No. 2110 [EN]

IMI Working Paper

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He Qing, Iikka Korhonen and Qian Zongxin

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Monetary Policy Transmission with Two Exchange Rates of a Single Currency: the Chinese Experience*

By* HE QING, IIKKA KORHONEN and QIAN ZONGXIN

October 2021

Abstract

In emerging market economies, transmission of monetary policy through the foreign exchange market is complicated by the coexistence of financial restrictions and arbitrages. Using China as an example, we show that the coexistence of exchange rate interventions, capital controls and an onshore-offshore exchange rate differential makes the long run equilibrium in the currency market nonlinear. Disturbances to this nonlinear long run equilibrium could offset the impact of monetary policy actions on domestic price stability. Omitting such nonlinearity leads to biased inference on the effectiveness of monetary policy.

JEL Classification: E52; F31; F41

Keywords: CNY; CNH; Monetary policy; Capital controls

* Published in International Review of Economics and Finance 75 (2021) 558-576.

* He Qing, Senior Research Fellow of IMI, School of Finance, Renmin University of China. Iikka Korhonen, Member of IMI International Committee, and the Bank of Finland Institute for Economies in Transition. Qian Zongxin, Senior Research Fellow of IMI, School of Finance, Renmin University of China.

1. Introduction

Over the past decades, participation of emerging market countries in global financial market has generated a great deal of attention in the field of international macroeconomics. Different from developed countries, the emerging market economies often suffer from financial repression and tend to be more vulnerable to international economic shocks, which can complicate the exchange rate channel of monetary policy transmission. As surveyed by Frankel (2010), literature usually distinguishes emerging market economies from advanced economies by the existence of an imperfect financial sector, capital controls and opportunities for international arbitrage. Therefore, ignoring these features can prove costly. However, data for international financial market arbitrage is usually unobserved for most emerging market countries, which makes these important issues yet to be fully understood.

Emergence of China's renminbi (RMB) offshore market provides a unique opportunity to explore the exchange rate channel of monetary policy transmission. China's rapid economic growth not only increases its economic impact on the rest of the world but also increases the importance of its currency in international financial market. According to the recent BIS triennial central bank survey (BIS, 2016), the daily average turnover of RMB transactions increased from almost nothing to $202 billion between 2007 and April 2016. RMB became the 8^{th} most traded currency, and account for about 4% of global foreign exchange transactions. The increasing transaction volume of RMB has also created a huge offshore RMB market. However, due to the existence of capital controls and currency market interventions in the onshore RMB market, there are usually gaps between the offshore (CNH) and onshore RMB exchange rates (CNY). These gaps lead to large arbitrage transactions on the differentials of the CNY-CNH exchange rate. In this paper, we study China's monetary policy transmission in the context of two exchange rates of a single currency, RMB.[1]

It is well-established that monetary policy actions transmit to inflation rate through the currency market. An expansionary monetary policy depreciates the home currency, leading to an increase in domestic price of imported goods and an expansion in exports. Over time this will expand output, and, consequently, price level. For a typical emerging market economy, this transmission channel is complicated in two aspects. First, many emerging market economies, like China, do not allow the exchange rate to adjust fully. When there is an explicit or implicit exchange rate target, the impact of monetary policy on the currency market is at least partially sterilized. Sterilization will at least partially offset the expansionary effect of the monetary policy actions. Second, for macro-prudential purpose, many emerging economies, like China, have capital controls. When monetary policy changes relative returns between the home currency and foreign currencies, limitations on capital flows will restrict changes in exchange rate and the subsequent changes in output and price level (In addition, barriers to capital mobility create asset price differentials between the home country and international financial market[2]. As emerging market countries are more integrated in global financial market, there are increasing arbitrage transactions that try to avoid a country's capital controls (Montecino, 2018). It is particularly relevant for China due to the coexistence of offshore and onshore RMB markets. Any shock that occurs either in the offshore or

[1] There are a few papers on the dynamic relationship between the offshore and onshore RMB exchange rates (e.g. Cheung and Rime, 2014; Owyong *et al*., 2015). However, no study has explored the implications of that dynamic relationship on the transmission of monetary policy.

[2] One example is that the multiple listings of same stock in home countries and international financial market.

onshore currency market, can create arbitrage opportunities between these two markets. These arbitraging activities can create unexpected fluctuations in the RMB exchange rates, which in turn, may push output and the aggregate price level to a direction which contracts the targets of the central bank.

Based on the discussions above, we first identify a long run equilibrium relationship between the offshore and onshore RMB exchange rates, economic fundamentals and capital control measures in China. On the basis of the identified long run equilibrium in the currency market, we further explore China's monetary policy transmission in the context of this long run equilibrium relationship. We also study the impact of a disequilibrium shock in the currency market on inflation expectations. Then, we discuss whether it is feasible for the central bank to use its monetary policy actions to offset such an impact.

Our study contributes to two separate strands of literature. First, in the literature on purchasing power parity, or PPP (Taylor and Taylor, 2004; Hong and Phillips, 2010), the long-run relationship between exchange rate and the PPP fundamental are usually assumed to be linear. Even if nonlinearity is considered, it is usually modeled as nonlinear adjustments to a linear long-run relationship. However, as we have discussed above, a typical feature of emerging market economies is the existence of capital controls. For a country with capital controls, the impact of changes in the fundamentals on the exchange rate might vary with the degree of capital account openness. In this scenario, the long-run relationship itself will be nonlinear. Moreover, as argued by Hong and Phillips (2010), a linear approximation to the nonlinear cointegration relationship is not meaningful because there are no constant means of the non-stationary time series around which we can calculate the linear approximation. Using the newly developed nonlinear cointegration test of Vogelsang and Wagner (2016), we formally identify a nonlinear long-run equilibrium relationship between the CNY-CNH exchange rate differential, capital controls and economic fundamentals implied by the purchasing power parity (PPP). We demonstrate that omitting nonlinearity leads to misleading conclusions about China's monetary policy transmission. Since many emerging market economies have similar capital control measures to China, our results demonstrate the importance of capturing the nonlinearity in the long run equilibrium of their currency markets. This has obvious implications for the analysis of their monetary policy as well.

Second, we extend the empirical literature on monetary policy transmission in open economies and especially in emerging markets. A popular empirical method used in monetary policy analyses is the linear vector autoregression (VAR) model. In the open economy context, Eichenbaum and Evans (1995), Kim and Roubini (2000), Faust and Rogers (2003), Scholl and Uhlig (2008), Bjornland (2009), Kim and Lim (2016) use VAR to study the responses of exchange rate to monetary policy shocks. Those studies focus on short run dynamics and do not explicitly identify the long run relationship between the exchange rate and PPP fundamentals. However, as we have discussed above, disturbances to the long run equilibrium in the currency market might lead to exchange rate fluctuations which affect the central bank's target variable such as inflation. Because those papers do not identify the long run equilibrium in the currency market, they are silent on the impact of the disequilibrium shocks in the currency market. Chong *et al*. (2012) extend the local projection method of Jorda (2005) to a cointegrated system and show that impulse response analysis of shocks to the long run equilibrium can be calculated even without imposing any structural restrictions on the VAR system. We extend and apply the method of Chong *et al*. (2012) to calculate the impulse responses of inflation expectations to a disturbance to the long run relationship in the currency market. Chong *et al*. (2012) focus on a linear cointegrated system. As a result, their approach is more suitable for advanced economies where the relationship between the exchange rate and PPP fundamental are more likely to be linear. However, , due to capital controls, the long run equilibrium in the currency market might be nonlinear. For this reason, we extend the approach to allow for a nonlinear long run equilibrium relationship in the currency market. We believe that this extension is useful for monetary policy analyses of other emerging market economies which have similar capital controls to China.

