No. 2213 [EN]

IMI Working Paper

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Lei Xiaowen, Lu Dong and Kenneth Kasa

INTERNATIONAL MONETARY INSTITUTE

Macroeconomic Dynamics, **26**, 2022, 833–884. Printed in the United States of America.
doi:10.1017/S1365100520000383

"WAIT AND SEE" OR "FEAR OF FLOATING"?

XIAOWEN LEI
University of Oxford

DONG LU
Renmin University of China

KENNETH KASA
Simon Fraser University

This paper studies the evolution of China's exchange rate policy using real options theory. With intervention costs and ongoing uncertainty, intervention involves the exercise of an option. Increased uncertainty increases the value of this option. This "wait and see" effect leads the Central Bank to widen its intervention band. However, increased volatility also produces larger fluctuations in welfare, which creates a "fear of floating." This induces the Central Bank to set a tighter band. To study this trade-off, our paper incorporates stochastic volatility into a new Keynesian target zone model and then calibrates it to data from China. We find that increased uncertainty leads to a tighter intervention band, both in the data and in the model. Hence, in China, "fear of floating" appears to dominate the "wait and see" effect.

Keywords: Exchange Rate, Target Zone, Option Value, Stochastic Volatility

China's exchange rate policy will consistently follow the principles of automony, controllability, and gradualism.

—Zhou Xiaochuan, Former Governor of China's Central Bank, February 13, 2016

1. INTRODUCTION

Exchange rate policy is often portrayed as a choice between fixed or flexible exchange rates. Reality, of course, is in between.[1] During the late 1980s and early 1990s, the target zone literature pioneered by Krugman (1991) seemed to

We would like to thank Klaus Adam, Paul Beaudry, Michael Devereux, Edouard Djeutem, Geoffrey Dunbar, Martin Ellison, Meiyu Li, Dimitry Matveev, Jun Nie, Luba Petersen, Walter Steingress, Michael Tseng, David Vines, Vivian Yue, Lei Zhang, Hang Zhou, and Wenyu Zhu for useful comments. We thank seminar participants at the 2018 European Econometrics Society Winter Meeting, Bank of Canada, Hong Kong University, and University of Oxford. We have also benefited from comments by Hua Guan, Wei Huo, Lifeng Peng, and Guofeng Sun at the People's Bank of China. Jialin Liu, Yu Xia, and Zinan Zhou provided expert research assistance. We appreciate the constructive comments provided by three anonymous referees and the associate editor, which improved the paper significantly. Dong Lu acknowledges that this research is supported by the Key Program of National Social Science Foundation (19AJY028). Corresponding author: Dong Lu, School of Finance, Renmin University of China, No.59 Zhongguancun St. Haidian District, Beijing, China. e-mail: donglu@ruc.edu.cn. Phone: +8613522150737.

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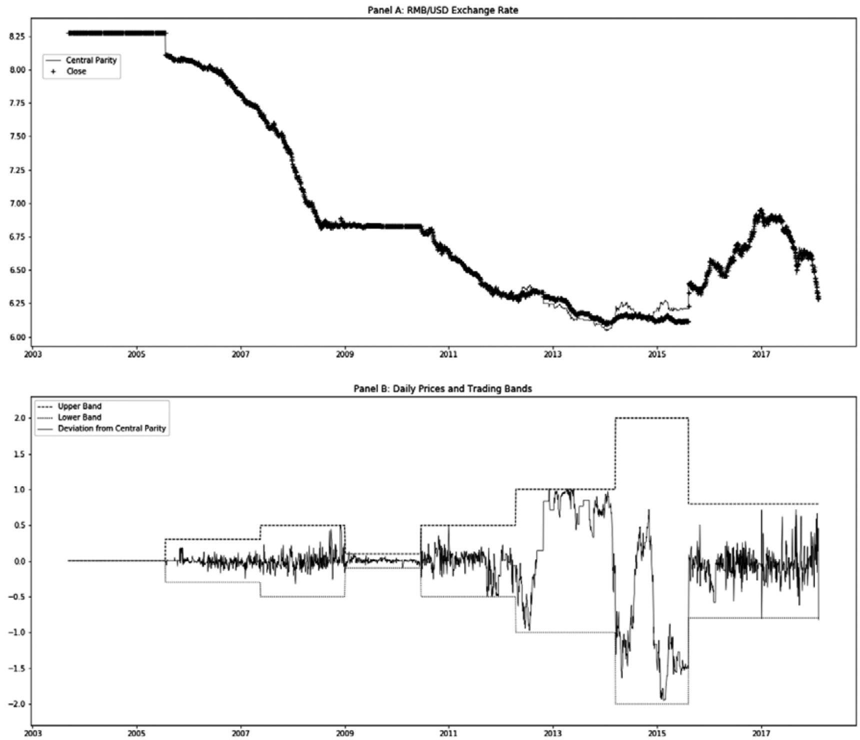

FIGURE 1. RMB/USD exchange rate and trading bands.

provide a useful way to model this middle ground. In a target zone, the Central Bank allows the exchange rate to fluctuate within a predetermined band but then intervenes at the boundary to keep the exchange rate from moving outside of the band. Target zone models were widely applied to the European Monetary System (EMS) in the years leading up to the euro, for example, Bertola and Caballero (1992), Bertola and Svensson (1993), and Miller and Zhang (1996).

After the euro was introduced, the target zone literature largely died out. Interest now focuses on China's exchange rate, given that China's currency Renminbi (RMB) is becoming ever more important in world trade and payments. In some respects, China's recent exchange rate policy seems to resemble that of the EMS. Since 2005, the People's Bank of China (PBOC) has allowed market forces to move the exchange rate, but only within limits. Hence, it is tempting to dust off a target zone model and see whether it can explain the recent behavior of the RMB/US Dollar (USD) exchange rate.

Even a cursory look at the data suggests that European-inspired target zone models might have trouble explaining China's recent exchange rate policy. Panel A of Figure 1 plots daily data since 2003 on the RMB central parity and

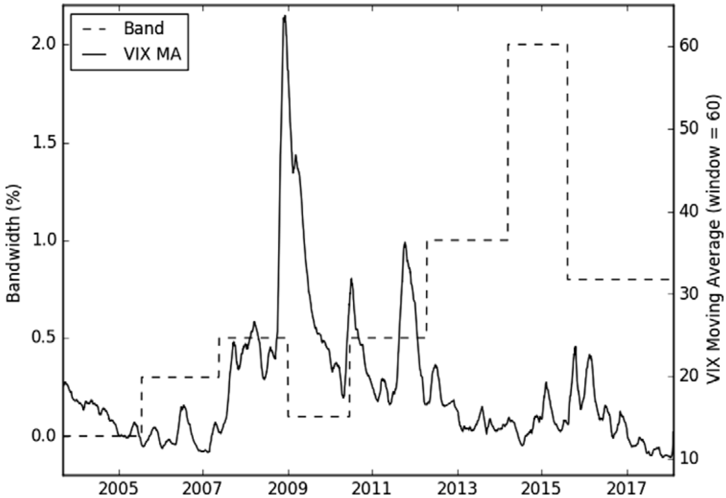

FIGURE 2. VIX and China's exchange rate band.

closing rate, while Panel B plots percentage deviations of the closing rate from the central parity, along with the trading band set by the PBOC.

Two discrepancies stand out. First, it is clear that the RMB does not fluctuate within a single, time-invariant band. From 2005 to 2015, the RMB steadily appreciated, with a brief respite during the financial crisis. Then, from August 2015 until 2017, it reversed course and began to depreciate. Second, Panel B shows that the *width* of the band has evolved over time. For example, it tightened during the financial crisis and then widened after 2012, only to be tightened again in 2016. In contrast, the band width in a traditional target zone model is time invariant.

Perhaps, we should not be too surprised that off-the-shelf target zone models do not work. After all, they failed to explain EMS data as well (Bertola and Caballero (1992)). However, the extensions that were developed to fit the EMS data seem ill-suited to China. In particular, there is little evidence of discrete realignments of the RMB. Instead, Figure 1 suggests that the PBOC controls the RMB's rate of drift, rather than its level. This suggests the need for a different sort of extension.

