

# **IMI Working Paper**

# Financial Investments and Commodity Prices

Liu Peng, Qiu Zhigang and Xu Xiaoyu

# **INTERNATIONAL MONETARY INSTITUTE**

For further information, please visit http://www.imi.ruc.edu.cn/en/





Weibo

WeChat

DOI: 10.1111/irfi.12361

## **ORIGINAL ARTICLE**



# Financial investments and commodity prices

Peng Liu<sup>1</sup> | Zhigang Qiu<sup>2</sup> | David Xiaoyu Xu<sup>3</sup>

<sup>1</sup>SC Johnson College of Business and School of Hotel Administration, Cornell University, Ithaca, New York, USA

<sup>2</sup>School of Finance, Renmin University of China, Beijing, China

<sup>3</sup>Department of Finance, Red McCombs School of Business, University of Texas, Austin, Texas, USA

#### Correspondence

Zhigang Qiu, School of Finance, Renmin University of China, Beijing, China. Email: zhigang.qiu@ruc.edu.cn

# Abstract

In this paper, we show that financial investments dilute the relationship between convenience yields (a proxy for fundamentals) and commodity prices. On average, the explanatory power of convenience yields on the movements of commodity prices decreased from 63% to 33% after 2004, when institutional investors rapidly started building their positions in commodity futures. We develop a model of the commodity market with financial investments and test the model predictions using futures prices of 21 US-traded commodities and index traders' positions on 12 agricultural commodities. Because of correlated financial demands for different commodity futures, we identify comovements in spot prices for fundamentally independent commodities.

#### **KEYWORDS**

commodities, convenience yield, financial demand

JEL CLASSIFICATION G13; D03; D53

#### 1 INTRODUCTION

Commodity financialization is new to the commodity market. Since 2004, commodities have enjoyed increasing recognition as a new financial asset class as an increasing number of institutions invest in commodity futures for the sake of portfolio diversification.<sup>1</sup> Prior to the 1990s, the Prudent Investor rule prohibited pension plans from buying commodity future contracts. However, the collapse of the equity market in 2000 and the discovery that there is no correlation between commodity performance and stock market movements have led money managers of pension funds, hedge funds, and other institutions to begin investing in the commodity market. For example, there was a total \$15 billion investment in commodity indexes in 2003, but the amount grew to \$319 billion in 2019.<sup>2</sup> Thus, commodity financialization should have had a significant impact on the commodity market in the last two decades.

© 2021 International Review of Finance Ltd.

International Review of Finance. 2021;1-25.

1

The rising volume of financial investments in commodity future markets, both at exchanges and over-thecounter (OTC) markets, changes the market structure significantly. For example, Tang and Xiong (2012) show that there was a structural break in the commodity market around 2004, and indexed commodities became more correlated. In fact, in this paper, we show that the relationship between convenience yields, a proxy for fundamentals, and commodity prices are diluted after 2004. The convenience yield is measured by a combination of convenience yield and storage cost, which is extracted from observable futures prices. For expositional simplicity, we still used the term "convenience yield" (hereafter) although it is net of the commodity's storage cost. On average, the convenience yields movement accounts for 63% of spot price movements before 2004; however, the effect changes to ~33% after 2004. It seems that increasing financial investments and unusual movements of commodity prices? This question is important and cannot be ignored. In this paper, we identify the effects of financial demand on commodity prices by providing both theoretical and empirical analyses.

We adopt a demand-based framework to analyze the effect of financial investments on commodity pricing. We use the convenience yield as a summary of the economy regarding real demand (or fundamentals) for a certain commodity. This indicates that real demand has less explanatory power regarding price movements after 2004. In traditional models on commodities such as Hirshleifer (1988 and 1990), there are two classes of players: hedgers and speculators. Hedgers short-sell futures to hedge their physical production (or inventory); speculators provide liquidity to hedgers. Since financial investors have become important in the commodity market, especially in recent years, we add financial investors on top of hedgers and speculators in the model. Both hedgers and speculators make trading decisions based on the maximization of utility, but trades from financial investors are assumed to be exogenous.<sup>3</sup>

The model predicts that the equilibrium commodity spot price is a combination of the convenience yield and financial demand; that is, stronger financial demand increases commodity spot prices since commodity futures have only zero net supply. Intuitively, when financial investors buy commodity futures for the sake of diversification; even though they buy commodity futures at a relatively higher price (than the value determined by fundamentals), they are better off because of the reduction in risks of their portfolios. Given that many commodity markets are much smaller than the mainstream financial market, financial demand from major financial institutions may cause a large price change in commodities. We also show that the volatility of commodity prices increases with the presence of financial investors<sup>4</sup>. Furthermore, our model predicts that the prices of two commodities can be strongly correlated if they are subject to a correlated financial demand, even when the underlying real demand is independent of each other, which is consistent with Tang and Xiong (2012).

To test the model predictions, we obtained data on 21 commodities traded in the US futures market to test model predictions. We find that, adjusted by the real demand (represented by convenience yields), commodity prices increased steeply after 2004. Moreover, the explanatory power of convenience yields on commodity prices is weaker after 2004 than before 2004. Considering commodity index investment as one type of financial demand, we show that commodity prices do depend on commodity index position. Moreover, we identify comovements in spot prices for fundamentally independent commodities because financial demands are correlated. Applying a principal component analysis, we shows that the first principal component explains 37.7% and 63.0% of the variation in spot prices before 2004 and after 2004, respectively.

In the literature, there have been heated debates about whether high commodity prices, especially during the 2007-2008 period, were caused by real demand or by financial demand. Masters and White (2008) argue that money inflows into the commodity market have sent oil and other commodity prices beyond their fundamental values. However, Krugman (2008) and Hamilton (2009) argue against the financial demand story, claiming that the inflated prices of commodities were due to high worldwide demand and the inelasticity of the world supply. Their main evidence is that if investment causes high oil prices then people would expect to see stock buildups; in contrast, the publicly announced oil inventory data (such as the one published by the Energy Information Administration of the USA) did not support the stockpile of inventory. From our point of view, however, it is very difficult to track

commodity stocks; for example, it is impossible to count the oil stocks in Asian countries or in tankers at sea. Hence, using inventory data would be problematic.

Traditionally, when inventory data are impossible to obtain, researchers such as Fama and French (1988) and many others use the convenience yield<sup>5</sup> instead because the convenience yield is the marginal value of holding one unit of inventory and hence reflects the extent of scarcity for a certain commodity. The theory of storage (see Brennan, 1958; Pindyck, 2001 and Telser, 1958) holds that a negative relationship exists between the convenience yield and inventory. For example, if the high oil price in mid-2008 was due to high demand and the inelasticity of supply, the "unobservable" inventory should be quite low, and hence the marginal value of holding one unit of inventory should have been quite high, and hence, the convenience yield should also have been high. However, the convenience yield did not shoot up in 2008, which indicates that holding oil inventory was inexpensive. Therefore, we conjecture that the "unobservable" oil inventory is likely to be quite large in 2008, which makes the benefit of holding it quite low. Hence, it is likely that financial demand drove up commodity prices in the 2004–2008 period.

Thus, our paper relates to the literature on the financialization of commodities. For example, Tang and Xiong (2012) argue that large amounts of index investment in commodity futures markets have precipitated the financialization process with respect to commodities. Singleton (2011) estimates money inflow into the oil market and shows that the money inflow indeed predicts price changes in the oil market. Gilbert (2009) documents that index-based investment in commodity futures Granger-caused a significant and bubble-like increase in energy and metal prices. Mou (2010) documents that the rolling activity of commodity index investors impacts futures prices. Basak and Pavlova (2016) study the impact of commodity financialization on futures prices. Ekeland et al. (2017) analyze commodity financialization using both the hedging pressure theory and the storage theory. Sockin and Xiong (2015), Goldstein et al. (2014), and Goldstein and Yang (2018) study the informativeness of asset prices by commodity financialization. Da et al. (2020) show that index trading affects return autocorrelations among commodities in that index. Our paper distinguishes from the literature by studying the relationship between convenience yields and commodity prices and how financial demands affect commodity prices.

Moreover, the general idea that the trading behavior of the various players in the commodity futures market can influence prices goes back to the theory of normal backwardation, proposed by Keynes (1923) and Hicks (1939), Hirshleifer (1988 and 1990), and Bessembinder (1992). Specifically, the theory of normal backwardation proposes that hedgers have to sell futures prices at relatively lower prices (i.e., offer a risk premium to speculators) to solicit speculators to come into the futures market.

For the behaviors of futures prices and convenience yields, Ng and Pirrong (1996) show a positive relationship between convenience yields and the volatility of commodity prices, and Liu and Tang (2008) document the hetero-skedasticity of convenience yields. Girma and Paulson (1999) investigate risk arbitrage opportunities for petroleum futures, and Chinn and Coibion (2009) investigate the predictive components of futures prices. Our paper differs from theirs in its focus on how the financial demand or a structural break affects the relationship between convenience yields and commodity prices. For the same reason, our paper is also distinct from the literature on returns and risk premium of commodities.<sup>6</sup>

Our paper treats financial investments as an exogenous demand shock, so it also falls into the literature on demand-based asset pricing models that have been applied to many financial products. For example, Garleanu et al. (2009) study demand-based option pricing in which risk-averse arbitrageurs face demand in the options market, but take the jump risk. Greenwood and Vayanos (2010) consider the fixed-income market. In their models, the exogenous demand is the supply of government bonds. Greenwood (2005) and Hau (2009) consider demand shocks on cross-sectional equity returns. In these papers, the demand shocks come from the revisions of the Nekkie or Morgan Stanley Capital International (MSCI) index, which in turn affects the portfolio holdings of institutional investors.

The remainder of this paper is organized as follows. Section 2 documents the stylized facts for conveyance yields and commodity prices. Section 3 presents the structural model and model predictions. Section 4 describes the data. Section 5 tests the model empirically and Section 6 concludes.