Chong *et al*. (2012) use a reduced-form vector error correction model (VECM) and show that this model is adequate for identification of impulse responses to disturbances to the long run equilibrium in the currency market. However, it is not suitable for the calculation of impulse responses to monetary policy shocks. Previous literature suggests that identification of structural policy shocks is important for policy analysis. A reduced-form VECM or VAR model usually generates misleading policy implications (Kuttner, 2001; Cochrane and Piazzesi, 2002; Bernanke and Kuttner, 2005). We use a combination of survey data and financial markets data to identify exogenous policy shocks. As suggested by the literature (Kuttner, 2001; Cochrane and Piazzesi, 2002; Bernanke and Kuttner, 2005), using survey data and financial markets data allows us to identify structural policy shocks. Thus, our study has clear policy implications.

We find a nonlinear long-run relationship between the CNY-CNH exchange rate difference and the PPP fundamental. The impact of the economic fundamentals on the exchange rate changes with the degree of capital account openness. More specifically, an increase in expected inflation in China relative to the US should depreciate the RMB against the US dollar (USD). However, capital account in mainland China is not fully open. As a result, depreciation of the onshore RMB is less than the depreciation of the offshore RMB. In other words, the onshore CNY exchange rate increases less than the offshore CNH exchange rate. Therefore, there is a negative relationship between the CNY-CNH difference and the PPP fundamental. This negative correlation is weaker when the capital account is more open because the reaction of CNY to the economic fundamental is less different from the reaction of CNH.

Based on the identified long-run relationship between the RMB exchange rates, capital controls and inflation expectations, we are able to study the implications of the deviations from this long-run equilibrium relationship on inflation expectations. Using a modified version of the local projection methods of Chong *et al*. (2012), we calculate impulse responses of inflation expectations to a disturbance to the long-run relationship. We find that when the CNY exchange rate is too high compared to its long-run equilibrium level, inflation expectations rise. Therefore, disequilibrium in the currency market can affect the price stability target of the central bank. Despite this, we still find that expansionary monetary policy effectively raises inflation expectations. Moreover, we do not find a significant impact of monetary policy on the equilibrium relationship in the currency market. Therefore, disequilibrium on the currency market does not completely prevent the Chinese monetary policy to affect inflation expectations. However, impact of a typical surprise in the monetary policy on inflation expectations is economically small. When disequilibrium in the currency market causes undesired changes in inflation expectations, it is difficult to offset the impact of such a shock by countervailing monetary policy actions. Capital control measures or currency market interventions might be needed to restore the currency market equilibrium so that inflation expectations can be stabilized.

The rest of this article is set out as follows. Section 2 presents the institutional setting and policy measures of the PBC. Section 3 introduces our methodology and describes the data. Section 4 presents the empirical results. Section 5 concludes.

2. The offshore RMB market and policy measures in China

2.1 The offshore RMB financial market

The pace of RMB internationalization accelerated after the financial crisis of 2008. To facilitate the external use of RMB, China announced a pilot scheme to ease the restrictions on cross-border trade settlement using RMB in mid-2009. This has created a RMB pool outside mainland China, and helped the development of the RMB offshore financial market.

Offshore delivery scheme for offshore RMB-linked products was launched in July 2010. The People's Bank of China (PBC) and Hong Kong Monetary Authority signed a supplementary memorandum of transactions of RMB products in Hong Kong in July 19, 2010. Since then, Hong Kong has become the prime offshore RMB center. The offshore market experienced a rapid expansion. Despite its initial restriction to Hong Kong market, this scheme was quickly broadened to other offshore financial centers over the next five years[3]. The daily average turnover of RMB transactions increases from almost nothing to 202 billion between 2007 and April 2016. RMB is already the 8th most actively traded currency in the 2016 BIS survey (BIS, 2016).

Despite the rapid growth of offshore RMB transactions, the flow of RMB between mainland China and the offshore market is still subject to a number of restrictions. China's degree of capital account openness was relatively low according to the Chinn-Ito index (Chinn and Ito, 2018). China has created a number of schemes to allow for a manageable opening up of capital account in both capital inflows and outflows, such as Qualified Foreign Institutional Investor (QFII) and Renminbi Qualified Foreign Institutional Investor (RQFII). The fund movements under these scheme need to be permitted by the PBC (Funke et al., 2015). In addition, despite the continuous market oriented reform, the onshore Chinese foreign exchange market (CNY market) is still highly regulated. PBC only granted a limited market participants access to the wholesale market. By the end of the sample period, the movements of exchange rate are subject to a trading band of +-2% around the central parity rate, announced by the PBC. By contrast, RMB is able to flow freely across different offshore financial centers outside mainland China. More importantly, unlike RMB transactions in the onshore Chinese foreign exchange market (CNY market), which are in many ways influenced by the PBC, the offshore market (CNH) is a free market. The offshore RMB is freely floating and accessible for all offshore participants. These facts mean that there are two distinct markets for a single Chinese currency, the RMB.

Figure 1 presents the time series graphs of the logarithm of CNY, the logarithm of CNH,

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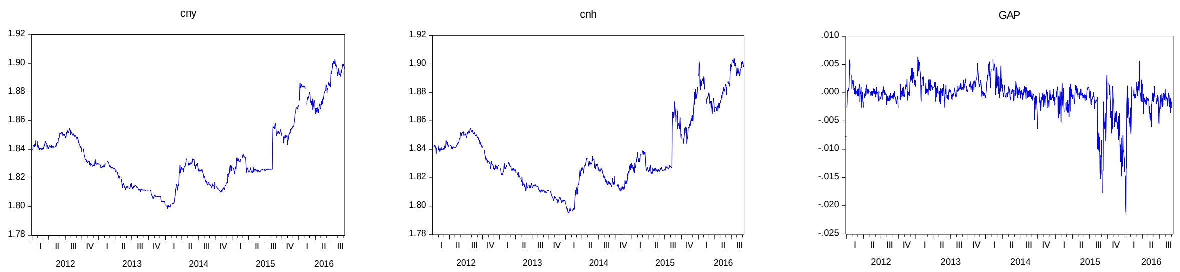
[3] In these years, the offshore use of RMB has spread to London, Singapore, Chinese Taipei, Frankfurt and other financial centers.

and the CNY-CNH difference since December 30 of 2011 when the Chinese government started to give quotas for investment in the mainland capital market using offshore RMB. The movements in CNH and CNY appear to have nonlinear trend components. In the first half of the sample, there is a downward trend in both CNY and CNH rates. With strong expectations of RMB appreciation, the CNY exchange rate was only allowed to vary in a narrow band on a daily basis. As a result, RMB appreciated gradually. Interestingly, even if there is no band imposed on the CNH market, we also observe a downward trend, which implies that arbitraging behaviors across markets closely link those two exchange rates. In the second half of the sample, both CNY and CNH experienced an upward trend. Although CNY and CNH rates have broadly similar trends in their movements, there are persistent deviations between CNY and CNH rates. Interestingly, we observe that the gap between CNY and CNH is positive on average when there is a downward trend, while it is negative on average when there is an upward trend. The CNY-CNH difference is on average 0.00072 (in logarithm) in the first half of the sample and -0.00206 in the second half of the sample. This results from the differences in capital controls and foreign exchange market interventions between the onshore and offshore market.   In the onshore market, capital flows are subject to quotas and the CNY is regulated to fluctuate in a narrow band. In the offshore market, capital movement is free and the CNH exchange rate can flexibly change. With those differences, CNH appreciates more than the CNY when there are appreciation expectations. In other words, the CNH exchange rate decreases more than the CNY, so the CNY-CNH difference tends to be positive. The CNH exchange rate also increases more than the CNY when there are depreciation expectations, leading to a negative CNY-CNH difference.

Figure 1 Time series of the exchange rates



Notes: CNY and CNH are log values. Gap is the difference between CNY and CNH.