In this paper, we argue that *stochastic volatility* is the key to understanding China's recent exchange rate policy. In particular, we show that the width of the PBOC's trading band evolves in response to changes in volatility.[2] For example, Figure 2 plots the width of the trading band against the Volatility Index (VIX) index, a common measure of aggregate uncertainty. A negative correlation is apparent. We show that this same negative correlation is present when other uncertainty measures are used and is robust to different effective bands, consistent with the findings in Marconi (2018).[3]

Adding stochastic volatility to a target zone model is straightforward in principle, but technically challenging in practice. When both the level and volatility of the exchange rate vary, the optimal band is characterized by a *partial* differential equation (PDE). This PDE does not have a closed-form solution. However, since the constant volatility case has a well-known analytic solution, we can obtain a first-order perturbation approximation. Using this approximation, we show that volatility has countervailing effects on the bandwidth. On the one hand, a higher *mean* level of volatility widens the band, for the usual option value reasons. Hence, if we compare across economies, those with higher mean volatility will have wider average bandwidths. However, if we look at bandwidth over time within a given economy, holding mean volatility constant, the band width decreases in response to (temporary) increases in volatility. When volatility is stochastic, it creates additional flow costs that increase the benefits of regulation.

Although stochastic volatility explains why the PBOC adjusts the width of its trading band, it does not explain the drift of the central parity that is apparent in Figure 1. The question of why and how the PBOC adjusts the central parity has recently been addressed by Jermann et al. (2019). They show that choice of the central parity after August 2015 adheres to a so-called "two pillars approach," which strikes a balance between stability and adjustment to market forces.[4] Two notable differences are: (1) from the end of 2008 to June 2010, China repegged to the dollar with a very narrow band of less than 0.1%, as announced by former Governor Zhou Xiaochuan and (2) after August 11, 2015, and the reform of the central parity, the PBOC implemented an effective band of 0.8% based on our estimation, similar in magnitude to Jermann et al. (2019)'s band of 0.5%. Appendix A.1 provides further institutional details. In particular, today's central parity is assumed to be a weighted average of yesterday's closing price (reflecting adjustment to market forces), and a rate that lends stability to a predefined currency basket. Although their model is successful at tracking changes in the central parity, they do not address the question of why the bandwidth changes. Hence, we view our paper as complementary to theirs. Their paper focuses on Panel A of Figure 1, while ours focuses on Panel B.[5]

The remainder of the paper is organized as follows. Section 2 briefly discusses the literature. Section 3 presents the model. We start with the case of constant volatility. We show that a permanent one-time increase in volatility produces a "wait-and-see" effect, which (permanently) widens the band. We then augment the model by incorporating stochastic volatility, using a Cox–Ingersoll–Ross (CIR) specification. Now the trading band responds continuously to ongoing changes in volatility. We show that stochastic fluctuations in volatility produce "fear of floating," which narrows the band in response to increased uncertainty. Section 4 uses Doob's optional stopping theorem to verify that our solution constitutes a rational expectations equilibrium. Section 5 studies the model's quantitative implications by calibrating it to data from China. We also decompose the "wait and see" and "fear of floating" effects quantitatively by separately varying permanent and transitory changes in volatility. A sensitivity analysis verifies that the calibration results do not depend on the choice of particular parameter values. Section 6 reports empirical evidence and confirms that the negative correlation between the bandwidth and volatility is robust to alternative volatility measures and alternative sample periods. Finally, Section 7 summarizes our conclusions and discusses possible extensions. A supplementary Appendix provides institutional background on China's exchange rate policy, derives technical results on the implementation of the target zone, fills in details of our New Keynesian target zone model, and reports additional figures and graphs supporting the sensitivity analysis.

2. LITERATURE REVIEW

Our paper contributes to the literature in several ways. First, this paper is related to the literature on exchange rate target zones (e.g., Krugman (1991), Bertola and Caballero (1992), Bertola and Svensson (1993), Miller and Zhang (1996), Mundaca and Øksendal (1998)). However, this previous literature is based on reduced form objective functions and, more importantly, does not allow the target zone to respond to changes in uncertainty. Our paper extends the classic target zone literature (summarized in Krugman and Miller (1992)) by incorporating stochastic volatility within a New Keynesian general equilibrium model. These extensions are crucial in helping us to understand how uncertainty influences the choice of target zone bandwidth.

Our paper also contributes more broadly to the understanding of China's monetary and exchange rate policy and, in particular, sheds light on optimal policy in the sort of intermediate exchange rate regimes that are widely used in developing countries.[6] Obstfeld (2006) studies the RMB exchange rate's exit strategy from pegging to US dollar and proposes a limited trading band for the RMB relative to a basket of major trading partner currencies. Cheng (2015) and Yu et al. (2017) both argue for a RMB target zone, but they do not study the optimal bandwidth choice under uncertainty. Our work is also related to recent work by Jermann et al. (2019) and Clark (2017), which studies RMB central parity. Our paper complements theirs by looking at the width of target zone band.

Finally, at a technical level, our paper is closely related to the real options literature, summarized by Dixit and Pindyck (1994) and Stokey (2009). Recently, real options theory has been successfully applied to a variety of macroeconomic questions where discrete actions and inertia are present. For example, Alvarez and Dixit (2014) examine the optimal timing of a potential eurozone breakdown; Stokey (2016) studies investment options under policy uncertainties; Lei and Tseng (2019) study the optimal timing and size of discrete interest rate adjustments. Recent advances in bringing stochastic volatility into the discussion of option exercise also shed lights on our analysis. Fouque et al. (2000) studies the pricing of American options under stochastic volatility. Their work was later applied to the real options literature by Sarkar (2000) and Tsekrekos and Yannacopoulos (2016). Our paper applies similar techniques to a target zone model.

3. THE MODEL

The original target zone literature was based on the monetary model of exchange rates, in which the Central Bank is assumed to influence the exchange rate by changing the money supply. It was never clear in these models why the Central Bank wanted to limit exchange rate fluctuations. These days, monetary policy is typically studied using the general equilibrium new Keynesian framework, in which Central Banks set interest rates to limit fluctuations in the output gap and inflation. They do this in order to maximize household welfare.[7] Besides incorporating stochastic volatility, another contribution of our paper is to study target zone dynamics using this more modern framework.

Our model is based closely on the prior work of Gali and Monacelli (2005) and Clarida et al. (2001) (henceforth CGG). We study a small open economy operating in a world of complete financial markets.[8] Goods are freely traded, and the Law of One Price holds. The key friction in the model, as in all New Keynesian models, is that firms cannot continuously adjust their prices. In an open economy, sluggish price adjustment implies that exchange rate fluctuations induce terms of trade fluctuations, which then produce output and inflation fluctuations. As a result, the Central Bank cares about exchange rate fluctuations.

The model is summarized by the following five log-linearized equations. Appendix A.4 provides a more detailed presentation of the model.

$$x_t = \mathbb{E}_t x_{t+1} - \frac{1}{\sigma_\gamma}\left(i_t - \mathbb{E}_t \pi_{H,t+1} - rr^0\right), \quad (1)$$

$$\pi_{H,t} = \beta \mathbb{E}_t \pi_{H,t+1} + \kappa_\gamma x_t + u_t, \quad (2)$$

$$s_t = \sigma_\gamma x_t, \quad (3)$$

$$s_t = e_t + p_t^* - p_{H,t}, \quad (4)$$

$$i_t = i_t^* + E_t(\Delta e_{t+1}). \quad (5)$$

As usual, x_t denotes the output gap, $\pi_{H,t}$ denotes *domestic* inflation (as opposed to Consumer Price Index inflation), rr^0 denotes the (constant) natural interest rate, s_t represents the terms of trade (defined as the relative price of foreign goods), e_t represents the nominal exchange rate (defined as the price of foreign currency), and p_t^* and i_t^* are the foreign price level and foreign nominal interest rate. Except for nominal interest rates and the natural interest rate, all variables are in units of percent deviation from the steady state. Equation (1) is the household's Euler equation, which plays the role of a dynamic Investment-Saving curve. Equation (2) is the New Keynesian Phillips curve, describing optimal price adjustment in the presence of Calvo price setting. We assume an exogenous cost push shock, u_t, enters the Phillips curve, which follows a random walk. The remaining three equations represent goods market clearing, the Law of One Price, and Uncovered Interest Parity (UIP), respectively.