# 2 | CONVENIENCE YIELDS AND COMMODITY PRICES BEFORE AND AFTER 2004

As documented by Tang and Xiong (2012), we regard 2004 as the structural break for the commodity market because after 2004, financial investments from institutional investors rapidly flowed into the commodity futures market. Considering the magnitude of financial investments, we conjecture that this investment flow leads to significant changes on commodity prices. Our first observation is on the convenience yield, which is regarded as a proxy for fundamentals and should have a stable relationship with commodity prices (e.g., Pindyck, 1993). To see whether financial investments have an impact after 2004, we show some preliminary results in Figure 1.

Figure 1 plots the relationship between commodity prices and convenience yields for four major commodities. Before 2004, convenience yields and spot prices comoved strongly, and the relationship was very stable. However, afterward, the comovement became weaker, and the two variables sometimes even moved in opposite directions. It is clear that the structural break has significant effects on commodities, and convenience yield does not appear to be the main driver for commodity prices. Since the structural break is due to financial investments, we conjecture that the demand shock from institutional investors is the reason for the dilution of the relationship. To examine our conjecture, in what follows, we analyze the effect of financial investment within a demand-based framework and empirically test the model predictions.

# 3 | A DEMAND-BASED MODEL

We regard financial investments as a demand shock and therefore adopt a demand-based framework for our analysis. The model setup is as follows.



**FIGURE 1** Prices and convenience yields of four commodities: Soybean, crude oil, copper, and live cattle, from January 1990 to December 2019

# 3.1 | Model setup

In the economy, there are three types of assets: a physical commodity, its associated commodity futures, and a risk-free money market account. A physical commodity with spot price  $S_t$  pays out benefits in the form of the convenience yield to its holder.<sup>7</sup> The cumulative convenience yield  $D_t$  is governed by the process

$$dD_t = \delta_t dt \tag{1}$$

and

$$d\delta_t = (b - l\delta_t)dt + \sigma_1 dB_{1,t}, \tag{2}$$

INTERNATIONAL

**REVIEW OF FINANCE** 

where  $\delta$  is the rate of the convenience yield and *B* denotes a Brownian motion. The convenience yield, summary of economic status, is assumed to be exogenous in the model as the determinants of the convenience yields are not focal points of this paper. Notably, although the convenience yield can be modeled endogenously,<sup>8</sup> the exogenously specified convenience yield bypasses the difficulty of modeling the consumption and production of commodities. This assumption is also consistent with existing studies such as Gibson and Schwartz (1990) and Pindyck (1993).

Following Breeden (1984) and Ho (1984), we assume a futures contract of instantaneous maturity (denoted as  $F_t = \lim_{T \to t} F(t,T)$ ) on spot price  $S_t$  as an asset with a payoff in the next instant that is equal to the spot price. When an instantaneous-maturity futures contract is assumed to exist at all times for a certain commodity, the implication is that at every instant a futures contract expires, and another one is created in the meantime. Note that we use instantaneous futures contracts in this paper mainly for the sake of parsimony.<sup>9</sup> From the cost of the carry relationship, futures with instantaneous maturity,  $F_t$ , follow

$$F_t = \lim_{T \to t} F(t,T) = \lim_{T \to t} \left[ S_t e^{r(T-t)} - \int_t^T dD_t \right]$$
(3)

Except for physical commodities and their associated futures, investors have access to a risk-free money-market account that pays out constant interest rates r > 0.

There are three types of agents in the economy: hedgers, speculators (or arbitrageurs), and index investors. Agents of the first type are hedgers, who are equipped with commodity containers and thus can hold physical commodities. There are many homogeneous hedgers in the model, with mass normalized to one. We assume that each hedger possesses inventory of a constant a unit of a physical commodity. In addition, she can write futures hedging on her physical commodity and invest in a money market account to achieve better lifetime utility. The second agent type is the speculator, who enters the commodity futures market as an arbitrageur who tends to arbitrage away any mispricing caused by hedgers and index investors and in the meantime allows the market to clear. The number of homogeneous speculators is w. Note that speculators only buy or sell "paper-based" futures contracts, since they do not have physical containers to hold physical commodities. Speculators maximize their utility by rebalancing their wealth using commodity futures and a money market account. Note that both hedgers and speculators know the status of the economy (i.e., the process of the convenience yields) and make their trading decisions based on this knowledge. Agents of the third type are homogeneous index investors with mass v, who take "paper-based" futures as a financial asset for investment. Index investors make their trading decisions typically based on financial reasons such as portfolio diversification needs and capital constraints, but not on fundamental real demand factors (i.e., convenience yields) in commodity markets. We assume that each index investor's trading position  $e_t$  is exogenous and follows an Ornstein-Uhlenbeck (OU) process,

WILEY

INTERNATIONAL Review of Finan

$$de_t = -ke_t dt + \sigma_2 dB_{2,t}, \tag{4}$$

$$dB_{1,t}dB_{2,t} = 0,$$
 (5)

where k and  $\sigma_2$  are constants, and the correlation between the increment of financial demand and the convenience yield is assumed to be zero because, as mentioned above, financial demand is exogenous to the commodity market.<sup>10</sup>

For hedgers and speculators, we use subscript i = 1 for a representative hedger and i = 2 for a representative speculator. Moreover, we assume that all agents maximize the mean-variance utility

$$\max_{\phi_{i,t}} \left[ E_t(dW_{i,t}) - \frac{\tau}{2} Var_t(dW_{i,t}) \right] \quad i = 1, 2,$$

$$\tag{6}$$

where  $\tau$  denotes the risk aversion parameter for both hedgers and speculators, and  $\phi_{i,t}$  denotes the position in futures contracts held by the *i*th (*i* = 1,2) agent.

The optimization problem is subject to the wealth processes of agents. The hedger's wealth follows

$$dW_{1,t} = r(W_{1,t} - aS_t)dt + a(dS_t + dD_t) + \phi_{1,t}dF_t$$
(7)

and the speculator's wealth follows

$$dW_{2,t} = rW_{2,t}dt + \phi_{2,t}dF_t.$$
 (8)

The market-clear condition is

$$\phi_{1,t} + w\phi_{2,t} + ve_t = 0, \tag{9}$$

which comes from the fact that the net supply of commodity futures is zero in the economy.

## 3.2 | Solution of equilibrium

In this section, we find an equilibrium where both hedgers and speculators maximize their utility, and the futures market clears. Appendix A shows that the process for futures prices with instantaneous maturity follows

$$dF_t = dS_t - rS_t dt + dD_t. \tag{10}$$

Hence, (7) and (8) are changed to

$$dW_{1,t} = r(W_{1,t} - aS_t)dt + a(dS_t + dD_t) + \phi_{1,t}(dS_t - rS_tdt + dD_t)$$
(11)

and

$$dW_{2,t} = rW_{2,t}dt + \phi_{2,t}(dS_t - rS_tdt + dD_t).$$
(12)

The following proposition shows the solution of the equilibrium.

⊥Wiley\_

**Proposition 1.** In equilibrium, the spot price  $S_t$  follows

$$S_t = \lambda_0 + \lambda_1 \delta_t + \lambda_2 e_t, \tag{13}$$

7

WILEY-

□.

INTERNATIONAL

**REVIEW OF FINANCE** 

where  $\lambda_0$ ,  $\lambda_1$ , and  $\lambda_2$  are all constants, as shown in Appendix A. The optimal demands for agents 1 and 2 are

$$\phi_{1,t} = \frac{\lambda_1 b - \lambda_2 (k+r) e_t - r \lambda_0}{\tau (\lambda_1^2 \sigma_1^2 + \lambda_2^2 \sigma_2^2)} - a, \tag{14}$$

and

$$\phi_{2,t} = \frac{\lambda_1 b - \lambda_2 (k+r) e_t - r \lambda_0}{\tau (\lambda_1^2 \sigma_1^2 + \lambda_2^2 \sigma_2^2)}.$$
(15)

Proof. Refer to Appendix A.

When solving for the equilibrium, we first conjecture that the price has the form of (13) and solve the optimal demands (14) and (15), taking  $\lambda_0$ ,  $\lambda_1$ , and  $\lambda_2$  as given. Then by the market-clearing condition, we solve  $\lambda_0$ ,  $\lambda_1$ , and  $\lambda_2$ . It can be shown that  $\lambda_0$  is a positive constant, and that  $\lambda_1$  is a function of  $\lambda_0$  and  $\lambda_2$ . We further show that, in Appendix A,  $\lambda_2$  is determined by a quadratic function.<sup>11</sup> The quadratic function can have two positive roots; therefore, there are multiple equilibria.

Although there are multiple solutions to our equilibrium, we can still obtain some useful information by looking at the properties of the equilibria. First, although there exist two solutions of  $\lambda_2$ , both are positive. From the expressions of (13), (14), and (15), it is easy for us to obtain the following properties:

$$1) \frac{\partial S_t}{\partial e_t} \ge 0 \tag{16}$$

2) 
$$\frac{\partial \phi_{i,t}}{\partial e_t} \leq 0 \ (i=1,2).$$
 (17)

The first property reflects that, others equal, when financial demand increases, the commodity spot price cannot decrease. For the second property, when the buying pressure of index investors is growing, as shown in proposition 3, the risk premium of buying futures tends to decrease. Therefore, both the hedgers and speculators should buy less (or short more) futures. These properties are useful for our empirical analysis.

In our paper, we assume that index investors trade commodity futures for reasons of diversification. For this reason, exogenous financial demand  $e_t$  is on the demand side. This is why we have market-clearing condition (9). Although this is true in most cases, some financial investors may trade commodity futures for other reasons, and  $e_t$  is not necessarily on the demand side. If it is on the supply side for some reason, the market-clearing condition becomes

$$\phi_{1,t} + \mathsf{w}\phi_{2,t} = \mathsf{v}e_t \tag{18}$$

With this new market-clearing condition, the solutions of  $\lambda_2$  are negative.<sup>12</sup> Thus, it is possible to have some opposite results in our original model if financial investors do not trade for the reasons of diversification.