2.2 Policy measures in China

The CNY exchange rate is determined by transactions in the China Foreign Exchange Trade System, which is effectively managed by the PBC[4]. At the start of each trading day, a reference CNY exchange rate (central parity rate) is announced and the daily fluctuations of the CNY exchange rate are restricted to a narrow band around this reference rate. To integrate the two markets better, the daily trading band of CNY was widened to ±1% relative to the reference rate in April 2012, and further widened to ±2% in March 2014. The CNY reference

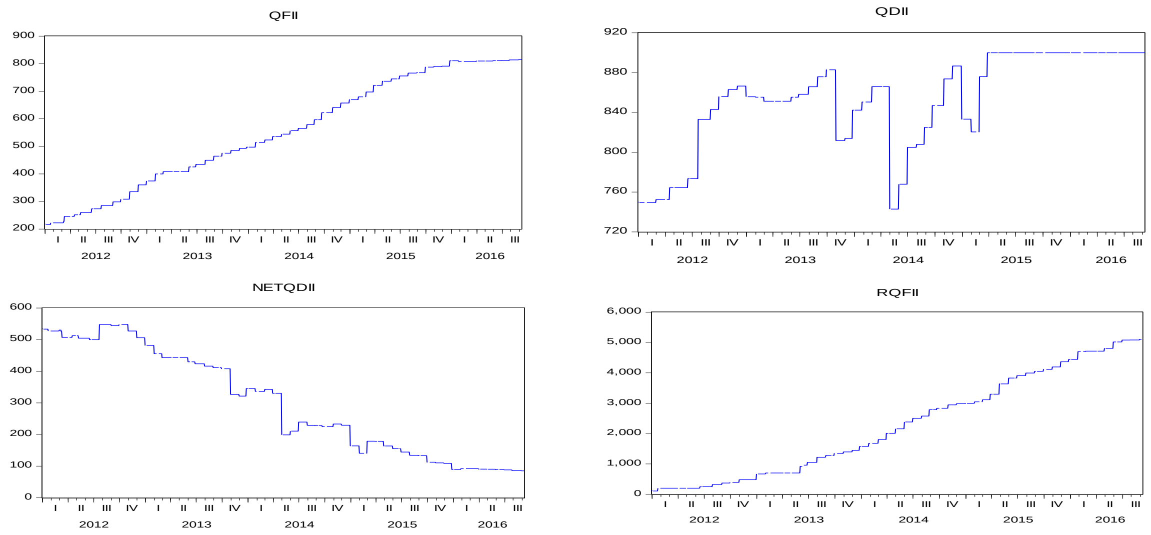
[4] According to the China Foreign Exchange Trade System (CFETS), the market participants are mainly composed of domestic banks, financial companies and subsidiaries of foreign banks.

exchange rate is a weighted average of major dealers' quotes. However, in practice, there was limited flexibility for the dealers to make the quote. On August 11 of 2015, the PBC announced a reform which gives more flexibility for the formation of the reference rate. But the daily change of CNY exchange rate still has to lie within the ±2% band.

The Chinese government launched a number of schemes to gradually open up the capital account in a controlled manner, which influences the availability and demand of RMB in the offshore market. Qualified Foreign institutional investor (QFII) scheme were launched in December 2002. Under this scheme, QFIIs are allowed to convert foreign currency to RMB and invest in a number of mainland RMB-denominated financial instruments. Qualified Domestic Institutional Investor (QDII), launched in 2006, allowed more domestic financial institutions to invest in offshore financial products. Since December, 2011, offshore RMB can be used to make investment in mainland China, through the RMB Qualified Foreign Institutional Investors (RQFII) program. This allows approved non-residents to participate in the onshore equity and bond market using offshore RMB, subject to an aggregate quota. This quota is expanded in 2013. Compared to QFII and QDII, RQFII investments do not need to convert between RMB and foreign currencies. However, the investment opportunities granted by the RQFII scheme can affect the incentive of offshore market participants to hold RMB. Therefore, changes in the RQFII quotas can also potentially affect the RMB exchange rates.

Figure 2 shows the time series graphs of QFII, QDII, and RQFII. We also report the difference between the QDII and the QFII (NETQDII). This difference reflects the net capital outflow allowed when currency conversions between RMB and foreign currencies are needed. The net capital outflow allowed through the QDII (net of QFII) window has a nonlinear trend. There was a declining trend until the end of 2015 when the level of capital market openness through this window stabilizes. There are still small changes after 2015, but daily changes compared to before are tiny. The initial declining trend in NETQDII was driven by the increasing trend in the QFII quota. Compared to QFII, the QDII quota was fluctuating in a narrow range before 2015 and there was no obvious trend in it. This reflects the more prudent attitude of the State Administration of Foreign Exchange (SAFE) towards capital outflows. Since March 26 of 2015, the QDII quota has remained constant. The changes in the QFII quota also have become quite small since 2016. The smaller changes in the quotas suggest that the Chinese government has become more conservative towards capital movements, which is related to the stock market turmoil in 2015 and the rising concern about financial stability. Moreover, there was large depreciation pressure on CNH in January 2016. The slowdown of capital account liberalization was also a reaction to the currency market movements. For RQFII, the growth of the Quota accelerated since the second quarter of 2013. The RQFII has an approved quota of 511.34 billion RMB by the end of Sep 2016 with 169 institutional participants.

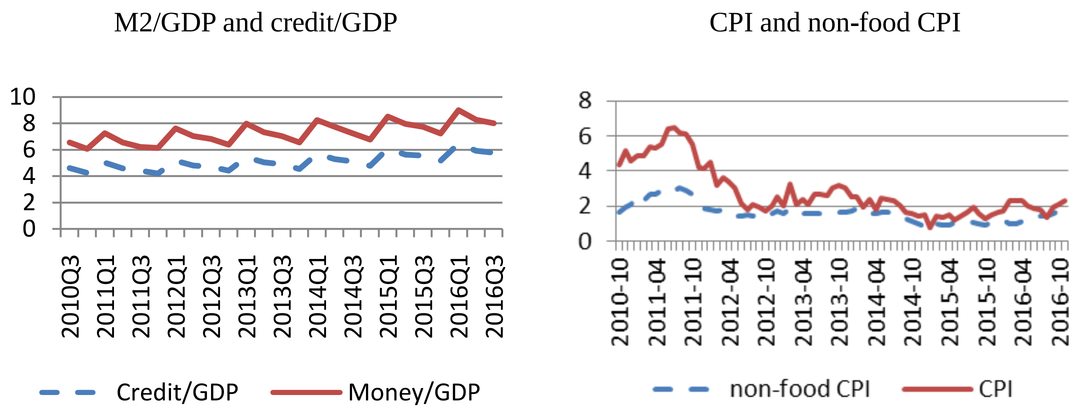
Figure 2 Time series of the capital control quotas



Notes: The units of QFII, QDII, and NETQDII are 100 million US dollars. The unit of RQFII is 100 million RMB.

These exchange rate and capital account policies might have influenced domestic economic objectives as well. Figure 3 depicts the time series of China's consumer price index (CPI) inflation, money supply (M2)-to-GDP ratio and credit-to-GDP ratio since the third quarter of 2010. Despite the obvious seasonality, there was clearly an upward trend in both the money supply-to-GDP and credit-to-GDP ratio. The expansion of money and credit supply relative to the economic size, however, does not bring inflation. The CPI inflation rate peaked in July 2011, and then declined from 6.5% to less than 3%. As is well-known, China's CPI inflation rate is dominated by food prices. The non-food CPI inflation rate declined to less than 2%. The producer price index (PPI) inflation was negative between March 2010 and August 2016, which implies that there was not enough effective demand for manufacturing goods. Why monetary and credit expansion seemed not very effective in containing deflation? We shall explore whether disequilibrium in the currency market was one of the reasons.

Figure 3 Time series of China's CPI, money and credit growth



Notes: data are from the Chinese Bureau of Statistics. Units for the price indices are percentages.