As emphasized by Gali and Monacelli (2005) and Clarida et al. (2001), the great virtue of this particular model is that it remains quite similar to the canonical closed-economy New Keynesian model. All that really changes is the parameter values. For example, $\sigma_\gamma = \frac{\sigma}{1+\gamma(\omega-1)}$, where γ is the share of foreign goods in domestic consumption. ω is an elasticity parameter that exceeds unity for plausible parameter values. Hence, output tends to respond more strongly to interest rate fluctuations in an open economy, because interest rate fluctuations produce exchange rate fluctuations, which then trigger expenditure switching effects. Similarly, the slope of the Phillips Curve, $\kappa_\gamma = \lambda(\sigma_\gamma + \phi)$, depends on the degree of openness as well. Since $\sigma_\gamma \leq \sigma$, domestic inflation is less responsive to output gap fluctuations in an open economy.

The innovation in our paper is to suppose that the Central Bank confronts a cost to changing either the interest rate or exchange rate. Lei and Tseng (2019) show that a fixed cost can explain observed infrequent interest rate changes at both the Fed and the Bank of Canada. Here, we adopt a similar assumption, although to be consistent with the absence of jumps in the RMB exchange rate, we instead suppose the intervention cost is *linear*, as in Miller and Zhang (1996). One interpretation of a linear intervention cost is that the Central Bank pays transaction costs, which increase with the size of the adjustment (e.g., bid/ask spreads). This produces an optimal barrier control policy. Absent intervention costs, an optimizing Central Bank would (counterfactually) continuously vary the interest rate in response to continuously arriving cost shocks.[9]

Rational expectations require the private sector to form expectations of the output gap and inflation that are consistent with Central Bank policy. Under discretion, the central bank chooses optimal monetary (exchange rate) policy by taking as given the private sector's expectations, with the resulting data-generating processes conforming to the private sector expectations. Pursuing a guess-and-verify solution strategy, we conjecture that private sector expectations are martingales:

$$E_t\pi_{t+1} = \pi_t, \tag{6}$$

$$E_t x_{t+1} = x_t, \tag{7}$$

$$E_t s_{t+1} = s_t, \tag{8}$$

$$E_t e_{t+1} = e_t. \tag{9}$$

Substituting equations (6) and (7) into equations (1) and (2), one obtains

$$x_t = a_{11}\tilde{i} + b_{11}u_t; \quad \pi_t = a_{22}\tilde{i} + b_{22}u_t, \tag{10}$$

where a_{11}, a_{22}, b_{11}, and b_{22} are constants[10], and $\tilde{i} = i - rr^0$. Evidently, our conjectured solution is confirmed as long as u_t is itself a martingale, and $\tilde{i}$ is a constant. Within the target zone, $\tilde{i}$ will indeed be constant, its particular value depending on the history of previous interventions, which in turn depends on the realization of past cost shocks. When the target zone is symmetric, its long-run average value will be zero. In the absence of adjustment costs, what would an optimizing

https://doi.org/10.1017/S1365100520000383 Published online by Cambridge University Press

Central Bank do? Employing a second-order approximation of social welfare, the problem would be summarized by the following

$$\min_{\tilde{i}} \frac{1}{2}(\lambda x_t^2 + \pi_t^2) = \frac{1}{2}[\lambda(a_{11}\tilde{i} + b_{11}u_t)^2 + (a_{22}\tilde{i} + b_{22}u_t)^2]$$
$$= \frac{1}{2}(\lambda a_{11}^2 + a_{22}^2)\tilde{i}^2 + \frac{1}{2}(\lambda b_{11}^2 + b_{22}^2)u_t^2 + (a_{11}b_{11}\lambda + a_{22}b_{22})\tilde{i}u_t$$
$$= \frac{1}{2}\tilde{\lambda}(\tilde{i} - \theta u_t)^2 + \frac{\lambda^2\beta(2-\beta)}{[\lambda(1-\beta)^2 + \kappa^2]^2}u_t^2, \quad \textbf{(11)}$$

where $\theta = \frac{\lambda(1-\beta)}{\lambda(1-\beta)^2+\kappa^2}$ and $\tilde{\lambda} = \lambda a_{11}^2 + 1$. Note that the second term is independent of policy. It cannot be mitigated by changing the interest rate or exchange rate. However, the Central Bank can minimize the loss arising from the first term by setting $\tilde{i}_t = \theta u_t$. Unfortunately, when u_t follows a Brownian motion, this policy would be disastrous in the presence of linear adjustment costs. Instead, the Central Bank must balance the costs of deviating from the target, θu_t, and the costs of adjusting to the target. This involves keeping the interest rate constant within an optimally chosen band and then intervening infinitesimally to keep the exchange rate from exiting the band.

3.1. Constant Volatility

Since we want to solve an optimal timing problem, it pays to work in continuous time. The continuous-time limit of a discrete-time martingale is a Brownian motion. Uncertainty is represented by a filtered probability space $(\Omega, P, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{F})$, induced by an observable one-dimensional standard Brownian motion $B(t)$, which satisfies the usual conditions. In the absence of any control, the cost shock $u = \{u_t, t \geq 0\}$ fluctuates as a Brownian Motion with standard deviation σ,[11]

$$du = \sigma dB. \quad \textbf{(12)}$$

The Central Bank faces two costs that it wants to minimize. First, there is the flow welfare cost $\lambda x^2 + \pi^2$, which arises from cost shocks. Second, there is an adjustment cost whenever the Central Bank intervenes, which is equal to m times the size of the adjustment. This cost will be discussed further in the calibration section.[12]

The Central Bank's objective is to find a policy that balances the expected discounted value of these two types of costs over an infinite planning horizon, when future costs are discounted at the rate of $\rho > 0$. A policy is defined as a pair of non-negative processes $L = \{L_t, t \geq 0\}$ and $U = \{U_t, t \geq 0\}$ that are non-decreasing and non-anticipating with respect to e_t. One can interpret L_t as the cumulative upward adjustment of e_t, and U_t as the cumulative downward adjustment to e_t. Therefore, under policy (U, L), the controlled/regulated process can be defined as

$$z_t = \tilde{i} - \theta u_t - U_t + L_t, \quad \textbf{(13)}$$

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which denotes the gap between the current level of the interest rate and the discretionary optimal interest rate. This implies the regulated process z_t follows.[13]

$$dz = \tilde{\sigma}\, dB, \tag{14}$$

within the inaction region, where $\tilde{\sigma} = -\theta\sigma$. The Central Bank's control problem can now be stated as

$$V(z_0) = \inf_{L,U} \mathbb{E}_{z_0}\left[\int_0^\infty e^{-\rho t}\tilde{\lambda} z^2 dt + m\int_0^\infty e^{-\rho t} dL + m\int_0^\infty e^{-\rho t} dU\right] \tag{15}$$

for a given initial exchange rate z_0.[14] Given the assumption of quadratic flow "carrying cost," it can be shown that the value function V is twice continuously differentiable and that it satisfies the following Hamilton–Jacobian–Bellman (HJB) equation within the inaction region,

$$\rho V = \tilde{\lambda} z^2 + \frac{1}{2} V_{zz}\tilde{\sigma}^2. \tag{16}$$

The general solution is

$$V(z) = Az^2 + K_1 e^{\beta_1 z} + K_2 e^{\beta_2 z} + C. \tag{17}$$

Due to symmetry, we have $K_1 = K_2 = K$, which simplifies the solution to

$$V(z) = Az^2 + K(e^{\beta_1 z} + e^{\beta_2 z}) + C, \tag{18}$$

where $A = \frac{\tilde{\lambda}}{\rho}$, $\beta_1 = \frac{\sqrt{2\rho}}{\tilde{\sigma}}$, $\beta_2 = -\frac{\sqrt{2\rho}}{\tilde{\sigma}}$, $C = \frac{\tilde{\lambda}\tilde{\sigma}^2}{\rho^2}$. The constants of integration K, and the optimal barriers, $(\bar{z}, -\underline{z})$, are determined by four boundary conditions: a pair of smooth-pasting/value-matching conditions, and a pair of higher-order contact conditions

$$V'(\bar{z}) = -m; \quad V'(\underline{z}) = m; \quad V''(\bar{z}) = V''(\underline{z}) = 0. \tag{19}$$