What happens if there is no financial demand? The following proposition shows the result.

**Proposition 2.** If and only if v = 0,  $\lambda_2 = 0$ .

Proof. Refer to (A21) in Appendix A.

□.

Thus, financial demand has no effect if and only if there is no index investor. Equation (16) and Proposition 2 indicate that since  $\lambda_2$  is normally nonzero, financial demand does influence commodity prices. In other words, investments in commodity markets can cause commodity prices to deviate significantly from their fundamentals. Note that our model is a partial equilibrium in that we do not model the source of financial demand directly. Intuitively, index investors are likely to buy commodity futures for the sake of portfolio diversification. In this case, they would achieve their goal of reducing portfolio risks in exchange for purchasing commodity futures at a relatively higher price (than the value determined by the fundamentals of commodities).

#### 3.3 Comparative statics and predictions of the model

For comparative statics, we mainly study the influence of the number of index investors and speculators on commodity prices and the influence of the amount of physical inventory held by hedgers on commodity prices. Because  $\lambda_2$  is a positive constant, (16) informs us that, as financial demand for futures grows in strength, commodity prices tend to increase. Thus, the model shows that, due to financial demand, commodity prices can rise considerably higher than their fundamental values.

We can find how  $\lambda_0$ ,  $\lambda_1$  and  $\lambda_2$  change in response to the amount of physical inventory held by hedgers *a* from their expressions in Appendix A.  $\lambda_0$  decreases in the amount of inventory, because the more inventory hedgers hold the more futures they need to short sell, and thus the greater they drag down commodity prices. This is consistent with the theory of normal backwardation proposed by Keynes (1923). Furthermore, since  $\lambda_2$  measures the sensitivity of the price change on financial demand  $e_t$ , and the amount of physical inventory is independent of financial demand in the model, we thus see that  $\lambda_2$  does not change with the amount of inventory.

From this analysis, we infer that financial demand is key to determining commodity price deviations from fundamentals. As shown in Table 1, Panel B, the long position of commodity index investors (only one type of index investors) accounts for ~30% of 12 agricultural commodities on average; thus, financial demand is an important force in determining commodity prices.

**Proposition 3.** The risk premium of futures  $\Lambda$  is

$$\Lambda = \lambda_1 b - r\lambda_0 - \lambda_2 (k+r)e_t \tag{19}$$

Proof. From (10) we derive the physical measure

$$E_t[dF_t] = [\lambda_1 b - r\lambda_0 - \lambda_2 (k+r)e_t]dt$$

Since futures should have zero drift under the risk-neutral measure, the drift of the futures in the physical measure is the risk premium of futures.

#### TABLE 1 Summary statistics

Panel A: Summar	y statistics of 21	commodity futures
-----------------	--------------------	-------------------

Commodity	Start date	Price unit	Exchange	Spot price mean	Spot Price std	Con. yield mean	Con. yield std	Num. of obs.
Chicago wheat	02/01/1990	Cents/bushel	CME	449.4	170.7	-22.7	56.5	1379
Kansas wheat	02/01/1990	Cents/bushel	KCBT	486.4	181.9	-15.4	52.2	1303
Corn	02/01/1990	Cents/bushel	CME	335.5	145.8	-7.8	43.6	1473
Oats	27/11/1990	Cents/bushel	CME	209.9	90.3	-1.8	38.7	1422
Soybeans	02/01/1990	Cents/bushel	CME	821.7	310.0	30.3	84.8	1451
Soybean oil	02/01/1990	Cents/pound	CME	29.6	11.3	-0.1	1.7	1470
Soybean meal	02/01/1990	Dollars/ton	CME	253.7	95.7	17.4	33.5	1309
Crude oil	02/01/1990	Dollars/barrel	NYMEX	47.2	29.7	0.9	5.2	1487
Heating oil	02/01/1990	Cents /gallon	NYMEX	137.3	89.9	1.5	11.3	1432
Natural gas	27/01/1992	Dollar/MMBtu	NYMEX	4.0	4.2	-0.2	2.1	1474
Cotton	02/01/1990	Cents/pound	ICE	69.1	22.4	1.7	12.1	1524
Orange juice	02/01/1990	Cents/pound	ICE	118.0	35.9	-2.2	11.5	1436
Cocoa	02/01/1990	Dollars/ton	ICE	1845.0	748.0	-28.8	76.6	1534
Sugar	02/01/1990	Cents/pound	ICE	12.8	5.6	0.4	1.7	1528
Coffee	02/01/1990	Cents/pound	ICE	118.6	47.7	-4.5	15.4	1542
Platinum	02/01/1990	Dollars/troy ounce	NYMEX	855.4	467.8	16.3	44.5	1487
Palladium	02/01/1990	Dollars/troy ounce	NYMEX	366.4	241.6	6.2	21.3	1371
Copper	02/01/1990	Cents/pound	NYMEX	180.8	111.0	7.1	13.1	1342
Lean hogs	02/01/1990	Cents/pound	CME	64.3	17.5	1.0	13.9	1389
Live cattle	02/01/1990	Cents/pound	CME	90.7	25.5	2.4	9.9	1486
Feeder cattle	30/01/1990	Cents/pound	CME	108.0	37.9	3.5	8.9	1448
Panel B: Summa	ry statistics of	12 agricultural co	mmodity fu	tures (sample	mean)			
Commodities	Long	Short	L	ong/OI (%)	Shoi	t/OI (%)	Num	. of obs.
Chicago Wheat	188.3	3 29.3	3	6.4	5.5		658	
Kansas Wheat	46.1	6.0	2	4.7	2.5		713	
Corn	422.2	63.3	2	4.0	3.4		657	
Soybeans	166.6	26.8	2	2.3	3.2		657	
Soybean oil	95.1	10.9	2	5.0	2.7		656	
Cotton	77.5	6.9	2	8.9	2.4		714	
Cocoa	33.6	8.1	1	5.4	3.1		716	
Sugar	271.6	45.8	2	8.0	4.6		716	
Coffee	48.7	6.8	2	3.1	2.6		716	
Lean hogs	86.3	6.1	3	3.9	2.1		653	
Live cattle	112.2	2 3.9	3	1.5	1.0		658	
Feeder cattle	9.0	0.9	2	1.0	2.0		655	

*Note:* Panel A of this table summarizes the characteristics of the 21 commodity futures from January 1990 to December 2019 in a weekly frequency. Spot prices are calculated with futures prices following Pindyck (2001), and convenience yields are annualized. Panel B reports the average long and short positions of the commodity index traders for the 12 agricultural commodities. The sample period covers from January 2006 to December 2019 in a weekly frequency. Long/OI and short/OI are index traders' long and short positions as fractions of total open interests.

LIU ET AL.

As shown before, more physical inventory held by hedgers increases the selling (hedging) pressure on futures and in turn causes a smaller  $\lambda_0$ , thus increasing the risk premium of commodity futures. This is also consistent with the theory of normal backwardation. In this theory, hedgers provide speculators with risk premiums to entice them to enter the market; the stronger the hedging pressure is, the higher the risk premiums. More importantly, (19) shows that the risk premium also depends on financial demand  $e_t$ , a more financial demand corresponds to a lower risk premium. This is because the financial demand offsets the hedging pressure of the hedgers and thus *lowers* the risk premium. Therefore, as shown in Brunetti and Reiffen (2010), the existence of index investors makes it easier for hedgers to hedge their physical positions.

**Proposition 4.** The volatility of spot price  $\sigma_{s}$  is

$$\sigma_{\rm S} = \sqrt{\lambda_1^2 \sigma_1^2 + \lambda_2^2 \sigma_2^2} \tag{20}$$

Proof. From (13).

$$\begin{aligned} d\mathbf{S}_{t} &= [\lambda_{1}(b - l\delta_{t}) - \lambda_{2}ke_{t}]dt + \lambda_{1}\sigma_{1}d\mathbf{B}_{1,t} + \lambda_{2}\sigma_{2}dB_{2,t} \\ &= [\lambda_{1}(b - l\delta_{t}) - \lambda_{2}ke_{t}]dt + \sqrt{\lambda_{1}^{2}\sigma_{1}^{2} + \lambda_{2}^{2}\sigma_{2}^{2}}dB_{3,t}, \end{aligned}$$

where  $B_{3,t}$  is a Brownian motion.

This proposition indicates that the uncertainty of financial demand impacts the volatility of the commodity spot price.  $\lambda_1 \sigma_1$  is the volatility of prices without index investors, and  $\sqrt{\lambda_1^2 \sigma_1^2 + \lambda_2^2 \sigma_2^2}$  is the volatility with index investors; hence, the presence of index investors increases the volatility of commodity spot prices.

**Proposition 5.** As an extension of the model, for an economy with multiple commodities, even though different commodities may have independent real demands, their prices can be correlated when both are subject to correlated financial demand.

Proof. The spot prices of two commodities are

$$\begin{aligned} dS_{t}^{1} &= \left[\lambda_{1}^{2} \left(b^{1} - l^{1} \delta_{t}^{1}\right) - \lambda_{2}^{1} k^{1} e_{t}^{1}\right] dt + \lambda_{1}^{1} \sigma_{1}^{1} dB_{1,t}^{1} + \lambda_{2}^{1} \sigma_{2}^{1} dB_{2,t}^{1} \\ dS_{t}^{2} &= \left[\lambda_{1}^{2} \left(b^{2} - l^{2} \delta_{t}^{2}\right) - \lambda_{2}^{2} k^{2} e_{t}^{2}\right] dt + \lambda_{1}^{2} \sigma_{1}^{2} dB_{1,t}^{2} + \lambda_{2}^{2} \sigma_{2}^{2} dB_{2,t}^{2} \end{aligned}$$

where superscripts of the variables identify the commodity. It is easy to show that, even though there is no correlation between the fundamentals of  $dB_{1,t}^1$  and  $dB_{1,t}^2$ , if there is a correlation between the increments of financial demand  $dB_{2,t}^1$  and  $dB_{2,t}^2$ , then the correlation between  $dS_t^1$  and  $dS_t^2$  can still be fairly large.