3. Data and Methodology

3.1 Data

Our sample use daily data in the period from December 30, 2011 to March 31, 2016 for which daily data are available. We use daily data for two reasons. First, the CNH exchange rate data starts from August 26, 2010. The RQFII quota data starts from December 30, 2011. Using monthly or quarterly sample generates small sample biases. Second, with daily data, we can precisely identify surprise macroeconomic news to market participants using the announcement date of the data. We shall explain this in more details below. Actually, using monthly data might even omit important policy impacts. For example, suppose a structure policy shock has significant positive impacts on the change rate of the exchange rate on a few days in a month while the impacts are not significant on the other days. A monthly-data analysis might tell that the policy has no impact on the exchange rate. However, those positive responses of the exchange rate returns actually raise the level of the exchange rate. Without following declines in the exchange rate returns, the level of the exchange rate will be persistently higher in that month. Very often, it is the level of the exchange rate that concerns the government. For example, the Chinese State Administration of Foreign Exchange pays attention to exchange rate movements because there is a concern that a depreciated currency leads to capital flight. In this case, it is the expectation on a prolonged period of higher exchange rate which matters.

We collect daily data of CNY, CNH, QFII, QDII, and RQFII quotas from the Wind database. To construct our policy surprise variables (explained in more details below), we also collect survey data on macroeconomic forecasts in China from the Wind database.

3.2. Modeling the long-run equilibrium relationship

We assume that the exchange rate between two currencies is determined by the purchasing power parity in the long run (for a discussion on this, see e.g. Taylor and Taylor, 2004). More specifically, if the CNY and CNH markets are fully integrated, we have

$$CNY_t = CNH_t = p_t^{CN} - p_t^{US}, \tag{1}$$

where CNY_t is the logarithm of the CNY exchange rate against dollar at time *t*, CNH_t is the logarithm of the CNH exchange rate, p_t^{CN} and p_t^{US} are respectively the log price level in China and the US.

However, due to capital controls and currency market interventions in mainland China, the CNY and CNH market are not fully integrated. More specifically, capital inflows to the mainland Chinese capital market are limited by the quotas of QFII and RQFII; capital outflows from the mainland Chinese capital market are limited by the quota of QDII. By contrast, capital market in Hong Kong is open to non-residents. Besides the QFII, QDII and

RQFII, currency exchange under current account and foreign direct investment (FDI) account are also more restricted in the CNY market (Funke *et al*., 2015). As we have discussed, there are also more interventions in the CNY market. As a result, market reactions to fundamental economic news are more constraint in the CNY market compared to the CNH market. This creates a gap between the CNY and CNH exchange rate. For brevity, let us call $p_t^{CN} - p_t^{US}$ the PPP fundamental. Suppose the PPP fundamental increases, both CNY and CNH exchange rate should increase. However, due to capital control, CNY will increase less than the CNH. Therefore, the CNY–CNH difference decreases. However, this decrease in the difference will be smaller, if there is less constraint on capital movement in the CNY market. Therefore, we have the following nonlinear long-run relationship:

$$\begin{aligned} CNY_t - CNH_t &= a_0 + a_1(p_t^{CN} - p_t^{US}) + a_2 NETQDII_t + a_3 RQFII_t \\ &+ a_4 NETQDII_t * (p_t^{CN} - p_t^{US}) + a_5 RQFII_t * (p_t^{CN} - p_t^{US}) + e_t, \end{aligned} \tag{2}$$

where $NETQDII_t$ is the difference between QDII and QFII quota, which measures the net capital flow allowed when the foreign investors have to convert between RMB and USD, $RQFII_t$ is the RQFII quota, which measures the net capital inflow allowed when the investment currency is in RMB. Because the necessity for currency conversion could matter for the impact of capital flows, we treat RQFII differently from QFII. There are also capital control measures under the current account and FDI account. Compared to the NETQDII and RQFII, changes in those measures are less frequent. Their impact will be absorbed in the vector of deterministic terms a_0. Because the PPP fundamental can still affect the CNY exchange rate even if capital flows through the capital market is not allowed, a_1 is not necessarily zero. Finally, e_t is the cointegration error.

Ideally, we would control for QDII and QFII separately as well. However, QDII, QFII and RQFII quotas are usually jointly determined by the SAFE's preferences over capital account openness, exchange rate stability and other macroeconomic concerns. Therefore, those measures are highly collinear. Controlling the three capital control measures separately on the right-hand side (RHS) thus causes identification problems. Actually, we encounter a matrix singularity problem when all three measures are put on the RHS. To circumvent this problem, we use QDII and QFII to construct the net capital outflow quota, NETQDII. We also regress RQFII on NETQDII and use the residual as the orthogonalized proxy of RQFII. Doing this better identifies the coefficient of NETQDII and RQFII. Our results from the impulse response analysis are robust if we do not use regression to orthogonalize RQFII.[5]

In practice, inflation data are only reported at a monthly frequency. However, market participants update their inflation expectations more frequently because they often have to trade more frequently than monthly. Hence, we use market-implied inflation expectations in our daily-data model. For the US, we use the treasury inflation protected securities

[5] Additional results are available upon request.

(TIPS)-implied 5-year inflation expectations which are directly available from the FRED database. As there is no TIPS market in China, we estimate the market inflation expectations using the term structure model of Rudebusch and Wu (2008). More specifically, we use their yields-only model for the obvious reason that the macroeconomic information used in their macro-finance model is not available at the daily frequency. Yao and Tan (2011) show that inflation expectations derived from this term structure model match survey-based inflation expectations data in China quite well at the monthly frequency. The Chinese term structure data are obtained from the China Central Depository & Clearing Co., Ltd.

Substituting $p_t^{CN} - p_t^{US}$ by the difference between the estimated inflation expectations of China and the US, $INFDIFF_t$, we obtain the following model:[6]

$$\begin{aligned} CNY_t &= CNH_t + a_0 + a_1 INFDIFF_t + a_2 NETQDII_t + a_3 RQFII_t \\ &+ a_4 NETQDII_t * INFDIFF_t + a_5 RQFII_t * INFDIFF_t + e_t. \end{aligned} \tag{3}$$

Model in equation (3) can also be justified from economic theory. In sticky-price models (Dornbusch, 1976; Frankel, 1979) of exchange rate determination, expected relative inflation rate affects the current-period exchange rate.

Note that, we restrict the coefficient of CNH in the equation (3) to 1. This identification has two advantages. First, the estimated model has clearer economic interpretation. It illustrates the impact of a change in the PPP fundamental or capital control measures on the CNY-CNH exchange rate difference. Second, from a theoretical perspective, the right hand side variables determine both CNY and CNH. Without the restriction we impose, the model suffers serious multicollinearity problems, because CNH is collinear with all the determinants of RMB exchange rates! Therefore, the identification of the coefficients is problematic. Even if we can still produce estimated coefficients, the economic interpretation of those coefficients is not clear.

As shown by Hong and Phillips (2010) and Vogelsang and Wagner (2016), the existence of nonlinear terms in the cointegration relationship is difficult to test because the potential endogeneity of the regressors in the cointegrating equation and error serial correlation requires biases corrections to the standard test statistics to allow for asymptotic chi-squared inference. Vogelsang and Wagner (2016) propose a Ramsey test which has an asymptotic chi-squared distribution on the basis of their Integrated Modified OLS (IM-OLS) estimator (Vogelsang and Wagner, 2014). Therefore, a Wald-type test can be applied. More specifically, consider the cointegrating regression as follows:

$$\begin{aligned} y_t &= X_t'\beta + u_t, \\ X_t &= X_{t-1} + v_t \end{aligned} \tag{4}$$

where the error terms u_t and v_t fulfill a functional central limit theorem, and are potentially correlated with each other. Obviously, when u_t and v_t are correlated, the regressors are

[6] Due to slow price adjustment, the relative price levels could be replaced by relative inflation rates for theoretical reasons as well (see Rossi (2013)).

endogenous.

