t^{bRPI}\\ \pi_{t,1}^{CPI}\\ \pi_{t,1}^{RPI}\end{pmatrix} = \mathbf{G}\begin{pmatrix}\tilde{\mathbf{a}}^{bRPI}+\tilde{\mathbf{B}}^{bRPI}\mathbf{x}_t\\ \delta\mathbf{x}_t\\ \bar{\delta}\mathbf{x}_t\end{pmatrix} = \mathbf{G}(\mathbf{a}+\mathbf{B}\mathbf{x}_t)$$

where

$$\boldsymbol{\pi}_t^{bRPI} = \begin{pmatrix}\pi_{t,n1}^{bRPI}\\ \dots\\ \pi_{t,nN}^{bRPI}\end{pmatrix},\quad \mathbf{a} = \begin{pmatrix}\tilde{\mathbf{a}}^{bRPI}\\ 0\\ 0\end{pmatrix},\quad \mathbf{B} = \begin{pmatrix}\tilde{\mathbf{B}}^{bRPI}\\ \delta\\ \bar{\delta}\end{pmatrix},$$

$$\tilde{\mathbf{a}}^{bRPI} = \begin{pmatrix}\tilde{a}_{n1}^{bRPI}\\ \dots\\ \tilde{a}_{nN}^{bRPI}\end{pmatrix},\quad \tilde{\mathbf{B}}^{bRPI} = \begin{pmatrix}\tilde{\mathbf{b}}_{n1}^{bRPI}\\ \dots\\ \tilde{\mathbf{b}}_{nN}^{bRPI}\end{pmatrix}$$

Following JSZ, we specify the dynamics of z_t under Q as

$$\mathbf{z}_{t+1} = \boldsymbol{\kappa}^{z\mathbb{Q}} + \boldsymbol{\Phi}^{z\mathbb{Q}}\mathbf{z}_t + \boldsymbol{\Sigma}^z\boldsymbol{\varepsilon}_{t+1}^{z\mathbb{Q}}$$
$$\boldsymbol{\varepsilon}_{t+1}^{z\mathbb{Q}} \sim \mathcal{N}(0,\mathbf{I})$$

where the parameters in the above equation can be inferred from those in Eq. (5) as

$$\boldsymbol{\kappa}^{z\mathbb{Q}} = \mathbf{B}_g\boldsymbol{\kappa}^{\mathbb{Q}} + \mathbf{a}_g - \boldsymbol{\Phi}^{z\mathbb{Q}}\mathbf{a}_g$$
$$\boldsymbol{\Phi}^{z\mathbb{Q}} = \mathbf{B}_g\boldsymbol{\Phi}^{\mathbb{Q}}\mathbf{B}_g^{-1}$$
$$\boldsymbol{\Sigma}^z = \mathbf{B}_g\boldsymbol{\Sigma}$$

We can derive the following general pricing model for CPI breakeven inflation rates with regard to z_t:

$$\pi_{t,n}^{CPI} = -\frac{1}{n}\left(a_n^{CPI} + \mathbf{b}_n^{CPI}\mathbf{z}_t\right) \tag{17}$$

where

$$a_n^{CPI} = a_{n-1}^{CPI} + \mathbf{b}_{n-1}^{CPI}\boldsymbol{\kappa}^{z\mathbb{Q}} + 0.5\mathbf{b}_{n-1}^{CPI}\boldsymbol{\Sigma}^z\boldsymbol{\Sigma}^{z\prime}\mathbf{b}_{n-1}^{CPI\prime} + a_1^{CPI} \tag{18}$$

$$\mathbf{b}_n^{CPI} = \mathbf{b}_{n-1}^{CPI}\boldsymbol{\Phi}^{z\mathbb{Q}} + \mathbf{b}_1^{CPI} \tag{19}$$

where $a_1^{CPI} = \delta B_g^{-1}\mathbf{a}_g$ and $\mathbf{b}_1^{CPI} = -\delta B_g^{-1}$, which are derived by solving the following equation:

$$\pi_{t,1}^{CPI} = -\left(a_1^{CPI} + \mathbf{b}_1^{CPI}\mathbf{z}_t\right) = \delta\mathbf{x}_t \tag{20}$$

Similarly, the n-period RPI breakeven inflation rate $\pi_{t,n}^{RPI}$ and that adjusted for the liquidity premium $\pi_{t,n}^{bRPI}$ are given as:

$$\pi_{t,n}^{RPI} = -\frac{1}{n}\left(a_n^{RPI} + \mathbf{b}_n^{RPI}\mathbf{z}_t\right)$$
$$\pi_{t,n}^{bRPI} = -\frac{1}{n}\left(a_n^{bRPI} + \mathbf{b}_n^{bRPI}\mathbf{z}_t\right)$$

where the scalars a_n^{CPI}, a_n^{bRPI} and vectors $\mathbf{b}_n^{RPI}$ and $\mathbf{b}_n^{bRPI}$ can be derived recursively as shown in Eqs. (18) and (19) the following initial conditions:

$a_n^{RPI} = \bar{\delta} B_g^{-1} \mathbf{a}_g, \mathbf{b}_n^{RPI} = -\bar{\delta} B_g^{-1};$

$a_n^{bRPI} = \bar{\delta}^b \mathbf{B}_g^{-1} \mathbf{a}_g, \mathbf{b}_n^{bRPI} = -\bar{\delta}^b \mathbf{B}_g^{-1}.$

The initial conditions are derived by solving the following equaitions:

$$\pi_{t,1}^{RPI} = -\left(a_1^{RPI} + \mathbf{b}_1^{RPI} \mathbf{z}_t\right) = \bar{\delta} \mathbf{x}_t \tag{21}$$

$$\pi_{t,1}^{bRPI} = -\left(a_1^{bRPI} + \mathbf{b}_1^{bRPI} \mathbf{z}_t\right) = \bar{\delta}^b \mathbf{x}_t \tag{22}$$

For the purposes of estimation, we assume that inflation swap breakeven inflation ($\pi_{t,n}{}^{RPI}$) and index linked bond breakeven inflations($\pi_{t,n}{}^{bRPI}$) are measured with errors:

$$\pi_{t,n}^{RPI} = -\left(a_n^{RPI} + \mathbf{b}_n^{RPI} \mathbf{z}_t\right)/n + e_{t,n} \tag{23}$$

$$\pi_{t,n}^{bRPI} = -\left(a_n^{bRPI} + \mathbf{b}_n^{bRPI} \mathbf{z}_t\right)/n + e_{t,n}^b, \tag{24}$$

where we the error terms $e_{t,n}$ and $e_{t,n}{}^b$ both follow independent normal distribution $N(0, \omega)$ with the same volatility.