As financial demand is typically influenced by common macroeconomic factors and investors' risk appetite, financial demand for different commodities is likely to be correlated. Moreover, as many index investors invest in commodity indexes (i.e., buy or sell different commodities simultaneously), financial demand among different commodities is likely to be correlated. This is consistent with Tang and Xiong (2012), who show that the correlation between fundamental-uncorrelated commodities such as oil and copper rose dramatically after 2004 because of financial demands. Hence, financial demand for different commodities can be considered as a driving source of the high correlations among different commodities observed in recent years.

□.

# 4 | DATA

Our data set consists of futures contracts for 21 US traded commodities, which can be classified into five groups. The grain group includes Chicago wheat, Kansas wheat, corn, oat, soybeans, soybean oil, and soybean meal; the energy group includes crude oil, heating oil, and natural gas; the soft group includes cotton, orange juice, cocoa, sugar, and coffee; the metal group includes platinum, palladium, and copper; and the live cattle group includes lean hogs, live cattle, and feeder cattle. Note that we do not include gold or silver in our sample since neither normally has a "real" usage and hence does not have a significant convenience yield. For all commodity futures, we use weekly data from January 1990 to December 2019. Commodity futures prices are obtained from the Deep History Package of the Individual Commodity Contracts Database of Pinnacle Data Corporation. For each weekly observation, we assign a near-month contract and a far-month contract when computing spot prices and convenience yields. To adjust the seasonality in many commodity futures (e.g., Richter & Sorensen, 2002), we generally used future contracts with a time to maturity of ~1 month and 13 months, except for several commodity futures whose 13-month futures price data are not available, for which use futures contracts with shorter time to maturity as far-month contracts instead.

We obtain historical weekly long and short positions of commodity index traders (CIT) from the CIT report from the commodity futures trading commission (CFTC, 2008), where futures traders are divided into three classes: commercial traders, index traders, and noncommercial traders. The CIT report shows historical long and short positions for 12 commodities: corn, soybean, Chicago wheat, Kansas wheat, soybean oil, coffee, cotton, sugar and cocoa, feeder cattle, lean hogs, and live cattle. The major commodity indexes are Standard & Poor's-Goldman Sachs Commodity Index (S&P-GSCI) and the Dow Jones-UBS Commodity Index (DJ-UBSCI) and nearly all are based on passive, long-only, fully collateralized commodity futures positions. The interest rate is based on data on 3-month Constant Maturity Treasury (CMT) yield from the Federal Reserve Board. Table 1 shows the summary statistics for the 21 commodities and index trader positions on 12 agricultural commodities. One can see that CIT mainly take long positions in commodity futures, as the average net long (long-minus-short) position is ~25% of the total open interest.

# 5 | HYPOTHESES AND EMPIRICAL EVIDENCE

Spot prices of commodities are usually not available, and one can use the prices of futures contracts for delivery in the current month (spot futures contract) as the proxies. However, even spot futures contracts are not available for every month. For this reason, we follow Pindyck (2001) and use the Equation (21) to infer the spot prices of commodities by extrapolating the traded futures prices:

$$S_{t} = F(t, T_{1}) \left[ \frac{F(t, T_{1})}{F(t, T_{2})} \right]^{\frac{T_{1} - t}{T_{2} - T_{1}}}$$
(21)

where  $F(t, T_1)$  and  $F(t, T_2)$  are prices of the near-month and far-month futures contracts with maturity  $T_1$  and  $T_2$  ( $T_1 < T_2$ ). We then extract the annualized convenience yield from spot and futures prices by discretizing (3),

$$\delta_t = \frac{\mathsf{S}_t + r\mathsf{S}_t\mathsf{T}_2 - \mathsf{F}(\mathsf{t},\mathsf{T}_2)}{\mathsf{T}_2} \tag{22}$$

$$=\frac{S_t - F(t, T_2)}{T_2} + rS_t \tag{23}$$

where *r* is the 3-month CMT interest rate. Note that the measure of convenience yield includes both the commodity's convenience yield and its storage cost. Because the data of storage costs for various commodities are usually unavailable, we

use the combination as a proxy for real convenience yields. Thus, although the extracted convenience yield measure includes the commodity's storage cost, for expositional simplicity we refer to it as convenience yield throughout the paper.

Pindyck (1993) shows that, by analogy to the stock market (refer to Campbell & Shiller, 1987), for commodities, the present value model of commodities is written as,

$$S_t = \lambda_0 + \lambda_1 \delta_t. \tag{24}$$

This is exactly the case when no financial demand (when  $\lambda_2 = 0$  in Equation (13)) exists in our model. Given the same convenience yields, financial demands have two effects on commodity prices when comparing (13) and (24). First, as shown in the comparative statics in Section 3.2,  $\lambda_0$  become larger with index investors coming into the market. Second, financial demand itself influences the price through the channel of  $\lambda_2 e_t$  in Equation (13). Hence, we propose the following hypothesis:

**H1.** Holding convenience yields the same, commodity prices rise when index investors buy commodity futures.

To test this hypothesis, we estimate the relationship between the level of commodity prices and financial investment. Before running the time-series test, we first check the order of integration of both spot prices and convenience yields. Using augmented Dickey-Fuller tests, we find that most commodity prices are I(1), while the convenience yields are I(0). Since some of the variables are non-stationary, it is necessary to apply a cointegration analysis to allow for a long-term relationship between commodity prices and convenience yields. However, the standard cointegration techniques, for example, Engle and Granger (1987) and Johansen (1991), require that all variables be integrated in the same order and thus are not applicable in this paper. Pesaran and Shin (1999) show that an autoregressive distributed lag (ARDL) model can be used in the presence of a long-run relationship between the underlying variables and to provide a consistent estimator of long-run parameters in the presence of the I(0) and I(1)variables in the estimation. The ARDL model is written as:

$$y_{t} = h_{0}z_{t} + \sum_{i=1}^{p} h_{i}^{y}y_{t-i} + \sum_{i=0}^{q} h_{i}^{x}x_{t-i} + \varepsilon_{t}$$
(25)

where *y* is the dependent variable, *x* are the regressors, *z* stands for the deterministic variables such as time trends and dummy variables, and  $\varepsilon_t$  is a noise variable. The long-run relationship between *z*, *x* and *y*,  $y_t = \alpha_z z_t + \alpha_x x_t$ , can be calculated with  $\alpha_z = \frac{h_0}{1 - \sum_{i=1}^{p} h_i^y}$  and  $\alpha_x = \frac{\sum_{i=0}^{q} h_i^x}{1 - \sum_{i=1}^{p} h_i^y}$ . Pesaran and Shin (1999) also show that the ARDL estimators are numerically identical to the OLS estimators of Bewley (1979):

$$\mathbf{y}_{t} = \alpha_{z} \mathbf{z}_{t} + \alpha_{x} \mathbf{x}_{t} + \sum_{i=0}^{p-1} \beta_{i} \Delta \mathbf{y}_{t-i} + \sum_{i=0}^{q-1} \gamma_{i} \Delta \mathbf{x}_{t-i}.$$
 (26)

To estimate the statistical inference of the long-term parameters, we employ the Bewley (1979) methodology. In our case, we choose p = 3 and q = 3 and estimate the following regression for each commodity:

$$\begin{aligned} & \text{SpotPrice}_{t} = \alpha_{0} + \alpha_{1} \text{After } 2004 + \alpha_{2} \text{ConYield}_{t} + \sum_{i=0}^{2} \beta_{i} \Delta \text{SpotPrice}_{t-i} \\ & + \sum_{i=0}^{2} \gamma_{i} \Delta \text{ConYield}_{t-i} + \Gamma' \text{Controls}_{t} + \epsilon_{t} \end{aligned}$$

$$(27)$$

where *After*2004 is a dummy variable that equals one when the time period is after January 1, 2004, and zero otherwise. We choose 2004 as a watershed for the presence of financial demand because, as shown in Masters and White (2008), Tang and Xiong (2012) and others, many financial institutional investors started to invest in the commodity futures market around 2004. We augment the ARDL model by controlling for changes in the 3-month Treasury Bill rate and changes in the 10-year term spread (to account for macroeconomic conditions), Shanghai Composite Index weekly returns (as a proxy for demand for commodities from China), and month of the year dummies to absorb the effects of seasonality.

Table 2 presents the results. Point estimate  $\hat{\alpha}_1$  is positive and significant for all 21 commodities, which shows that the dramatic increases in commodity prices are not related solely to the convenience yield (real commercial demands); instead, they are, to a large extent, determined by other factors, such as financial demands. This finding shows that there is a structural break in the price-convenience yield relationship around the year 2004, which is consistent with H1 and Figure 1. The ratio of the first two coefficients,  $\alpha_1/\alpha_0$ , can be seen as the percentage increment of prices that is unexplained by convenience yields. Across the 21 commodities, the average estimated  $\alpha_1/\alpha_0$  is close to 127.1%, with Heating Oil being the highest (413.9%) and Cotton being the lowest (16.2%).

If we compare the expressions of (13) and (24), we can see that the relationship between convenience yields and commodity prices is diluted due to the inclusion of financial demand. We hence propose.