Applied optimally, the marginal cost of control should be the same as the marginal present value benefit at both barriers.[15]

PROPOSITION 1. *With constant volatility, the Central Bank's optimal exchange rate policy features barrier control, with a target zone defined by constant lower and upper boundaries $\underline{z}$ and $\bar{z}$, respectively. The width of the target zone around the normalized central parity $z = 0$ is approximately*

$$\bar{z} = \left(\frac{3m\tilde{\sigma}^2}{4\tilde{\lambda}}\right)^{1/3}. \tag{20}$$

The target zone is symmetric around zero, with $\underline{z} = -\bar{z}$. To keep the exchange rate within the target zone, the Central Bank exerts instantaneous control, lowering the domestic interest rate when z hits $\underline{z}$ and raising the domestic interest rate when z hits $\bar{z}$.

Proof. See Dixit (1993) Section 4.6 and Stokey (2009) Proposition 10.10 for solution method. ■

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Evidently, with constant volatility, a one-time permanent increase in uncertainty increases the value of "wait and see." In response, the Central Bank optimally adopts a wider (time invariant) target zone. This seems contradictory at a first glance. However, there are two opposing forces underlying the determination of the target zone. On one hand, increased uncertainty raises the option value of waiting, so that the Central Bank would favor a wider band in order to avoid constantly paying the regulation cost. On the other hand, increased uncertainty also makes domestic variables more volatile, which is something the Central Bank wants to minimize. It turns out that with constant volatility, the former effect dominates. This is opposite to what we observe in the data. In the following section, we resolve this puzzle by incorporating stochastic volatility.

3.2. Stochastic Volatility

In this section, we develop and solve a model of an exchange rate target zone under stochastic volatility. This allows us to examine the dynamics of the optimal bandwidth. Now the volatility of the state variable z_t is itself random. Note that this does not violate the martingale property of the the *level* of the exchange rate. Exchange rate changes within the band remain unpredictable. Exchange rate dynamics are now governed by

$$dz = \tilde{\sigma}\, dB, \tag{21}$$

$$d\tilde{\sigma} = \kappa(\theta - \tilde{\sigma}) + \sqrt{\xi}\sqrt{\tilde{\sigma}}\, dB_\sigma. \tag{22}$$

The CIR specification for volatility is attractive, since it constrains volatility to be non-negative. For simplicity, we assume that $corr(dB, dB_\sigma) = 0$. The long-run mean of volatility is θ, and the speed of mean reversion is equal to κ.[16]

Note that volatility now becomes a state variable, and the Central Bank's barrier control problem is given by

$$V(z_0, \sigma_0) = \inf_{L,U} \mathbb{E}\left[\int_0^\infty e^{-\rho t}\tilde{\lambda} z^2 dt + m\int_0^\infty e^{-\rho t} dL + m\int_0^\infty e^{-\rho t} dU)\right]. \tag{23}$$

Once again, within the inaction region the Central Bank's policy must respect the HJB equation, which is now a PDE

$$\rho V = \tilde{\lambda} z^2 + \frac{1}{2} V_{zz}\tilde{\sigma}^2 + V_{\tilde{\sigma}}\kappa(\theta - \tilde{\sigma}) + \frac{1}{2} V_{\tilde{\sigma}\tilde{\sigma}}\xi\tilde{\sigma}. \tag{24}$$

Fortunately, this is linear, so the general solution is the sum of a particular solution and the homogeneous solution. The particular solution can be written as

$$V^p(z, \sigma) = \frac{\tilde{\lambda}}{\rho} z^2 + A_1\tilde{\sigma}^2 + A_2\tilde{\sigma} + B_1. \tag{25}$$

Matching coefficients gives $A_1 = \frac{\tilde{\lambda}}{\rho(\rho+2\kappa)}$; $A_2 = \frac{2\kappa\theta+\xi}{\rho+\kappa} A_1$; and $B_1 = \frac{\kappa\theta}{\rho} A_2$. The homogeneous part of the solution must satisfy the following PDE

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$$\rho V = \frac{1}{2} V_{zz} \tilde{\sigma}^2 + V_\sigma \kappa(\theta - \tilde{\sigma}) + \frac{1}{2} V_{\tilde{\sigma}\tilde{\sigma}} \xi \tilde{\sigma}. \quad (26)$$

One can guess and verify the following functional form for the solution

$$V^h(z, \tilde{\sigma}) = e^{f(z,\tilde{\sigma})}, \quad (27)$$

where $f(z, \sigma)$ must satisfy the following nonlinear PDE

$$\rho = \frac{1}{2} \left(f_{zz} + f_z^2\right) \tilde{\sigma}^2 + f_\sigma \kappa(\theta - \tilde{\sigma}) + \frac{1}{2} \left(f_{\tilde{\sigma}\tilde{\sigma}} + f_{\tilde{\sigma}}^2\right) \xi \tilde{\sigma}. \quad (28)$$

We solve this PDE using a perturbation approximation around the constant volatility benchmark, where $\kappa = \xi = 0$,

$$f(z, \tilde{\sigma}) = f^0(z, \tilde{\sigma}) + \kappa f^\kappa(z, \tilde{\sigma}) + \xi f^\xi(z, \tilde{\sigma}). \quad (29)$$

Taking derivatives and evaluating at the perturbation expansion point, one obtains

$$f^0(z, \tilde{\sigma}) = \beta_{1,2} z; \quad \beta_{1,2} = \pm \frac{\sqrt{2\rho}}{\tilde{\sigma}}, \quad (30)$$

$$f^\kappa(z, \tilde{\sigma}) = \frac{1}{2} p_1 z^2 + q_1 z; \;\; p_1 = -\frac{\beta_{\tilde{\sigma}}(\theta - \tilde{\sigma})}{\beta \tilde{\sigma}^2}; \;\; q_1 = \frac{\beta_{\tilde{\sigma}}(\theta - \tilde{\sigma})}{2\beta^2 \tilde{\sigma}^2}, \quad (31)$$

$$f^\xi = -\frac{1}{6} z^3 + \left(\frac{1}{4\beta} - \frac{1}{2}\right) z^2 - \left(\frac{1}{4\beta^2} - \frac{1}{2\beta}\right) z. \quad (32)$$

Therefore, a first-order perturbation approximation of the general solution is

$$V(z, \tilde{\sigma}) = \frac{\alpha}{\rho} z^2 + A_1 \tilde{\sigma}^2 + A_2 \tilde{\sigma} + B_1 + K_1 e^{f(z,\tilde{\sigma}|\beta_1)} + K_2 e^{f(z,\tilde{\sigma}|\beta_2)}. \quad (33)$$