Vogelsang and Wagner (2016) show that the OLS estimator of the following equation is consistent and has a zero mean Gaussian mixture limiting distribution.

$$SY_t = SX_t'\beta + SM_t'\gamma + X_t'\alpha + w_t, \tag{5}$$

where $SY_t = \sum_{j=1}^{t} y_j$, $SX_t = \sum_{j=1}^{t} X_j$, SM_t is similarly defined as a partial sum of the cross products of elements in X_t, w_t is the error term.

After estimating equation (5), chi-squared tests can be applied to test the significance of β and γ.[7] Obviously, when the null hypothesis of $\gamma = 0$ is rejected, we can conclude that the long-run equilibrium relationship is nonlinear. We shall apply this IM-OLS test to our model (3) in the empirical analysis.

3.3. Impulse response analysis

Chong *et al.* (2012) suggest that conditional on that a cointegration relationship is found in the data, we can calculate the impulse responses of the economic system by the following two local projections.

$$e_{t+h} = A_1^h e_t + \Phi_1^h \Delta Y_t + \dots + \Phi_p^h \Delta Y_{t-p+1} + \eta_{t+h} \tag{6}$$

$$\Delta Y_{t+h} = B_1^h e_t + \Psi_1^h \Delta Y_t + \dots + \Psi_p^h \Delta Y_{t-p+1} + \varepsilon_{t+h},$$

where h=1,…,H is the forecast horizon. More specifically, the first equation in (6) describes linear projections of the h-step-ahead equilibrium errors on the current equilibrium error and the current and past values of the endogenous variables. Similarly, the second equation in (6) describes linear projections of the h-step-ahead values of the endogenous variables on the same set of variables. η_{t+h} and ε_{t+h} are error terms. Jorda (2005) introduces the local projection method as an alternative to calculate the impulse response functions using VAR. This method consistently estimates the impulse responses of a system of stationary variables. Its advantage over traditional VAR is that it does not require a specific model specification, so avoids potential specification errors. The two local projections in (6) are extensions of the Jorda (2005) approach to non-stationary dynamic systems. One nice property of the local projection system in (6) is that the h-step impulse responses of endogenous variables to a disturbance to the long-run equilibrium relationship can be simply calculated as $B_1^h + \Psi_1^h\beta$, where β is the vector of cointegration coefficients. The impulse responses to the shocks in ε_{t+1} are $B_1^h\beta' + \Psi_1^h$. To control for the nonlinearity, the interaction term

[7] See the appendix for technical details of the test.

$NETQDII_t * INFDIFF$ is added to the right hand side of the local projection systems in (6).

In a typical exchange rate model as the one studied by Chong *et al.* (2012), the interest rate difference between two countries is added to ΔY_t to capture the uncovered interest rate parity (UIP) effect. In this context, impulse response functions (IRFs) to the interest rates may be interpreted as the impact of the interest rate policy. However, the error terms in the reduced-form system (6) are not structural. In other words, they might be a combination of deeper structural economic shocks. Therefore, the economic interpretation of the IRFs is difficult. Moreover, Cochrane and Piazzesi (2002) point out current changes in the interest rates may have been anticipated by the market participants. In this case, the current exchange rates already contain information on those anticipated interest rate changes and we will not observe any responses of the exchange rates to the interest rate changes. To overcome those difficulties, we substitute the usual interest rates in the exchange rate models by two surprise monetary policy measures.

The US monetary policy shock is estimated as the difference between the announced federal funds rate changes and the anticipated changes implied by the futures market for federal funds. Detailed explanation of the construction of this variable is given by Kuttner (2001) and Bernanke and Kuttner (2005).

Unfortunately, there is no futures market for the Chinese interbank funds. Instead, we use survey data to construct our surprise monetary policy indicator of China. As is well-known (He *et al*., 2013; Cheung *et al*., 2016; He *et al*., 2016), interest rate policy in China was not as frequently used as quantity-based policies. Money supply and credit supply are closely monitored and regulated by the central bank. The Wind database surveys the major financial institutions in China on key macroeconomic variables including M2 growth and the flow of credit supply on a monthly basis. Hence, we can construct the money supply shock and credit supply shock by the difference between the realized data and the median survey of the forecasts. Since the People's Bank of China has better control over credit supply than the money aggregate, we use the credit supply shock in our benchmark model. However, our results are robust if we use the M2 shock as the monetary policy indicator.

In addition to the two monetary policy variables, we also control for two surprise indicators of real activities. Scotti (2016) demonstrates that an aggregate index of surprise news on real activities significantly affects asset prices. We use his US surprise index, *Ussurp*, as a control variable in our model. There is no aggregate index of surprise news on real activities in China. We use the difference between realized industry production and survey median forecast of industry production, *Industry_cn*, instead. Data on *Ussurp* is from Scotti (2016). Survey median forecast of China's industry production is from the Wind database.

In summary, our extended system of local projections is as follow:

$$e_{t+h} = A_1^h e_t + \Phi_1^h \Delta Y_t^{ex} + \ldots + \Phi_p^h \Delta Y_{t-p+1}^{ex} + \eta_{t+h}, \qquad (7)$$

$$\Delta Y_{t+h} = B_1^h e_t + \Psi_1^h \Delta Y_t^{ex} + \ldots + \Psi_p^h \Delta Y_{t-p+1}^{ex} + \varepsilon_{t+h}$$

where ΔY_{t+h} is a vector of the following variables, ΔCNY_{t+h}, ΔCNH_{t+h}, $\Delta INFDIFF_{t+h}$,

$\Delta NETQDII_{t+h}$, $\Delta RQFII_{t+h}$; ΔY_t^{ex} adds $\Delta INFDIFF_t * NETQDII_t$, *Credit_cn*, *Interest_us*, *Ussurp*, and *Industry_cn* to ΔY_t. Here, we use *Credit_cn* and *Interest_us* to denote the surprise credit supply in China and surprise federal funds rate (FFR) change in the US, respectively. Note that, because the surprise variables are by construction exogenous, we do not need to put them on the left hand side of the equations[8].