**H2**. The explanatory power of convenience yields for spot prices is weaker in the presence of index investors.

To test this hypothesis, for each commodity, we compare the extent to which convenience yields explain variation in spot prices. Specifically, we regress the changes in spot prices on the changes in the convenience yields before and after 2004 separately:

$$\Delta SpotPrice_t = a_0 + a_1 \times \Delta ConYield_t + \varepsilon_t.$$
(28)

We consistently use changes in all time-series regressions except for in the ARDL model in Equation (27), which we use for testing the hypothesis on price levels. Since the change of convenience yield is stationary for all commodities, doing so helps avoid spurious results.<sup>13</sup>

Table 3 reports the regression results.<sup>14</sup> The explanatory power of convenience yields on prices is highly significant before 2004. Specifically, the average adjusted  $R^2$  of all commodities is 63% before 2004, which is strongly consistent with our notion and Pindyck (1993) in that the convenience yield can be thought of as the key fundamental variable underlying commodity prices. More importantly, the adjusted  $R^2$  before 2004 is larger than that after 2004 for all of the 21 commodities, with an average difference of around 30%. This is consistent with H2. For a robustness check, we further control for lagged changes in convenience yield and find qualitatively and quantitatively similar results.

Generally, it is difficult to pin down the exogenous financial demand for commodities. However, since 2006, CFTC has been publishing long and short positions of CIT for 12 agricultural commodities. As mentioned in the introduction section, commodity index investment is one of the most important demands in commodity futures. A key fact is that commodity index investment is a passive investment; its position change is typically determined by the needs of portfolio diversification, capital constraints, and so forth, which are orthogonal to commodity fundamentals. Hence, the positions of index investors can be considered as exogenous to commodity fundamentals. Therefore, the CIT data allow us to develop a direct tests for our model. We then propose the following hypothesis:

H3. Changes in index traders' positions positively affect changes in spot prices.

To test this hypothesis, we estimate a discretized version of Equation (13):

TABLE 2 Commo	odity prices before	e and after 2004								
Commodity	Intercept	After2004	C۲,	$\Delta SP_t$	ΔCY <sub>t</sub>	$\Delta SP_{t-1}$	$\Delta C Y_{t-1}$	$\Delta SP_{t-2}$	$\Delta C Y_{t-2}$	R <sup>2</sup>
Chicago wheat	316.5***	280.9***	$1.1^{***}$	0.5	$-1.0^{**}$	0.3	$-1.1^{**}$	0.4	-1.0*	0.55
	(22.4)	(37.9)	(14.5)	(1.4)	(-2.1)	(1.1)	(-2.4)	(1.2)	(-1.8)	
Kansas wheat	324.3***	292.7***	1.4***	0.3	$-1.0^{*}$	0.3	$-1.2^{**}$	0.4	$-1.0^{*}$	0.45
	(20.3)	(37.2)	(18.6)	(1.0)	(-1.7)	(0.9)	(-2.2)	(1.2)	(-1.9)	
Corn	241.4***	202.3***	2.1***	0.7**	-1.6***	0.5*	-1.0***	0.4	-0.9**	0.77
	(38.4)	(51.4)	(39.1)	(2.5)	(-4.1)	(1.9)	(-2.7)	(1.6)	(-2.3)	
Oats	138.7***	136.1***	0.8***	0.5**	-0.7***	0.5**	-0.7***	0.4*	-0.5**	0.67
	(30.1)	(46.3)	(20.4)	(2.0)	(-3.0)	(2.1)	(-3.2)	(1.7)	(-2.6)	
Soybeans	540.1***	431.0***	1.8***	0.7**	-1.4***	0.6**	$-1.2^{**}$	0.6**	-1.4**	0.73
	(35.9)	(46.5)	(16.4)	(2.5)	(-2.6)	(2.1)	(-2.0)	(2.2)	(-2.4)	
Soybean oil	20.9***	17.7***	1.3***	0.5	-0.4	0.3	-0.3	0.4	-0.7	0.50
	(25.2)	(36.6)	(14.1)	(1.3)	(-0.6)	(0.8)	(-0.3)	(1.1)	(-0.9)	
Soybean meal	155.6***	119.5***	1.7***	0.8***	-1.4***	0.6***	$-1.0^{***}$	0.5***	-0.8***	0.79
	(31.6)	(43.6)	(31.6)	(3.5)	(-4.6)	(3.0)	(-3.7)	(2.9)	(3.3)	
Crude oil	15.3***	58.0***	2.0***	0.4	$-1.5^{***}$	0.4	-1.4***	0.4	$-1.1^{**}$	0.87
	(11.4)	(69.4)	(26.3)	(1.4)	(3.0)	(1.3)	(-2.7)	(1.5)	(-2.2)	
Heating oil	43.3***	179.2***	2.5***	0.2	-1.4**	0.2	$-1.3^{**}$	0.3	-0.8	0.80
	(10.7)	(64.5)	(22.2)	(0.7)	(-2.1)	(0.8)	(-2.0)	(1.0)	(-1.3)	
Natural gas	1.8***	3.1***	1.7***	0.5	$-1.2^{**}$	0.4	-0.9*	0.4	-0.7	0.83
	(7.4)	(14.4)	(6.5)	(1.4)	(-2.0)	(1.0)	(-1.7)	(1.0)	(-1.5)	
Cotton	61.2***	9.9***	1.7***	0.3*	-0.7***	0.3*	-0.7***	0.3*	-0.7***	0.85
	(73.0)	(21.3)	(88.6)	(1.9)	(-3.4)	(1.7)	(3.3)	(1.8)	(-3.2)	
Orange juice	107.3***	37.2***	2.2***	0.6***	$-1.2^{***}$	0.6***	$-1.1^{***}$	0.4***	-0.9***	0.46
	(57.1)	(41.8)	(51.4)	(5.3)	(-5.5)	(2.6)	(-5.5)	(4.4)	(-4.1)	
Cocoa	1368.2***	1058.8***	2.9***	0.4***	$-1.8^{***}$	0.5***	$-1.7^{***}$	0.4***	-1.6***	0.65
	(28.7)	(42.0)	(18.3)	(2.7)	(-3.3)	(3.7)	(-3.2)	(3.0)	(-2.9)	

LIU ET AL.

Commodity	Intercept	After2004	CY <sub>t</sub>	$\Delta SP_t$	$\Delta C Y_t$	$\Delta SP_t = 1$	$\Delta C Y_{t-1}$	$\Delta SP_{t-2}$	$\Delta C Y_{t-2}$	R <sup>2</sup>
Sugar	7.5***	8.4***	2.0***	0.3*	$-1.0^{***}$	0.2	-0.9***	0.3	-0.8**	0.75
	(25.4)	(54.3)	(51.1)	(1.7)	(-3.0)	(1.2)	(2.8)	(1.3)	(-2.5)	
Coffee	96.0***	62.7***	1.9***	0.5*	-1.4***	0.5*	-1.4**	0.4	$-1.2^{**}$	0.52
	(31.8)	(33.0)	(35.4)	(1.8)	(-3.3)	(1.8)	(2.8)	(1.5)	(-2.3)	
Platinum	416.0***	826.8***	0.9***	0.4	-0.8**	0.6*	-0.3	0.5	-0.2	0.75
	(16.7)	(53.0)	(3.5)	(1.2)	(-2.5)	(1.7)	(-1.4)	(1.5)	(-1.3)	
Palladium	179.5***	343.9***	4.4**	0.3	-2.9***	0.4	-2.5**	0.2	-2.3**	0.52
	(8.8)	(26.7)	(7.2)	(0.7)	(-2.6)	(1.2)	(-2.4)	(0.7)	(-2.4)	
Copper	94.0***	200.4***	0.1	0.3	-0.7	0.2	-0.5	0.3	-0.6	0.79
	(16.1)	(59.6)	(0.5)	(0.9)	(-0.8)	(0.7)	(-0.6)	(1.1)	(-0.6)	
Lean hogs	47.5***	24.4***	0.7***	0.6***	-0.4***	0.5***	-0.4**	0.4***	-0.2**	0.63
	(52.4)	(39.5)	(24.3)	(3.9)	(-3.4)	(3.9)	(-3.7)	(2.9)	(-2.0)	
Live cattle	66.7***	42.4***	0.8***	0.4	-0.2	0.4	-0.2	0.2	-0.1	0.63
	(40.1)	(44.3)	(14.9)	(1.2)	(-0.8)	(1.1)	(-0.5)	(0.7)	(-0.2)	
Feeder cattle	74.8***	58.1***	0.9***	0.5	-0.4	0.3	0.1	0.4	-0.3	0.56
	(28.3)	(38.0)	(8.9)	(0.9)	(-0.6)	(0.6)	(0.1)	(0.7)	(-0.5)	
lote: This table reports r	egressions of com	modity spot prices	on convenience	yields (CY), and	d changes in both	spot prices (ΔSP)	), and convenien	ce yields (ΔCY), a	s well as their la	gged

January 1, 2004. Additional control variables include changes in 3-month Treasury bill rate and 10-year-3-month Treasury yield spread, Shanghai composite index return, and month of terms, for the 21 commodities. The sample is from January 1990 to December 2019 in a weekly frequency, and After2004 is a dummy variable that equals one for observations after the year dummies. Superscripts \*\*\*, \*\*, and \* indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

TABLE 2 (Continued)