As before, to pin down $(K_1, K_2, \bar{z}, \underline{z})$, we need two smooth-pasting and two higher-order contact conditions

$$V_z(\bar{z}) = -m; \quad V_z(\underline{z}) = m, \quad (34)$$

$$V_{zz}(\bar{z}) = V_{zz}(\underline{z}) = 0. \quad (35)$$

A detailed derivation of these boundary conditions under stochastic volatility is provided in the Appendix. The key difference now is that the optimal barriers, $\left(\bar{z}(\tilde{\sigma}), \underline{z}(\tilde{\sigma})\right)$, become functions of $\tilde{\sigma}$. When volatility changes, so does the target zone. Due to symmetry, we know again that $K_1 = K_2 = K$. To an $\mathcal{O}(z^2)$ approximation, we know that the threshold $\bar{z}(\tilde{\sigma})$, and its mirror image $-\bar{z}(\tilde{\sigma})$ are characterized by the following two conditions[17]:

$$V_z = -\frac{2\tilde{\lambda}}{\rho} z + K \left(a_1 + a_2 + 2z(\frac{a_1^2 + a_2^2}{2} + b_1 + b_2) + \frac{3}{2} z^2 (a_1 b_1 + a_2 b_2) \right) = -m, \quad (36)$$

$$V_{zz} = -\frac{2\tilde{\lambda}}{\rho} + K \left(2(\frac{a_1^2 + a_2^2}{2} + b_1 + b_2) + 3z(a_1 b_1 + a_2 b_2) \right) = 0, \quad (37)$$

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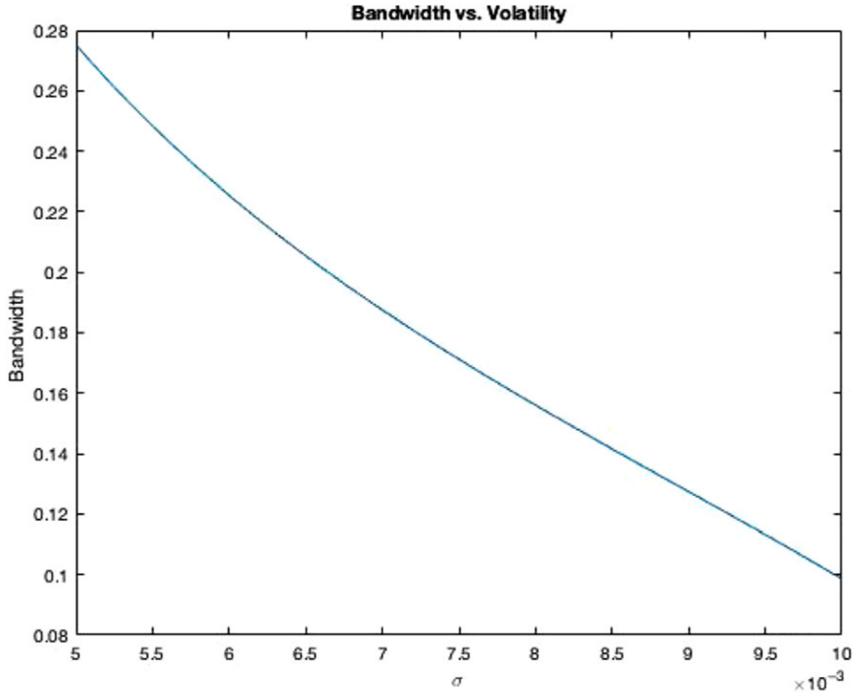

FIGURE 3. Bandwidth versus volatility.

where the subscripts $(1, 2)$ denote the values corresponding to the positive and negative root $\beta_{1,2}$, respectively.

Figure 3 plots the model-implied bandwidth as a function of the currently prevailing level of volatility.

We use the benchmark parameters from the following table to compute the barrier as a function of volatility. The values of these parameters will be discussed in detail in the next section. From the graph, we can see that an increase in volatility now *reduces* the bandwidth. Stochastic volatility serves as an extra source of risk, which induces the Central Bank to tighten the band. In the Appendix, we show this result is robust to parameter changes.

3.3. Comments on Stochastic Volatility

We have seen that by considering stochastic volatility, the relation between bandwidth and uncertainty is reversed. This result is not generated by adding any additional assumptions other than letting uncertainty evolve stochastically. "Fear of floating," in the words of Calvo and Reinhart (2002), is generated endogenously. Although it is common that stochastic volatility has real effects in models with at least a third-order approximation, here the effect kicks in with only a second-order approximation. To understand this result, recall that the Central

Bank is trading off two types of costs: the flow welfare cost and the regulation cost. On one hand, a higher regulation cost leads to widening the band due to the usual "wait and see" effect. On the other hand, a higher welfare cost leads the Central Bank to tighten the band, due to "fear of floating." Stochastic volatility implies a higher welfare cost relative to the constant volatility, due to additional variation coming from volatility itself. This can be shown more rigorously below.

Proof. One can show that the weight on the "welfare cost" part of the objective function is higher under stochastic volatility. Let $\tilde{\sigma}_0$ be drawn from its long-run stationary distribution, so that the long-run mean of $\tilde{\sigma}_t$ is equal to θ, which eases our comparison of the constant volatility economy with the stochastic volatility economy. Note that $E(\tilde{\sigma}_t^2|\tilde{\sigma}_0) = var(\tilde{\sigma}_t|\tilde{\sigma}_0) + E^2(\tilde{\sigma}_t|\tilde{\sigma}_0) = var(\tilde{\sigma}_t|\tilde{\sigma}_0) + \theta^2$.

Let $z_0 = 0$. Since $z_t = z_0 + \int_0^t \tilde{\sigma}_s dB_s$, we have $E(z_t^2) = E[(\int_0^t \tilde{\sigma}_s dB_s)^2|\sigma_0] = E\left[\int_0^t \quad (\tilde{\sigma}_s^2|\tilde{\sigma}_0)ds\right] = E\left[\int_0^t E(\tilde{\sigma}_s^2|\tilde{\sigma}_0)ds\right]$, where the second equality comes from Ito Isometry. Therefore, $E(z_t^2|\tilde{\sigma}_0) = E\int_0^t E(\tilde{\sigma}_s^2|\tilde{\sigma}_0)ds = E[\int_0^t var(\tilde{\sigma}_t|\tilde{\sigma}_0)ds] + \int_0^t \theta^2 ds \geq \int_0^\infty \theta^2 ds$. Therefore, $E^{stoch}\int_0^\infty e^{-\rho t}\tilde{\lambda}z_t^2 dt \geq E^{constant}\int_0^\infty e^{-\rho t}\tilde{\lambda}z_t^2 dt$. Note that this result does not rely on any specific structure of stochastic volatility and holds more generally than the CIR specification. ■

3.3.1. Comments on regulation costs. As noted earlier, the Central Bank maintains the target zone by using high frequency, infinite variation, changes in the instantaneous interest rate. The Appendix provides the details. Each time the Central Bank changes the interest rate, it must pay a linear marginal cost of m. How big are the resulting costs? Given the nature of Brownian motion, it pays the cost "many" times, but each time it does so, the adjustment is infinitesimal, so it is not at all obvious how big the product of the two turns out to be over finite time intervals. The appendix uses results on Brownian local times from Stokey (2009) to compute the regulation cost as a percentage of total welfare cost, using our benchmark parameter values. It turns out that regulation costs are quite small, being only about 0.0037715% of the long-run total welfare cost.

4. RATIONAL EXPECTATIONS

In this section, we prove that private sector expectations of the dynamics of x_t, π_t, e_t, s_t are consistent with the Central bank's policy, so that we conclude that the equilibrium defined above satisfies rational expectations. To do this, recall that private sector expectations regarding the two processes satisfy a local martingale-type condition once we convert the discrete-time system into its continuous-time counterpart. Define the stopping times $\tau_k < \tau < \tau' < \tau_{k+1}$, where τ_k is the kth intervention time, the private sector's expectaiton can be written as

$$E(x_{\tau'}|\mathcal{F}_\tau, i_k) = x_\tau, \tag{38}$$

$$E(\pi_{\tau'}|\mathcal{F}_\tau, i_k) = \pi_\tau, \tag{39}$$

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$$E(s_{\tau'}|\mathcal{F}_\tau, i_k) = s_\tau, \tag{40}$$

$$E(e_{\tau'}|\mathcal{F}_\tau, i_k) = e_\tau. \tag{41}$$

From Doob's optional stopping theorem,

$$E(x_{\tau'}|\mathcal{F}_\tau, i_k) = a_{11} i_k + b_{11} u_\tau = x_\tau, \tag{42}$$

$$E(\pi_{\tau'}|\mathcal{F}_\tau, i_k) = a_{22} i_k + b_{22} u_\tau = \pi_\tau, \tag{43}$$

$$E(s_{\tau'}|\mathcal{F}_\tau, i_k) = \sigma_\gamma x_\tau = s_\tau, \tag{44}$$

$$E(e_{\tau'}|\mathcal{F}_\tau, i_k) = E(e_{\tau'}|\mathcal{F}_\tau, i_k) = e_\tau. \tag{45}$$

Thus, under the Central Bank's policy, the private sector's expectations are confirmed to be consistent with the true data-generating process.