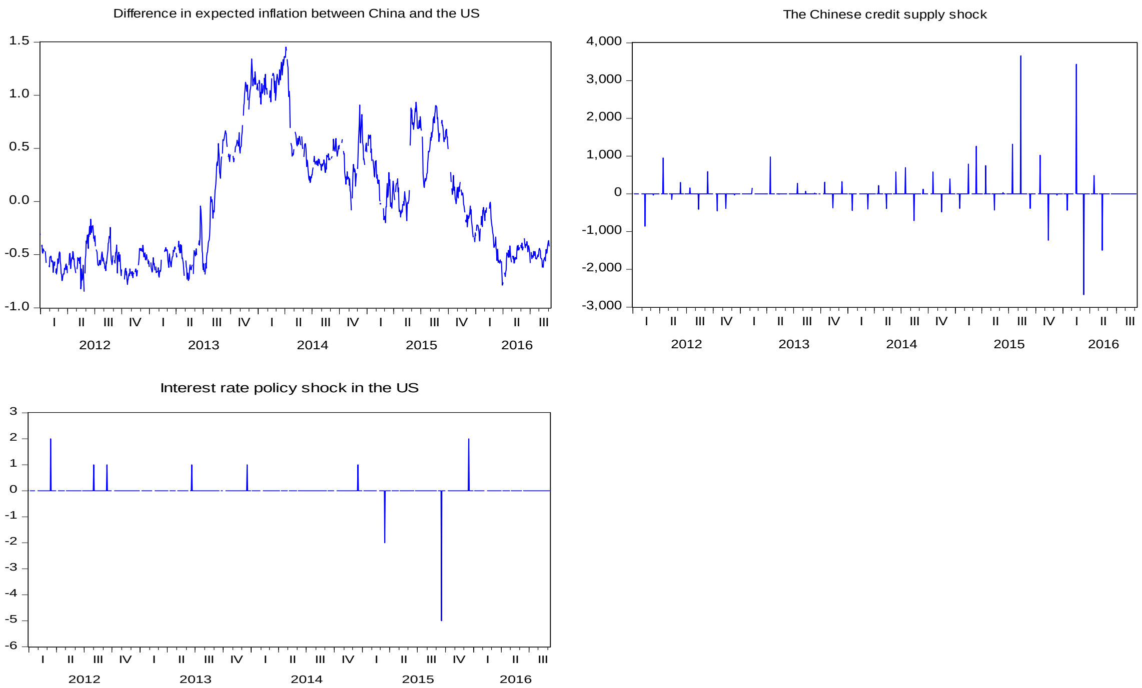
4. Empirical findings

4.1 The expected inflation difference and surprise policy variables

The upper-left panel of Figure 4 plots the time series of $INFDIFF_t$. The expected inflation difference between China and the US in the first half of the sample was, on average, negative (-0.09 percent). This implies that the exchange rate should decrease. However, the average expected inflation difference turned positive (0.10 percent) in the second half of the sample. The relatively high inflation expectation in China should leads to a higher exchange rate. However, due to the exchange rate and capital account regulations, the adjustment is a gradual process. Therefore, we observe downward trends in in the CNY and CNH exchange rates in the first half of the sample and upward trends in the second half of the sample. This reasoning is consistent with the time series plots of the CNY and CNH exchange rates in Figure 2.

Figure 4 also shows the time series plots of the Chinese credit supply shock and US interest rate policy shock.

Figure 4 Expected inflation difference and policy shocks



[8] We control the interest rate of the US. Someone may be concerned that the interest rate gap between China and US is more relevant. To check the robustness, we use the interest rate gap to re-estimate our regressions. It turns about that our primarily results are qualitatively unchanged.

Notes: The unit of INFDIFF is percentage. The policy shocks data are based on authors' calculations. The unit of the Chinese credit supply shock is 100 million RMB. The unit of the US interest rate policy shock is basis point.

4.2. Long run equilibrium relationship

Before cointegration test, it is necessary to test whether the variables in model (3) are truly I(1) variables. Table 1 summarizes the unit root test results. Clearly, the unit root hypothesis is not rejected for all the level variables in model (3). On the other hand, the first differences are shown to be I(0) variables. Therefore, it is suitable to perform a cointegration test for model (3). We remove quadratic deterministic trends from CNY, CNH, NETQDII and RQFII before our cointegration analysis.

Table 1 Unit root test results

	CNY	CNH	NETQFII	RQFII	INFDIFF
Level	-0.7669	-1.5545	-2.6308	-2.8331	-1.4497
First difference	-29.23***	-30.61***	-24.14***	-31.19***	-30.46***
Trend in test	Yes	Yes	Yes	Yes	No

Notes: The test is the augmented Dicky-Fuller test. The null hypothesis assumes a unit root. The lag length of the test is selected by the Schwarz information criterion. We denote statistical significance at the 1%, 5%, 10% percent by ***, **, *, respectively. The row "Level" corresponds to the test t-statistics for the level of the variables. The row "First difference" corresponds to the test t-statistics for the first difference of the variables. The row "Trend in test" tells whether the test includes a deterministic trend. All tests include an intercept.

Using the nonlinear cointegration technique of Vogelsang and Wagner (2016), we identify the following long run relationship:[9]

$$CNY_t = CNH_t - \underbrace{0.0017}_{(p=\ 0.0104)} INFDIFF_t - \underbrace{0.0588}_{(p=0.1054)} NETQDII_t - \underbrace{0.0963}_{(p=0.8270)} RQFII_t + \underbrace{0.3392}_{(p=0.0030)} NETQDII_t * INFDIFF_t. \tag{8}$$

The *p*-values in the parentheses under the estimated coefficients are the ones for the tests of the zero-coefficient null hypothesis. The interaction term between *RQFII* and *INFDIFF* is not significant if it is included. Moreover, *RQFII*INFDIFF* is not only correlated to the level variables, *RQFII* and *INFDIFF*, but also correlated to the other interaction term. Therefore, the addition of this variable also makes the multi-collinearity problem more serious. As a result, all individual coefficients appear to be insignificant when two interaction terms are added. Therefore, we report the estimated model without *RQFII*INFDIFF* here. As expected, an increase in the expected inflation difference raises the RMB exchange rates, but to a less

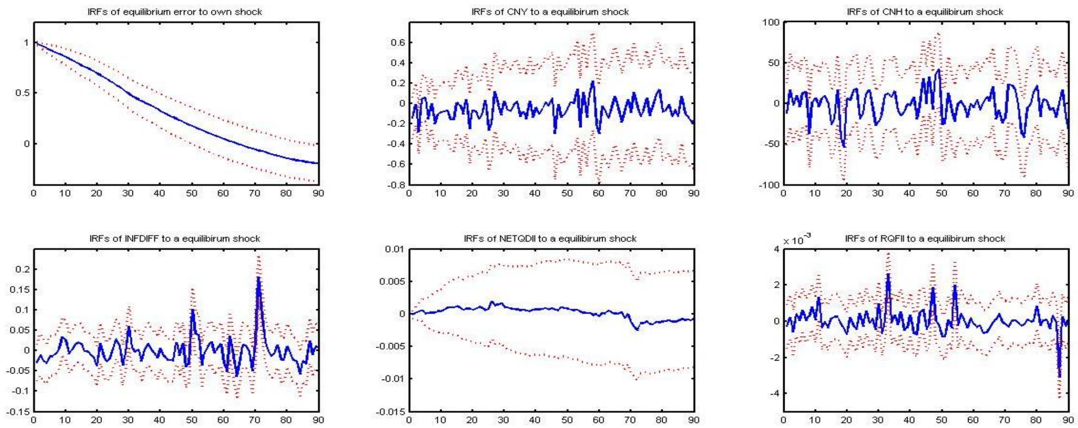
[9] Note that, to make the coefficients of the quotas visible within four digits, we have changed the units of the NETQDII and RQFII quotas to 10000 US dollar and 10000 RMB, respectively.

extent on *CNY* because of the capital control, so *INFDIFF* has a negative sign. The negative impact of *INFDIFF* is smaller if more capital outflow is allowed for. This is because for a given level of expected inflation increase, the CNY exchange rate can increase more. One concern is that the traditional cointegration test may not work for the case with interaction terms of two unit roots, as they are not unit roots anymore. To provide further evidence on the nonlinear relationship, we implement an additional test on the stationary of the residuals in Equation (8). The unreported unit root test for the residuals rejects the unit root hypothesis[10].

4.2. The impact of a disturbance to the long-run equilibrium relationship

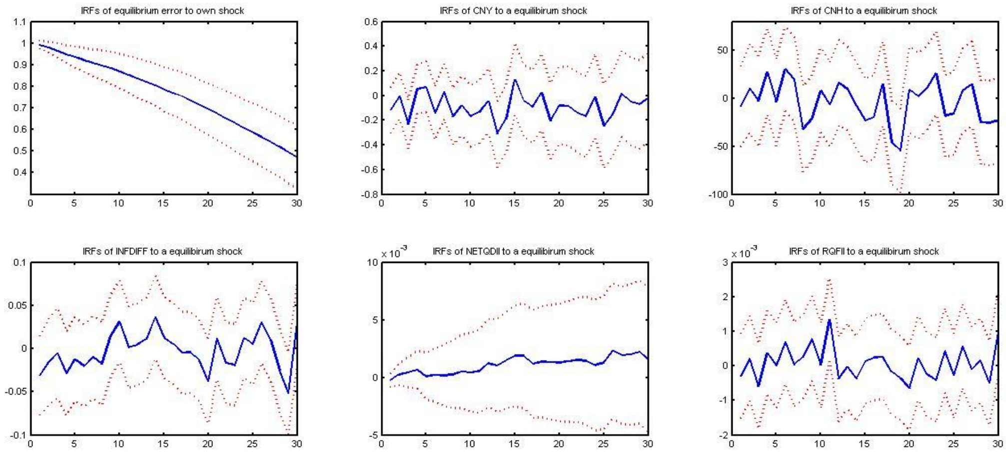
Figure 5 and Figure 6 present impulse response functions of the error correction terms and first differences of *CNY, CNH, INFDIFF, NETQDII* and *RQFII* to a one-unit shock to the equilibrium relationship. Figure 5 present the IRFs up to 90 days. To facilitate the interpretation of the more immediate responses, we separately report the IRFs up to 30 days in Figure 6.