ΊLΕΥ

# TABLE 3 Decrease in explanatory power of convenience yield on commodity prices

	Before 2004	4		After 2004			
Commodity	Intercept	$\Delta CY_t$	R <sup>2</sup>	Intercept	$\Delta CY_t$	R <sup>2</sup>	Difference in $R^2$
Chicago wheat	-0.02	1.18***	0.64	0.39	1.09***	0.28	0.36
	(-0.06)	(13.52)		(0.40)	(7.64)		
Kansas wheat	-0.00	1.23***	0.64	0.64	0.18***	0.20	0.44
	(-0.00)	(13.95)		(0.64)	(10.07)		
Corn	0.01	1.07***	0.58	0.39	0.95***	0.37	0.21
	(0.04)	(7.95)		(0.62)	(11.32)		
Oats	-0.01	0.77***	0.63	-0.02	0.75***	0.50	0.13
	(-0.07)	(18.36)		(-0.04)	(17.69)		
Soybeans	0.04	1.04***	0.61	1.20	0.97***	0.39	0.22
	(0.07)	(23.35)		(1.01)	(7.99)		
Soybean oil	0.00	1.20***	0.49	0.02	0.08***	0.03	0.46
	(0.10)	(25.97)		(0.34)	(8.20)		
Soybean meal	-0.00	1.00***	0.68	0.53	1.08***	0.64	0.04
	(-0.01)	(27.52)		(1.42)	(20.37)		
Crude oil	0.03	1.31***	0.85	0.11	1.37***	0.33	0.52
	(1.45)	(35.48)		(0.77)	(13.59)		
Heating oil	0.08	1.31***	0.81	0.18	0.38*	0.11	0.70
	(1.38)	(32.59)		(0.55)	(1.80)		
Natural gas	0.01	2.04***	0.86	-0.01	1.13***	0.66	0.20
	(0.17)	(9.94)		(-0.52)	(12.02)		
Cotton	-0.02	1.14***	0.79	0.03	1.15***	0.73	0.06
	(-0.39)	(40.89)		(0.39)	(21.42)		
Orange juice	-0.07	1.34***	0.54	0.16	0.18***	0.17	0.37
	(-0.57)	(7.71)		(0.62)	(6.41)		
Сосоа	0.55	4.09***	0.41	-0.84	0.11***	0.18	0.23
	(0.16)	(3.50)		(-0.24)	(3.67)		
Sugar	-0.00	1.37***	0.75	0.01	1.43***	0.68	0.07
	(-0.42)	(33.58)		(0.28)	(18.93)		
Coffee	0.05	1.11***	0.51	0.10	1.88***	0.21	0.30
	(0.26)	(7.75)		(0.47)	(5.89)		
Platinum	0.51	0.44***	0.54	0.40	1.29***	0.07	0.47
	(1.01)	(66.76)		(0.23)	(3.50)		
Palladium	0.12	1.65***	0.26	0.66	-0.05	0.00	0.26
	(0.14)	(3.67)		(0.69)	(-0.20)		
Copper	0.03	1.33***	0.66	0.43	2.68***	0.31	0.35
	(0.47)	(30.35)		(0.88)	(7.46)		
Lean hogs	0.03	0.69***	0.66	0.09	0.38***	0.47	0.19
	(0.40)	(12.20)		(0.86)	(5.42)		
Live cattle	0.03	1.05***	0.72	0.06	0.40***	0.35	0.37

## TABLE 3 (Continued)

	Before 2004			After 2004			
Commodity	Intercept	$\Delta CY_t$	R <sup>2</sup>	Intercept	$\Delta CY_t$	R <sup>2</sup>	Difference in $R^2$
	(0.60)	(16.52)		(0.60)	(6.41)		
Feeder cattle	0.03	0.76***	0.55	0.08	0.35***	0.22	0.33
	(0.74)	(12.49)		(0.67)	(5.53)		
Average			0.63			0.33	0.30

*Note:* This table reports regressions of changes in commodity prices on changes in convenience yield for 21 commodities before and after the year of 2004. The sample is from January 1990 to December 2019 in a weekly frequency. The last column is the difference in adjusted  $R^2$  between the two subsamples before and after the year 2004. The superscripts \*\*\*, \*\*, and \* indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

TABLE 4 Relationship between index trading positions and commodity prices

Commodity	Correlation	
Chicago Wheat	0.008	
Kansas Wheat	0.027	
Corn	0.056	
Soybeans	0.091	
Soybean Oil	-0.028	
Cotton	0.016	
Сосоа	0.034	
Sugar	-0.060	
Coffee	0.067	
Lean Hogs	0.042	
Live Cattle	0.046	
Feeder Cattle	-0.027	

#### Panel B: Commodity-by-commodity regressions

	Intercept	ΔCY <sub>t</sub>	$\Delta NetLong_t$	R <sup>2</sup>
Chicago Wheat	0.37	1.08***	0.79**	0.29
	(0.36)	(7.61)	(2.47)	
Kansas Wheat	0.37	0.18***	3.56***	0.24
	(0.36)	(11.58)	(6.75)	
Corn	0.42	0.94***	0.17**	0.38
	(0.61)	(11.11)	(2.10)	
Soybeans	1.09	0.93***	1.65***	0.42
	(0.88)	(7.65)	(4.51)	
Soybean Oil	0.01	0.08***	0.08***	0.06
	(0.18)	(9.16)	(3.73)	
Cotton	0.01	1.14***	0.09	0.73
	(0.15)	(21.16)	(1.52)	
Сосоа	-1.21	0.11***	10.33***	0.21
	(-0.34)	(3.59)	(4.68)	

17

# TABLE 4 (Continued)

# Panel B: Commodity-by-commodity regressions

		Intercer	ot	$\Delta CY_t$		ΔNetLo	ongt	
Sugar		0.00		1.43**	*	0.01		0.69
C .		(0.06)		(18.71)		(1.63)		
Coffee		0.10		1.81**	*	0.63*	**	0.23
		(0.47)		(5.71)		(3.75)		
Lean Hogs		0.11		0.37**	**	0.31*	**	0.49
		(0.99)		(5.39)		(5.09)		
Live Cattle		0.03		0.39**	*	0.24*	**	0.38
		(0.27)		(6.37)		(3.57)		
Feeder Cattle		0.06		0.34**	*	1.00*	*	0.23
		(0.45)		(5.16)		(2.10)		
Panel C: Panel re	egressions w	vith lagged c						
	∆SpotPric	ce <sub>t</sub>			SpotPrice	t + 3 - SpotPr	ice <sub>t</sub>	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept	0.25	0.06	0.14	0.48	0.68	0.74	0.75	0.72
	(0.59)	(0.15)	(0.38)	(1.06)	(0.93)	(1.02)	(1.04)	(0.84)
$\Delta ConY$ ield <sub>t</sub>		0.12***	0.12***	0.12***		-0.07***	-0.07***	-0.07***
		(40.24)	(40.88)	(40.87)		(-11.41)	(-11.87)	(-11.87)
$\Delta ConY ield_{t-1}$			-0.02***	-0.02***			-0.03***	-0.03***
			(-5.43)	(-5.45)			(-3.40)	(-3.40)
$\Delta \text{ConY ield}_{t\ -\ 2}$			-0.03***	-0.03***			0.01*	0.01*
			(-10.74)	(-10.73)			(1.70)	(1.70)
$\Delta \text{ConY ield}_{t\ -\ 3}$			-0.03***	-0.03***			-0.01	-0.01
			(-10.72)	(-10.73)			(-0.95)	(-0.95)
$\Delta NetLong_t$	0.78***	0.75***	0.76***	0.93***	-0.10	-0.08	-0.09	-0.11
	(7.26)	(7.62)	(7.90)	(6.06)	(-0.55)	(-0.46)	(-0.51)	(-0.37)
$\Delta NetLong_{t-1}$	-0.17	-0.16	-0.15	-0.15	-0.08	-0.08	-0.07	-0.06
	(-1.53)	(-1.62)	(-1.49)	(-1.54)	(-0.42)	(-0.43)	(-0.35)	(-0.35)
$\Delta NetLong_{t-2}$	-0.03	-0.08	-0.08	-0.09	-0.22	-0.18	-0.19	-0.19
	(-0.25)	(-0.84)	(-0.85)	(-0.89)	(-1.16)	(-0.99)	(-1.01)	(-1.01)
$\Delta NetLong_{t-3}$	-0.09	-0.08	-0.06	-0.06	-0.01	-0.03	-0.02	-0.02
	(-0.86)	(-0.80)	(-0.63)	(-0.63)	(-0.03)	(-0.14)	(-0.12)	(-0.12)
$\Delta \textit{NetLong}_t  imes \textit{I}(\Delta$	NetLong <sub>t</sub> > 0	))		-0.31				0.02
				(-1.36)				(0.06)
Observations	8399	8399	8399	8399	8103	8103	8103	8103
R-squared	0.01	0.17	0.19	0.19	0.00	0.02	0.02	0.02

*Note:* Panel A of this table reports Pearson correlation coefficients between changes in convenience yields and changes in index traders' net long positions for the 12 agricultural commodities. Panel B reports commodity-by-commodity regressions of changes in spot prices on changes in both convenience yields and index traders' net long positions. Panel C reports panel regressions that further include lagged changes. The sample is in weekly frequency from January 2006 to December 2019. Superscripts \*\*\*, \*\*, and \* indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

WILEY

19

(29)

#### $\Delta$ SpotPrice<sub>t</sub> = $b_0 + b_1 \times \Delta$ ConYield<sub>t</sub> + $b_2 \times \Delta$ NetLong<sub>t</sub> + $\varepsilon_t$ .

where *NetLong* represents the net long position of the index traders, which is their aggregate long position minus their short position. In Panel A of Table 4, we first show that the correlations between  $\Delta ConY$  ield<sub>t</sub> and  $\Delta NetLong_t$ are tiny for all 12 commodities. This fact is consistent with the assumption of our theoretical model in which the correlation of convenience yield and financial demand is zero.

Panel B in Table 4 reports the results of commodity-by-commodity regressions. All point estimates  $b_2$  are positive, and 10 out of 12 commodities are significant at the 95% or higher levels. This result confirms the hypothesis that commodity prices positively depend on financial demands.

In Panel C, we estimate panel regressions using the sample of all 12 commodities to examine the persistence and potential asymmetry of the effect. In Columns (1)–(3), we re-estimate Equation (29) with additional lagged changes in both convenience yield and index traders' net long positions. The estimates show that, unlike changes in convenience yield whose effect on spot prices diminishes subsequently, there is no significant association between lagged changes in index positions and current changes in spot prices. These results suggest that index investment has a persistent impact on commodity prices. In Column (4), we further test for differential effects between the increase and decrease in index positions by including an interaction term that equals the change in index positions if and only if the change is positive. The point estimate shows that the effect of a reduction in index positions is moderately larger, but the difference is statistically indistinguishable from zero. In Columns (5)–(8), we replace the dependent variable to the change in spot prices during the three subsequent weeks. This allows us to detect potential reversal of index investment's impact on spot prices over relatively longer horizons. Consistent with Columns (2)–(4), the effect of changes in convenience yield on spot prices diminishes over time. In contrast, past changes in index positions do not predict subsequent changes in spot prices. This further supports the interpretation that index investment has a persistent impact on commodity prices.