5. CALIBRATION

In this section, we take the above model to Chinese data and examine its quantitative implications for the evolution of the RMB exchange rate trading band. Since there is a one-to-one mapping from the nominal exchange rate to the terms of trade in the model, we use monthly data for export and import prices from China's National Bureau of Statistics to estimate the parameters governing the dynamics of stochastic volatility for China's terms of trade. The base year is 2000, and the terms of trade are normalized to 100 in 2000. To deal with seasonality, we follow the literature and consider year-over-year (YoY) changes.[18] The data span from January 2001 to December 2017. A graph depicting changes in the terms of trade is presented in the Appendix. We estimate terms of trade volatility (σ) using a Generalized Autoregressive Conditional Heteroskedasticity (GARCH)(1,1) model. Maximum likelihood estimation is implemented to estimate the CIR parameters for the volatility process using equation (22) and then converted to their continuous-time counterparts. We find the mean reversion parameter $\kappa = 0.6475$, the long-run mean of volatility $\theta = 0.0767$, and the volatility of stochastic volatility parameter $\xi = 0.0278$.

For the other parameters, the monthly discount rate ρ is set to be 0.33%, so that the steady-state annual risk-free interest rate is about 4%. To calibrate the weight on "carrying cost," $\tilde{\lambda}$, we set the weight on output gap deviation to be 0.5, and we use the structural parameters of the new Keynesian model described in the Appendix, where we select the "openness" parameter $\hat{\alpha}$ to be 0.2171, which matches China's average import/Gross Domestic Product ratio from 2006 to 2016. All other structural parameters assume the same values as in Galí (2015).[19] Next, the labor supply elasticity is set to 5, which implies that its inverse is $\phi = 0.2$, and the monthly discount factor β is 0.9967, which is consistent with $\rho = 0.33\%$, to match the annual risk-free rate of 4%. The intervention cost parameter, m, is calibrated to match the average one-year bandwidth (whose definition will become clearer in the next paragraph) of $\pm 0.4601\%$. This implies that $m = 9e-05$. Finally, we use terms of trade data from China to generate a model-predicted

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TABLE 1. Benchmark parameter values

Parameters	λ	β	κ_γ	ρ	m	κ	ξ	θ
value	0.5	0.9967	0.38	0.33%	9e-05	0.6475	0.0767	0.0278

target zone and then compare it with various empirical measures of the observed target zone. The parameters are summarized in Table 1.

When comparing the model-implied band with the data, it is better to use a *De Facto* continuous band, rather than the discretely adjusted *De Jure* band depicted earlier in Figure 1. It is widely believed that the announced (*De Jure*) band is wider than the effective (*De Facto*) band. We compute *De Facto* bands using different time windows and check which one provides the best fit. In particular, we compute the *De Facto* band by selecting the percentage deviation from the daily central parity that is effectively hit for at least 80% of the time during the past 3 months, 6 months, and 1 year, respectively.[20] More specifically, based on the methodology used in the *De Facto* exchange rate regime literature (e.g., Reinhart and Rogoff (2004) and Ilzetzki et al. (2017)), we construct the continuous *De Facto* band using the following algorithm:

$$P(\epsilon < b\%) \geq 80\%,$$

where ϵ is the daily absolute percentage change of the nominal exchange rate from the central parity. This probabilistic approach has been widely used in the exchange rate classification literature.[21] We also experiment with a higher threshold probability, for example, 95%, and the results are robust.

We then compare these observed bands with the model-implied 3-month, 6-month, and 1-year moving average bands. Since the empirical *De Facto* band is computed using an 80% threshold, we must also revise the model-implied band by taking its historical average. Figure 4 plots both the actual and model-implied 6-month bands. For reference, it also plots the terms of trade volatility that was used to compute the theoretical band. The left scale denotes the bandwidth and the right scale denotes volatility.

Except during the two years before the financial crisis, one can see that the model captures time variation in the band reasonably well.[22] It is clear that both the actual and model-implied bandwidth moves inversely with terms of trade volatility. The most noticeable discrepancy is that the actual band is more volatile. The relatively poor pre-crisis fit could perhaps be due to learning-induced policy inertia. Given that China only started to increase exchange rate flexibility in July 2005, after more than a decade with a rigid peg, the PBOC might have been initially reluctant to set a wide band, despite the relatively low level of uncertainty that prevailed at the time. Another explanation might be related to financial market underdevelopment. During this period, China was in the process of introducing various Foreign Exchange (FX) financial products to help investors hedge exchange rate risk, such as FX forward contracts, FX swaps, and options. Thus, they might have hesitated to embrace a wider band before

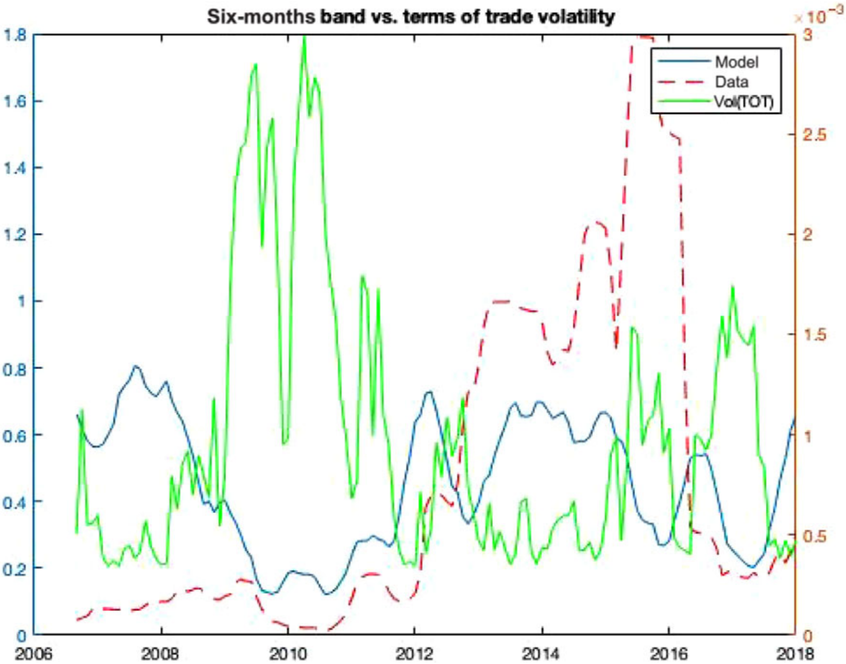

FIGURE 4. Bandwidth: model versus actual.

these important FX financial market products were introduced.[23] More detailed numerical comparisons are contained in Tables 2 and 3.

To assess model fitness, we compare the mean, variance, correlation, and the persistence of the two bandwidth series for each of the three time horizons. Recall that we choose the marginal cost parameter m to match the 1-year mean bandwidth between the model and the data, which is 0.4601%. The predicted 3-month mean bandwidth is 0.4765%, which is close to the observed value of 0.4300%, while the corresponding 6-month mean bandwidth is 0.4749%, which is again close to the observed value 0.4574%. Since we attribute *all* the variation in the bandwidth to changes in volatility, it is perhaps not surprising that the model-implied variance is smaller than in the data. Relatedly, one can also see that the correlation between the bandwidth and volatility is significantly higher in the model than in the data. Again, this reflects the fact that we have a "one-factor" model, whereas in reality multiple factors likely influence bandwidth. On a more positive note, one can see that the model does quite well matching the persistence of the band, for all three measures. Finally, Table 3 reports simple correlation coefficients between the actual and predicted bandwidths. Although the full-sample correlations are rather low, one can see that the fits improve substantially if we omit the short pre-crisis subperiod. As noted above, one might argue that during this initial "burn-in" period additional factors were at work that weakened the relationship between uncertainty and bandwidth.