Figure 5 Impulse responses to the disequilibrium error (90 days)



[10] This result is available upon request.

Figure 6 Impulse responses to the disequilibrium error (30 days)



The first observation from those Figures is that a temporary shock to the long-run relationship has a very persistent impact. The IRF of the error correction term does not converge within 90 days, consistent with the literature (Taylor and Taylor, 2004; Chong *et al*., 2012).

Second, disequilibrium on the currency market seems to have little impact on the inflation expectations in the first month following the disequilibrium shock. The impulse responses are not significantly different from zero on most days within that month. However, the shock significantly raises inflation expectations within a quarter. More specifically, our indicator of expected inflation difference rises by 0.1 percent on day 50 and 0.18 percent on day 71 after a 1 percent over-depreciation of the *CNY*. Note that there are no subsequent significant drops in the *first difference* of *INFDIFF*. Therefore, the *level* of inflation expectations in China has been persistently higher than the before-shock periods since those two days. Those movements in the expected inflation are helpful for the restoration of the long-run equilibrium. The disequilibrium shock makes the *CNY* exchange rate too high relative to the level implied by fundamentals. Rises in expected inflation close the gap.

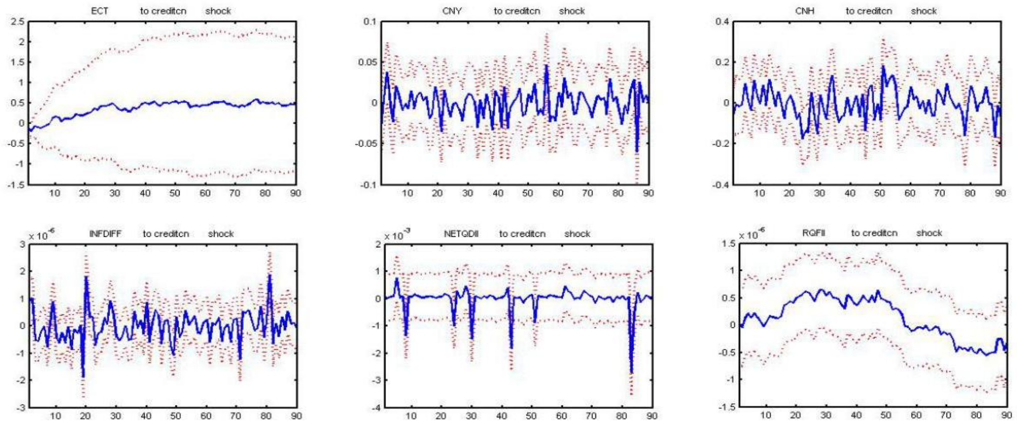
By contrast, there are no notable significant responses of the exchange rates to the disequilibrium shock. Note that the IRFs of *CNH* have big magnitudes, but the intervals are also very wide and almost always cover zero except on day 18 and 19. On those two days, the upper bounds of the 95% confidence interval are negative, which implies that *CNH* appreciates on those two days. These responses are in the "wrong" direction because the shock makes the *CNY* exchange rate too high relative to the *CNH* and we need the *CNH* exchange rate to rise as well to restore equilibrium. Therefore, the equilibrium is obviously not restored by the movements in the *CNH*.

The IRFs of *CNY* are also mostly insignificant except on day 3 when the upper bound of the 95% confidence interval is negative. This implies that *CNY* appreciates on that day. Because the equilibrium error over-depreciates the *CNY* relative to the level implied by the long-run equilibrium relationship, a *CNY* appreciation corrects the error. However, the magnitude of the response is small (in the range from -0.4947% to -0.0690%).

4.3. The impact of a surprise credit expansion in China

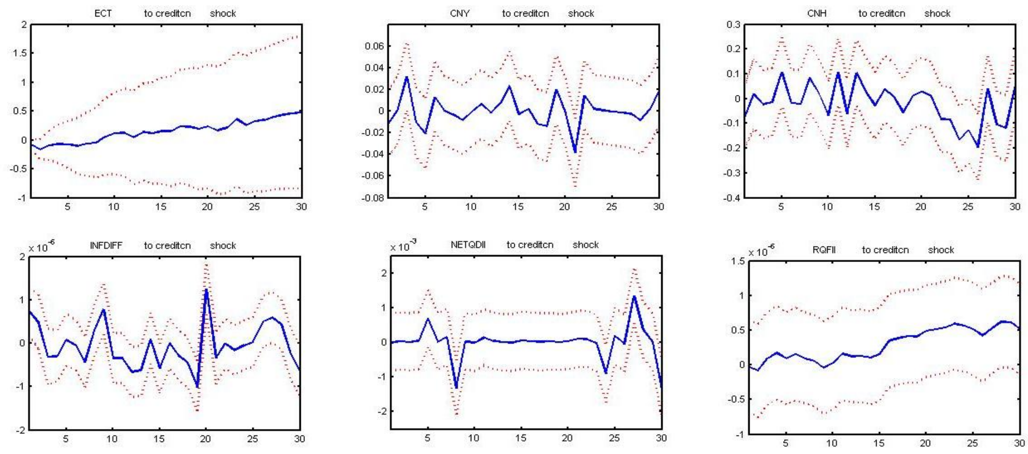
Figure 7 and Figure 8 summarize the IRFs of equilibrium error and endogenous variables to a one-unit increase in the credit supply in China up to 90 and 30 days, respectively.

Figure 7 Impulse responses to a one-unit shock of credit supply in China (90 days)



Notes: ECT denotes the disequilibrium error; CNY, CNH, INFDIFF, NETQDII and RQFII are first differences. Values on the horizontal axis are the numbers of days after the shock. The variables on the vertical axis are the responses. The dashed lines are the 95% intervals.

Figure 8 Impulse responses to a one-unit shock of credit supply in China (30 days)