Proposition 5 shows that correlated financial demands for different commodity futures can give rise to comovement in spot prices, even if commodity fundamentals are independent of each other. To test this unique prediction, we first investigate the comovement of commodity spot prices using a principal component analysis. We perform the analysis using commodity prices before and after 2004 separately. Table 5 shows that the first principal component explains 37.7% and 63.0% of variation in spot prices before 2004 and after 2004, respectively. The cumulative explained proportion of commodity prices by principal components is consistently higher after 2004 for up to five principal components. Combined with our earlier findings, this result indicates that index trading increases the comovement of commodities despite limited correlations among their fundamentals.

Next, we examine the relationship between pairwise correlation in index trading and pairwise correlation in commodity prices. For each of the two variables, we estimate the contemporaneous correlation for every pair of the 10 commodities for which we observe index traders' positions.<sup>15</sup>

Figure 2 presents a scatter plot of all estimated pairwise correlations. Clearly, commodity pairs with a higher correlation in index trading also tend to have a higher correlation in commodity spot prices. The fitted line reflects a regression coefficient of 1.93 (t = 5.17), which is consistent with our prediction.

# 6 | CONCLUSION

Commodities are gradually being accepted as a new financial asset class as institutional investors increasingly invest in commodity futures for financial reasons, such as portfolio diversification and capital constraints. Thus, financial demand is becoming an important force in the commodity futures market. This paper shows that the relationship between convenience yields and commodity prices can be diluted by financial investments.

	Before 2004		After 2004	
	Explained proportion	Cumulative	Explained proportion	Cumulative
First principal component	37.7%	37.7%	63.0%	63.0%
Second principal component	21.6%	59.3%	12.9%	75.9%
Third principal component	11.4%	70.7%	7.6%	83.5%
Fourth principal component	7.9%	78.6%	4.0%	87.5%
Fifth principal component	7.1%	85.6%	3.0%	90.5%

#### TABLE 5 Comovement of commodity prices: Principal component analysis

*Note:* This table reports the results of a principal component analysis (PCA) separately performed using time series of commodity prices before and after year 2004.



Pairwise Correlation in Index Trading

**FIGURE 2** Relationship between pairwise correlation in index trading and pairwise correlation in commodity prices. The fitted line has a regression coefficient 1.93 (t = 5.17) and the shaded area indicates confidence interval at the 95% level

To analyze the impact of financial investments on commodity prices, we develop a demand-based model in which there are three types of traders—hedgers, speculators, and index investors—in the economy. Only hedgers hold physical stocks of commodities, and all players trade commodity futures. Both hedgers and speculators determine their positions by maximizing their utility and index investors decide their positions for reasons that are exogenous to commodity markets. Convenience yields are regarded as representative of the fundamentals underlying commodities. The equilibrium commodity price in this model is a combination of the convenience yield and financial demand. Therefore, the explanatory power of the convenience yield for commodity prices is diluted when index investors enter the commodity futures market. Furthermore, the volatility of commodity prices increases in the presence of index investors because the inflows and outflows of money cause prices to move accordingly.

Empirically, we show that the convenience yield-price relationship exhibits a structural change in 2004; since that year financial investors have been actively trading in the commodity futures market. We find that convenience yields have less power in explaining commodity prices after 2004 than before 2004. Considering commodity index

INTERNATIONAL

21

investment as one type of financial demand, we document that financial demand influences commodity prices for 12 agricultural commodities. We show that stronger financial demand corresponds to higher commodity prices, and larger uncertainty of financial demand contributes to larger volatility in commodity prices.

## ACKNOWLEDGMENT

We are grateful to ALLAUDEEN HAMEED (the Editor) and an anonymous referee for their helpful comments and suggestions. Zhigang Qiu acknowledges financial support from the National Natural Science Foundation of China (Grant No. 71773127). All errors are our own.

## **ENDNOTES**

- <sup>1</sup> Major financial institutions invested in the commodity market through individual commodity futures, commodity futures indexes, commodity-linked notes and exchange traded products (ETPs).
- <sup>2</sup> The figure for 2003 comes from the Commodity Futures Trading Commission (CFTC), and the figure for 2019 is estimated by Barclays.
- <sup>3</sup> Because institutional investors trade commodities mainly for diversification that is almost independent with the commodity market, we regard financial investments as exogenous demand shock to commodity market.
- <sup>4</sup> Due to the re-balancing of financial demand through the flows of money in and out, the volatility of commodities rose after 2004.
- <sup>5</sup> The convenience yield is defined as the benefit or premium yielded by withholding an underlying product or physical good, rather than holding associated futures. The benefits include production smoothing, stockout avoidance, and so on. Convenience yields arise because of the mismatch in demand for and supply of a certain commodity.
- <sup>6</sup> For example, Erb and Harvey (2006), Gorton et al. (2007), Hong and Yogo (2009) and Acharya et al. (2009). These papers focus on the roles of producer's open interest, the convenience yield, and cross-sectional behavior of commodity risk premium.
- <sup>7</sup> Note that the convenience yield here refers to the net dollar convenience yield, which equals the actual dollar convenience yield less the storage cost. It is different from the percentage convenience yield as defined in Brennan (1958), which equals the net dollar convenience yield divided by spot commodity prices.
- <sup>8</sup> For example, Routledge et al. (2000) make risk-neutral investors determine an optimal inventory level. The equilibrium convenience yield is thus determined endogenously.
- <sup>9</sup> In the case where futures have a constant maturity, the control problem normally does not have an analytical solution.
- <sup>10</sup> Note that, in this model, physical commodities are traded among different hedgers but cannot be traded between hedgers and speculators or hedgers and index investors since speculators and index investors cannot hold physical commodities.
- <sup>11</sup> See (A21) in Appendix A.
- <sup>12</sup> If  $e_t$  is on the supply side, equation (A21) in the Appendix A becomes  $\tau v \sigma_2^2 \lambda_2^2 + (1+w)(k+r)\lambda_2 + \tau v \lambda_1^2 \sigma_1^2 = 0$ . The solutions of  $\lambda_2$  for this new equation are negative.
- <sup>13</sup> That said, results based on levels are gualitatively similar.
- <sup>14</sup> For a robustness check, we also consider using commodity inventory as a control variable. However, due to data limitation, we only have proxy for aggregate inventory level for one commodity, Crude Oil. Adding this variable into our existing econometric specifications do not change our results.
- <sup>15</sup> To mitigate the impact of mechanically high correlations between highly similar commodities, we exclude Kansas Wheat and Feeder Cattle from the sample in this exercise.

#### REFERENCES

Acharya, Viral, Lars Lochstoer and Tarun Ramadorai (2009), Limits to Arbitrage and Hedging: Evidence from Commodity Markets, Working Paper, NYU Stern.

Basak, S., & Pavlova, A. (2016). A model of Financialization of commodities. Journal of Finance, 71, 1511-1556.

Bessembinder, H. (1992). Systematic risk, hedging pressure, and risk premiums in futures markets. Review of Financial Studies, 4, 637-667.

- Bewley, R. A. (1979). The direct estimation of the equilibrium response in a linear dynamic model. *Economic Letters*, *3*, 357–361.
- Breeden, D. (1984). Futures markets and commodity options: Hedging and optimality in incomplete markets. Journal of Economic Theory, 32, 275–300.

Brennan, M. (1958). The supply of storage. American Economic Review, 48, 50-72.

- Brunetti, Celso and David Reiffen (2010), Commodity Index Trading and Hedging Costs, Working Paper, John Hopkins University.
- Campbell, J. Y., & Shiller, R. J. (1987). Cointegration and tests of present value models. *Journal of Political Economy*, 95, 1062–1088.
- CFTC (2008), Staff Report on Commodity Swap Dealers & Index Traders with Commission Recommendations.
- Chinn, M. D., & Coibion, O. (2009). The predictive content of commodity futures. *Journal of Futures Markets*, 34(7), 607-636.
- Da, Zhi, Ke Tang, Yubo Tao and Liyan Yang (2020). Financialization and Commodity Market Serial Dependence, Working Paper.
- Ekeland, I., D. Lautier, and B. Villeneuve (2017), Hedging Pressure and Speculation in Commodity Markets, Working Paper.
- Engle, R. F., & Granger, C. W. J. (1987). Co-integration and error correction: Representation, estimation, and testing. Econometrica, 55, 251–276.
- Erb, C., & Harvey, C. (2006). The strategic and tactical value of commodity futures. Financial Analysts Journal, 62, 69-97.
- Fama, E., & French, K. (1988). Business cycles and the behavior of metals prices. Journal of Finance, 43, 1075-1094.
- Garleanu, N., Pedersen, L., & Poteshman, A. (2009). Demand-based option pricing. *Reviews of Financial Studies*, 22, 4259–4299.
- Gibson, R., & Schwartz, E. (1990). Stochastic convenience yield and the pricing of oil contingent claims. Journal of Finance, 45, 959–976.
- Gilbert, Christopher L. (2009), Speculative Influences on Commodity Futures Prices 2006–2008, Working Paper, UNCTAD and University of Trento.
- Girma, P. B., & Paulson, A. S. (1999). Risk arbitrage opportunities in petroleum futures spreads. *Journal of Futures Markets*, 19(8), 931–955.
- Goldstein, I., Li, Y., & Yang, L. (2014). Speculation and hedging in segmented markets. *Review of Financial Studies*, 27, 881-822.
- Goldstein, I. and L. Yang (2018), Commodity Financialization and Information Transmission, Working Paper.
- Gorton, Gary, Fumio Hayashi and Geert Rouwenhorst (2007), The Fundamentals of Commodity Futures Returns, Working Paper, Yale University.
- Greenwood, R. (2005). Short- and long-term demand curves for stocks: Theory and evidence on the dynamics of arbitrage. *Journal of Financial Economics*, 75, 607–649.
- Greenwood Robin, Vayanos D. (2010), Bond supply and excess bond returns, Working Paper, Harvard University.
- Hamilton, James D (2009), Causes and Consequences of the Oil Shock of 2007–2008. Working paper, UC San Diego.
- Hau H. (2009), Global versus local asset pricing: evidence from arbitrage of the MSCI index change. Working Paper, INSEAD.
- Hicks, J. R. (1939). Value and capital: An inquiry into some fundamental principles of economic theory. Claredon Press.
- Hirshleifer, D. (1988). Residual risk, trading costs and commodity futures risk Premia. Review of Financial Studies, 1, 173–193.
- Hirshleifer, D. (1990). Hedging pressure and futures price movements in a general equilibrium model. *Econometrica*, 58, 411–428.
- Ho, T. (1984). Intertemporal commodity futures hedging and the production decision. Journal of Finance, 39, 351–377.
- Hong, H., & Yogo, M. (2009). Digging into commodities, working paper. Princeton University and University of.
- Johansen, S. (1991). Estimation and hypothesis testing of Cointegration vectors in Gaussian vector autoregressive models. *Econometrica*, 59, 1551–1580.
- Keynes, John M. (1923), Some Aspects of Commodity Markets, Manchester Guardian Commercial: European Reconstruction Series.
- Krugman, Paul (2008), The Oil Nonbubble, The New York Times, May 12.
- Liu, P., & Tang, K. (2008). The stochastic behavior of commodity prices with Heteroskedasticity in the convenience yield. Journal of Empirical Finance, 18(2), 211–224.
- Masters, Michael and Allen K. White (2008), The Accidental Hunt Brothers: How Institutional Investors are Driving up Food and Energy Prices, White Paper.
- Mou, Yiqun (2010), Limits to Arbitrage and Commodity Index Investments: Front-running the Goldman Roll. Technical Report, Columbia Business School.