TABLE 2. Calibration result (using terms of trade data)

Measure	3-month data	3-month model
Mean	0.4300%	0.4765%
Variance	0.2307	0.04554
Corr(band, uncertainty)	−0.2867	−0.8867
Autoregression (AR(1)) coefficient	0.95536	0.95211
Measure	6-month data	6-month model
Mean	0.4574%	0.4749%
Variance	0.2523	0.0383
corr(band, uncertainty)	−0.3238	−0.9107
AR(1) coefficient	0.98279	0.97438
Measure	1-year data	1-year model
Mean	0.4601%	0.4601%
Variance	0.2572	0.0299
corr(band, uncertainty)	−0.3598	−0.9391
AR(1) coefficient	0.99103	0.99145

TABLE 3. Correlation coefficient (model versus data)

Sample period/band frequency	3 months	6 months	1 year
2005.7–2017.12	0.1624	0.1961	0.2386
2008.10–2017.12	0.3616	0.4547	0.5805

5.1. A Decomposition of "wait and see" and "fear of floating"

In the model, a permanent increase in long-run mean volatility (an increase in θ) creates a "wait and see" effect which widens the band, whereas a temporary increase in volatility (an increase in σ fixing θ) generates a "fear of floating" effect, which tightens the band. In this section, we illustrate these two effects quantitatively by decomposing volatility changes into separate "wait and see" and "fear of floating" components. To isolate the wait-and-see effect, we should not just change θ by itself, since θ also influences the fear-of-floating effect. This is due to the fact that it influences the variance of volatility. Specifically, given the following CIR process of volatility,

$$d\tilde{\sigma} = \kappa(\theta - \tilde{\sigma})dt + \sqrt{\xi}\sqrt{\tilde{\sigma}}dt, \qquad (46)$$

the long-run variance of the process is equal to $\frac{\theta\xi}{2\kappa}$. Fear of floating relates to the variance of volatility, so it responds to θ. Notice, however, that we can isolate the wait-and-see effect by simply scaling ξ in inverse proportion to the scaling of θ. Doing this holds the variance of volatility constant, while changing the long-run mean. Figure 5 plots the bandwidth as a function of $\tilde{\sigma}$ for two alternative values of θ, a high value (θ_h) and a low value (θ_l), where in each case ξ is scaled inversely so as to keep the long-run variance of volatility constant at its benchmark value.

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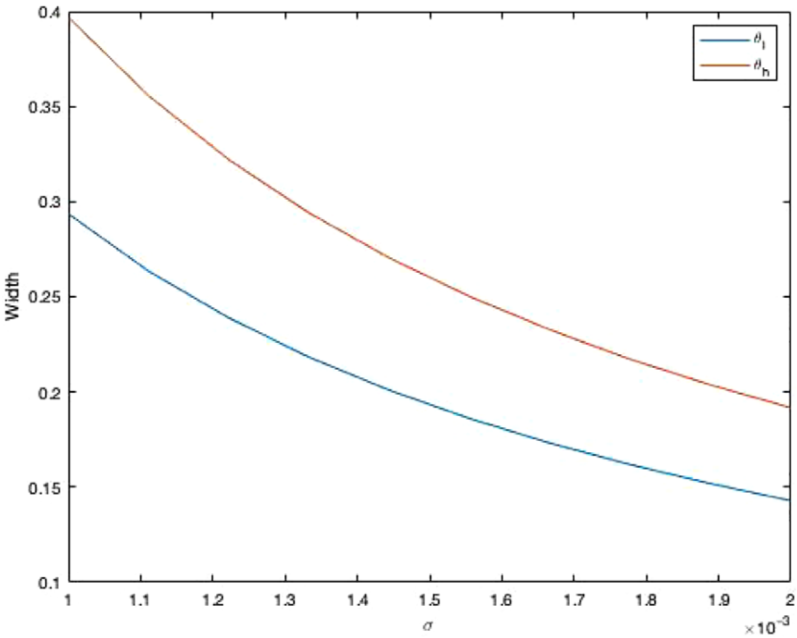

FIGURE 5. Bandwidth versus volatility.

Notice that the red line pertaining to θ_h is higher than the blue line for every given level of $\tilde{\sigma}$. The vertical distance illustrates the pure "wait and see" effect, whereas the slope along a given line illustrates the pure "fear of floating" effect.

To further illustrate the quantitative exercise of the decomposition, the following table depicts how the bandwidth changes in σ and compensated changes in θ. Denoting the benchmark value of θ as θ_0, we have the following Table 4.

The table can be interpreted as follows: Reading across columns for a given row illustrates the "fear of floating" effect. Evidently, for all values of θ, the bandwidth decrease with temporary increases in σ. Conversely, reading down rows for a given column illustrates the wait-and-see effect. Evidently, the bandwidth increases with volatility, and this is the case no matter what the current short-run volatility is.

6. EMPIRICAL IMPLICATIONS

In this section, we take the negative correlation between the bandwidth and volatility seriously and empirically examine if this correlation is robust. With a focus on China, we empirically test how the RMB/USD trading band changes in response to various uncertainty measures. We first look at the discrete *De Jure* daily bands as depicted in Figure 1. We then further examine various *De Facto* bands calculated by using the method in Reinhart and Rogoff (2004). We find

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TABLE 4. Bandwidth under “wait and see” versus “fear of floating”

θ/σ	0.0010	0.0011	0.0012	0.0013	0.0014	0.0016	0.0017	0.0018	0.0019	0.0020
1/10 θ_0	0.0969	0.0870	0.0788	0.0718	0.0659	0.0607	0.0560	0.0518	0.0480	0.0444
1/5 θ_0	0.1367	0.1228	0.1113	0.1018	0.0936	0.0865	0.0803	0.0748	0.0699	0.0654
1/2 θ_0	0.2128	0.1912	0.1735	0.1587	0.1461	0.1353	0.1259	0.1177	0.1103	0.1037
θ_0	0.2935	0.2634	0.2388	0.2183	0.2009	0.1861	0.1732	0.1618	0.1518	0.1429
$2\theta_0$	0.3969	0.3556	0.3219	0.2939	0.2703	0.2501	0.2325	0.2172	0.2037	0.1917
$5\theta_0$	0.5728	0.5127	0.4638	0.4233	0.3892	0.3600	0.3349	0.3131	0.2938	0.2768
$10\theta_0$	0.7702	0.6929	0.6301	0.5781	0.5343	0.4970	0.4648	0.4367	0.4119	0.3900

three empirical patterns: first, the bandwidth is tightened in response to higher uncertainty and this pattern is very robust to how effective bandwidth is estimated. The negative correlation is also qualitatively unchanged in response to a series of robustness tests. Second, central parity changes do not seem to affect the bandwidth, therefore justifying our modeling strategy that abstracts away from interaction between the bandwidth and central parity. Lastly, we find that the effect of uncertainty on the bandwidth is more pronounced during depreciation episodes.[24]

6.1. Uncertainty Measures

To empirically examine the correlation between bandwidth and uncertainty, we use five different uncertainty measures: First, we look at terms of trade uncertainty. Terms of trade shocks have been shown in the literature to be important for emerging market economies.[25] As illustrated in Section 5, we estimate the volatility of terms of trade shocks using a GARCH(1,1) model. Figure A.3 in Appendix A.2 shows that the trading band tends to narrow when terms of trade volatility increases.