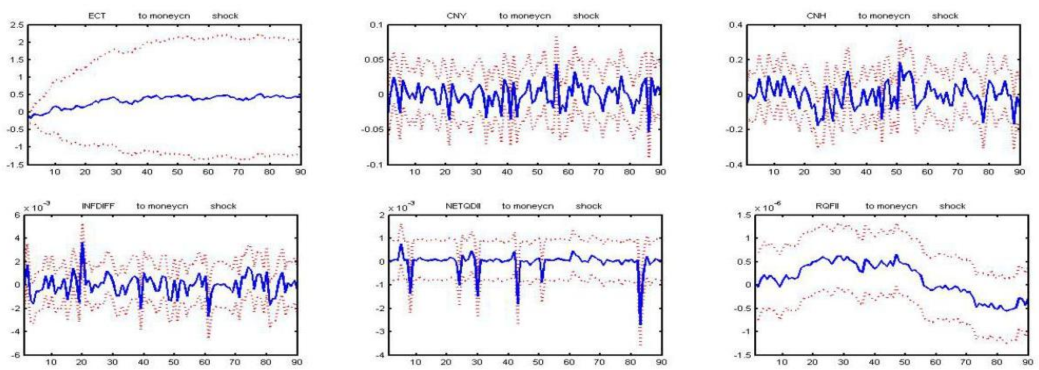


Obviously, the expansionary credit shock has no significant impact on the equilibrium error. Hence, there is no evidence that credit policies in China contribute to currency market disequilibrium. Moreover, the credit shock effectively raises inflation expectations. The impact responses are largest on day 20 and 81. On day 20 and 81 after a 1-trillion RMB increase in the credit supply, inflation expectations in China (relative to the US) are raised by 0.0182 percent and 0.0189 percent, respectively. The magnitudes of the impacts are small but persistent. Note that the IRFs are in first differences. Without significant negative IRFs after those two days, the level of inflation expectations is persistently higher. Those findings suggest that disequilibrium in the currency market does not have a significant impact on the transmission of credit policy shocks to inflation expectations. However, these results do not mean that disequilibrium in the currency market does not affect the central bank's ability to maintain price stability. As we have found in the last subsection, a one-percentage over-depreciation of *CNY* can create daily changes in inflation expectations to a scale about 10 times bigger than a one-trillion RMB credit surprise. Although monthly credit increase in China can be larger than one-trillion, creating a surprise credit supply is far more difficult because a large part of the credit supply could be anticipated by the market. The largest Chinese credit supply shock in our sample is 365.68 billion RMB (Figure 4). This means that a moderate shock to the equilibrium of the currency market can easily offset the impact of a large-scale credit policy on the inflation expectations. This finding helps us understand the phenomenon shown in Figure 3. Despite the persistent increase in money and credit supply, China's inflation rate remains low. Currency market disequilibrium might have disturbed the expansionary monetary and credit policy from working.

4.4. Robustness of the impulse responses functions

So far, we have used credit supply as the monetary policy variable in China. In this subsection, we show that our qualitative results on monetary policy transmission are unchanged if shocks to aggregate money supply growth rate are used. Figures 9 reports the IRFs of the equilibrium error and endogenous variables to a one-unit increase in the M2 growth rate in China up to 90 days (to save space, we do not separately report the IRFs of the first 30 days for the robustness tests). A surprise increase in the money growth rate leads to a significant increase in inflation expectations on day 20. By contrast, no significant IRFs of the exchange rates are found. Therefore, monetary policy is effective in affecting inflation expectations and does not bring significant distortions to the currency market. However, as what we find before, the magnitude of the respones of the inflation expectations are small. The largest response of inflaton expectations to a one-percent money growth shock is only 0.006 percent.

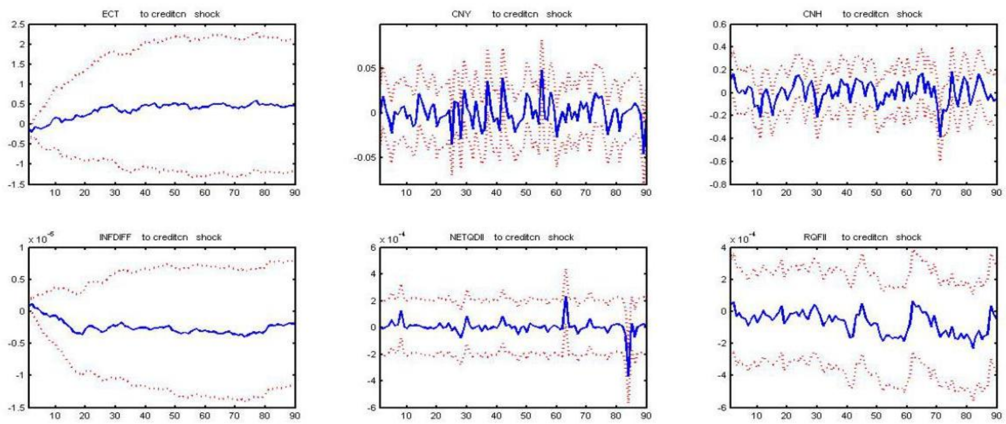
Figure 9 Impulse responses to a one-unit shock of money supply in China (90 days)



4.5. The importance of the nonlinearity

Most empirical studies on the long-run relationship between the exchange rate and PPP fundamental assume a linear relationship. In this subsection, we demonstrate that in a country like China, where capital control exists, the potential nonlinear relationship might affect the qualitative results of the subsequent policy analysis and lead to faulty inferences and policy decisions. Figure 10 reports the IRFs of the equilibrium error and endogenous variables to a one-unit increase in the credit supply in China up to 90 days in a linear model. Those IRFs suggest that the credit supply shock has no impact on the inflation expectations in China which we have seen to be false.

Figure 10 Impulse responses to a one-unit shock of credit supply in China (linear model)



Notes: ECT denotes the disequilibrium error; CNY, CNH, INFDIFF, NETQDII and RQFII are first differences. Values on the horizontal axis are the numbers of days after the shock. The variables on the vertical axis are the responses. The dashed lines are the 95% intervals.

5. Conclusion

With rapid development of RMB as an international currency, offshore RMB markets were created for financial transactions among non-residents since 2010. Over the past several years, the CNH market experienced a rapid development, but the persistent difference between CNY and CNH rates has led to massive speculations and complicates the aggregate environment in which the PBC's policy instruments work.

In this paper, we find a nonlinear long-run relationship between the onshore and offshore RMB exchange rates as well as the expected inflation. The nonlinearity is caused by China's capital control policies and currency market regulations in the mainland. We demonstrate that failure to capture the nonlinearity generates misleading conclusions on the transmission of monetary policy shocks to inflation expectations.

Based on the identified long-run relationship, we calculate the impulse responses of inflation expectations to a disturbance to the long-run relationship. It shows that disequilibrium in the currency market can affect the price stability target of the central bank. More specifically, although monetary policy shocks in China can still effectively change inflation expectations, the magnitudes of the effects are small. Discretionary monetary policy might fail to fight deflation and recession when the currency market is in disequilibrium. This is because the impact of a moderate-size equilibrium error in the currency market on inflation expectations is much larger than the impact of a typical surprise credit supply or money shock. Therefore, measures have to be taken to maintain currency market equilibrium if the central bank want its policy instrument to effectively manage inflation expectations.

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Appendix. Technical notes for the test for nonlinear cointegration relationship

In this appendix, we briefly introduce the technical details of the test for nonlinear cointegration relationship of Vogelsang and Wagner (2016). Define the vector of regression coefficients in equation (5) as θ and denotes its OLS estimator by $\widetilde{\theta}$. Proposition 1 of Vogelsang and Wagner (2016) shows that as the sample size goes to infinity, the limiting distribution of $\widetilde{\theta} - \theta$ is normal distribution with mean zero and variance matrix V_{IM}. The variance matrix can be estimated by $\widehat{\omega}_{uv} A_{IM}^{-1} (\widetilde{SX}' \widetilde{SX})^{-1} C' C (\widetilde{SX}' \widetilde{SX})^{-1} A_{IM}^{-1}$, where $\widetilde{SX}$ is the vector of regressors in equation (5), A_{IM} is a scaling matrix, $C = [c_1, \ldots, c_T]', c_t = S\widetilde{SX}_T - S\widetilde{SX}_{t-1}, S\widetilde{SX}_t = \sum_{j=1}^{t} \widetilde{SX}_j . u_t$, $v_t$in equation (4) follows a functional central limit theorem $T^{-1/2} \sum_{t=1}^{[rT]} \eta_t \Rightarrow B(r) = \Omega^{1/2} W(r)$, where $\eta_t = [u_t, v_t']'$, r lies in [0,1]. $\omega_{uv} = \Omega_{uu} - \Omega_{uv} \Omega_{vv} \Omega_{vu}$.

Give the above results, Vogelsang and Wagner (2016) show that the linearity of the cointegrating relationship could be tested by the following Wald-type tes statistics:

$W_R = \widetilde{\gamma}' [R A_{IM} \widehat{V}_{IM} A_{IM} R']^{-1} \widetilde{\gamma}$, where R is the selection matrix corresponding to $\gamma \in \theta$. Proposition 2 of Vogelsang and Wagner (2016) shows that this statistics has a chi-squared distribution.