Ng, V. K., & Pirrong, S. C. (1996). Price dynamics in refined petroleum spot and futures markets. *Journal of Empirical Finance*, 2(4), 359–388.

INTERNATIONAL

**REVIEW OF FINANCE** 

- Pesaran, M. H., & Shin, Y. (1999). An autoregressive distributed lag modelling approach to Cointegration analysis. In S. Strom (Ed.), *Econometrics and economic theory in the twentieth century*. Cambridge University Press.
- Pindyck, R. (1993). The present value model of rational commodity pricing. Economic Journal, 103, 511–530.
- Pindyck, R. (2001). The dynamics of commodity spot and futures markets: A primer. The Energy Journal, 22, 1-29.
- Richter, M. and C. Sorensen (2002): Stochastic volatility and seasonality in commodity futures and options: The case of soybeans, Working paper, Copenhagen Business School.
- Routledge, B. R., Seppi, D. J., & Spatt, C. S. (2000). Equilibrium forward curves for commodities. *Journal of Finance*, 55(3), 1297–1338.
- Singleton, Kenneth J. (2011), Investor Flows and the 2008 Boom/Bust in Oil Prices, Working paper, Stanford University.
- Sockin, M., & Xiong, W. (2015). Informational frictions and commodity markets. Journal of Finance, 70, 2063–2098.
- Tang, K., & Xiong, W. (2012). Index investment and the Financialization of commodities. Financial Analysts Journal, 68, 54–74.
- Telser, L. (1958). Futures trading and the storage of cotton and wheat. Journal of Political Economy, 66, 233–255.

How to cite this article: Liu, P., Qiu, Z., & Xu, D. X. (2021). Financial investments and commodity prices. *International Review of Finance*, 1–25. https://doi.org/10.1111/irfi.12361

#### Appendix A: MODEL

#### Futures process with instantaneous maturity

We assume the futures price with instantaneous maturity follows

$$\lim_{T \to t} F(t,T) = S_t e^{r(T-t)} - \int_t^T dD_s$$
(A1)

Using Ito's lemma on F(t, T), we have

$$dF(t,T) = e^{r(T-t)}dS_t - rS_t e^{r(T-t)}dt + dD_s$$
(A2)

Defining  $\Delta := T - t$ , if  $T \rightarrow t$ , that is,  $\Delta \rightarrow 0$ , then

$$dF_t \coloneqq \lim_{t \to \infty} dF(t, T) \tag{A3}$$

$$= (1+r\Delta)dS_t - rS_t(1+r\Delta)dt + dD_t$$
(A4)

$$= dS_t - rS_t dt + dD_t \tag{A5}$$

#### Solution of the equilibrium

In the equilibrium, we conjecture that the spot price  $S_t$  follow

$$\mathbf{S}_t = \lambda_0 + \lambda_1 \delta_t + \lambda_2 \mathbf{e}_t, \tag{A6}$$

WILEY

WILEY— Review of Finance

For three constants  $\lambda_0$ ,  $\lambda_1$ , and  $\lambda_2$ . Then the spot price  $dS_t$  can be specified as

$$dS_t = \lambda_1 d\delta_t + \lambda_2 de_t \tag{A7}$$

$$= (\lambda_1 b - \lambda_1 l \delta_t - \lambda_2 k e_t) dt + \lambda_1 \sigma_1 dB_{1,t} + \lambda_2 \sigma_2 dB_{2,t}$$
(A8)

Thus, the wealth processes for both hedgers and speculators, (7) and (8), are

$$dW_{1,t} = [rW_{1,t} + (\phi_{1,t} + a)(\lambda_1 b - (\lambda_1 l + r\lambda_1 - 1)\delta_t - \lambda_2 (k + r)e_t - r\lambda_0)]dt$$
(A9)

$$+(\phi_{1,t}+a)(\lambda_1\sigma_1dB_{1,t}+\lambda_2\sigma_2dB_{2,t}) \tag{A10}$$

$$dW_{2,t} = \left[ r dW_{2,t} + \phi_{2,t} (\lambda_1 b - (\lambda_1 l + r\lambda_1 - 1)\delta_t - \lambda_2 (k + r)e_t - r\lambda_0) \right] dt$$
(A11)

$$+\phi_{2,t}(\lambda_1\sigma_1d\mathsf{B}_{1,t}+\lambda_2\sigma_2d\mathsf{B}_{2,t})\tag{A12}$$

By the FOC with respect to the utility, (6), we can derive the optimal demands

$$\phi_{1,t} = \frac{\lambda_1 b - (\lambda_1 l + r\lambda_1 - 1)\delta_t - \lambda_2 (k + r)e_t - r\lambda_0}{\tau (\lambda_1^2 \sigma_1^2 + \lambda_2^2 \sigma_2^2)} - a$$
(A13)

and

$$\phi_{2,t} = \frac{\lambda_1 b - (\lambda_1 l + r\lambda_1 - 1)\delta_t - \lambda_2 (k+r)e_t - r\lambda_0}{\tau (\lambda_1^2 \sigma_1^2 + \lambda_2^2 \sigma_2^2)}$$
(A14)

Together with the expression of  $\lambda_1$ , (A19), we can obtain (14) and (15) in the proposition.

With the optimal demands (A13) and (A14), the future market-clearing condition (9) can be arranged as

$$\begin{cases} (1+w)(\lambda_1 b - r\lambda_0) - \tau(\lambda_1^2 \sigma_1^2 + \lambda_2^2 \sigma_2^2)a - \\ (1+w)(\lambda_1 l + r\lambda_1 - 1)\delta_t + [\tau(\lambda_1^2 \sigma_1^2 + \lambda_2^2 \sigma_2^2)v - (1+w)\lambda_2(k+r)]e_t \end{cases} = 0$$
(A15)

which is an affine function of  $\delta_t$  and  $e_t$ . Thus, the constant and the coefficients of  $\delta_t$  and  $e_t$  need to be zero.

First, the coefficient of  $\delta_t$  needs to be zero, which means

$$(1+w)(\lambda_1 l + r\lambda_1 - 1) = 0$$
 (A16)

We can derive  $\lambda_1$  as

$$\lambda_1 = \frac{1}{r+l} \tag{A17}$$

Second, the constant needs to be zero, which means

$$(1+w)(\lambda_1 b - r\lambda_0) - \tau \left(\lambda_1^2 \sigma_1^2 + \lambda_2^2 \sigma_2^2\right) a = 0$$
(A18)

We can derive  $\lambda_0$  as

$$\lambda_0 = \frac{(1+w)\lambda_1 b - \tau (\lambda_1^2 \sigma_1^2 + \lambda_2^2 \sigma_2^2) a}{(1+w)r}$$
(A19)

INTERNATIONAL Review of Finance

Third, the coefficient of  $e_t$  needs to be zero, which means

$$\tau \left(\lambda_1^2 \sigma_1^2 + \lambda_2^2 \sigma_2^2\right) v - (1+w)\lambda_2(k+r) = 0 \tag{A20}$$

which can be rearranged as

$$\tau v \sigma_2^2 \lambda_2^2 - (1+w)(k+r)\lambda_2 + \tau v \lambda_1^2 \sigma_1^2 = 0$$
(A21)

Thus,  $\lambda_2$  is determined by a quadratic function (A21). Define  $F(\lambda_2) = \tau v \sigma_2^2 \lambda_2^2 - (1+w)(k+r)\lambda_2 + \tau v \lambda_1^2 \sigma_1^2$ , so the equilibrium exists iff  $F(\lambda_2) = 0$  has a positive root. Note that F(0) > 0 and the solution to  $F'(\lambda_2) = 0$  is positive, any solutions of  $\lambda_2$  to the equilibrium is positive.

(A19) shows that  $\lambda_0$  is a function of  $\lambda_1$  and  $\lambda_2$ , so  $\lambda_0$  is determined once we know  $\lambda_1$  and  $\lambda_2$ . This concludes our proof.

WILEY