Second, we consider foreign exchange market uncertainty. We look at the parallel foreign exchange market in Hong Kong, or the so-called Chinese Yuan in Hongkong (CNH) market, where the RMB is traded offshore against many other currencies without regulatory restrictions.[26] Specifically, we use the one-year RMB/USD NDF (nondeliverable forwards) rate in the Hong Kong offshore market, which is among the most active forward contracts. The data are from Thomson Reuters and span from September 2003 to February 2018. After taking the daily percentage change, we fit it into a GARCH(1,1) model and predict the variance. We smooth it using a 60-day rolling window. A negative correlation between CNH exchange rate volatility and bandwidth is evident in Figure A.4 in Appendix A.2.

Third, we consider global risk appetite, measured by the VIX index of the S&P500 using daily data from the Federal Reserve Economic Data (FRED) St.Louis dataset, which spans from January 2003 to February 2018. VIX is the implied volatility from S&P500 option prices. We use rolling windows of 60 days to compute moving averages of daily VIX-implied volatility. Figure 2 illustrates the negative correlation between the bandwidth and VIX.

Fourth, we use China's sovereign credit default swap (CDS) spread, which is a proxy for China-specific macroeconomic uncertainty.[27] We get daily time-series data from Bloomberg which spans from January 2003 to February 2018. We calculate percentage changes of the CDS spreads and fit them into a GARCH(1,1) model, and then predict the variance. We calculate the 60-day moving average of the volatility of CDS spreads. Figure A.5 in Appendix A.2 clearly shows the negative correlation between the bandwidth and the CDS spreads.

Lastly, we use the implied volatility of 3-month RMB/USD option prices (at the money options) in the Hong Kong offshore market following the method in

Jermann et al. (2019). The data are from Bloomberg. As evident from Figure A.6 in Appendix A.2, there is also a negative correlation between bandwidth and the implied volatility from RMB option prices. A summary of variable construction and data sources can be found in Table A.1 in Appendix A.2.

All the correlations between the bandwidth and various uncertainty measures above are negative and statistically significant. As all uncertainty measures show, the two obvious and well-known spikes of uncertainty are the 2008–2009 global financial crisis and the 2015 turmoil in China's financial markets. However, there are other variations in uncertainty measures, for example, uncertainty measures jumped in 2011–2013 due to heightened European sovereign debt default risk. We discuss this issue in detail in the robustness checks subsection.

6.2. Empirical Test

We now empirically test to what extent the exchange rate bandwidth is affected by various measures of uncertainty, among many other factors.[28] In the following regression, we denote the dependent variable, bandwidth, as w_t, and denote the key explanatory variable, the uncertainty measure, as $Uncertainty_t$. For the control variables, we first consider policy inertia, which is captured by 12-month moving average of bandwidth, denoted as $BandMA_t$. Another key control variable is the central parity. The percentage change of the central parity, for example, the central parity's appreciation or depreciation, is denoted as $CentralParity_t$. We also add an interaction term between uncertainty and exchange rate changes, which is denoted as $Uncertainty_t * ExRate_t$. We want to see whether uncertainty's effect on the bandwidth is symmetric. More precisely, we perform linear regressions of bandwidth on the uncertainty measures as follows:

$$w_t = \alpha + \beta_0 Uncertainty_t + \beta_1 BandMA_t + \beta_2 CentralParity_t + \beta_3 Uncertainty_t * ExRate_t + \epsilon_t. \quad (47)$$

Columns (1) and (2) in Table 5 show a significant negative correlation between bandwidth and uncertainty. For the five different measures of uncertainty, the coefficients are all negative and statistically significant.[29] This finding is consistent with the recent empirical work in Marconi (2018), which shows that the width of RMB/USD band is negatively related to the volatility of RMB/USD exchange rate. It is also consistent with the Central Bank's official mandate. Further, the bandwidth has strong inertia. Interestingly, the change of the level of RMB central parity does not have significant effects on the bandwidth, in most of the regressions. This lends support to our modeling strategy that abstracts away from interaction between the bandwidth and central parity. Moreover, we use two different RMB exchange rates to interact with the uncertainty measures: the Hongkong offshore RMB exchange rate and the central parity rate. We find that if we use the central parity's change to measure the currency depreciation/appreciation, the coefficient of the interaction term is not statistically significant. However, if we use the exchange rate in the Hong Kong offshore

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Table 5. Regression of exchange rate bandwidth on various uncertainty measures

Dependent variables	(1) Bandwidth	(2) Bandwidth	(3) Band (3M)	(4) Band (3M)	(5) Band (6M)	(6) Band (6M)
(a) Terms of trade uncertainty						
Terms of trade volatility	−1.13***	−1.04***	−0.81***	−0.60***	−0.74***	−0.46***
BandMA	1.03***	0.93***	0.82***	0.72***	0.91***	0.81***
CentralParity	−0.88	0.83	−0.32	0.23	1.04**	0.96
Uncertainty*CNH exchange rate change	−34.60***		−34.32***		−34.42***	
Uncertainty*Central parity change		−23.76**		−8.92		0.98
R^2	0.8144	0.8048	0.7241	0.7040	0.8420	0.8214
N	169	169	166	166	163	164
(b) Exchange rate uncertainty						
CNH volatility	−92.41***	−108.02**	−27.67***	−32.75***	−22.23***	−28.65***
BandMA	0.98***	0.96***	0.95***	0.94***	0.96***	0.95***
CentralParity	−0.08	−0.13	−0.08**	0.03	−0.03	−0.03
Uncertainty*CNH exchange rate change	−7.83***		−2.29***		−3.11***	
Uncertainty*Central parity change		65.17		−143.53*		−0.11
R^2	0.8587	0.8567	0.7523	0.7526	0.8241	0.8237
N	3013	3013	3013	3013	2989	2989
(c) Global uncertainty						
VIX	−0.005***	−0.005***	−0.001***	−0.002***	−0.0027***	−0.002***
BandMA	0.97***	0.95***	0.94***	0.93***	0.95***	0.94***
CentralParity	−0.07	−0.12	−0.08**	−0.02	−0.02	−0.04
Uncertainty*CNH exchange rate change	−0.0003***		−0.0001***		−0.0001***	
Uncertainty*Central parity change		0.002		−0.004		0.001
R^2	0.8381	0.8368	0.7485	0.7484	0.8228	0.8225
N	2978	2978	2933	2933	2904	2904

TABLE 5. Continued

Dependent variables	(1) Bandwidth	(2) Bandwidth	(3) Band (3M)	(4) Band (3M)	(5) Band (6M)	(6) Band (6M)
(d) China macroeconomic uncertainty						
China CDS volatility	−0.06***	−0.01***	−0.01***	−0.01***	−0.02***	−0.01***
BandMA	0.96***	0.96***	0.94***	0.94***	0.95***	0.95***
CentralParity	−0.09	−0.09	−0.08**	−0.08**	−0.03	−0.03
Uncertainty*CNH exchange rate change	−0.007***		−0.001**		−0.003***	
Uncertainty*Central parity change		−0.01		−0.003		−0.02
R^2	0.8320	0.8310	0.7484	0.7483	0.8211	0.8209
N	3063	3063	3018	3018	2989	2989
(e) Option price implied RMB volatility						
RMB option implied volatility	−0.03**	−0.08***	−0.02	−0.08***	−0.06***	−0.10***
BandMA	0.96***	0.92***	0.95***	0.88***	0.98***	0.94***
CentralParity	0.25**	0.02	0.01	0.24	0.03	−0.34
Uncertainty*CNH exchange rate change	−0.01***		−0.01***		−0.01**	
Uncertainty*Central parity change		0.03		−0.06		0.06
R^2	0.7527	0.7444	0.6831	0.6694	0.8061	0.8020
N	259	259	259	259	259	259

Note: *, **, *** mean the 10%, 5%, 1% level of significance, respectively.

market, the coefficient of the interaction term becomes negative and statistically significant.[30]

6.3. Robustness Checks

We calculate the continuous *De Facto* band that Central Bank allows the exchange rate to fluctuate, following Reinhart and Rogoff (2004)'s method. More specifically, we use the same methodology as in Section 5, if 80% of the daily absolute changes of the RMB exchange rate during the past two years lie within the band, we will use that band as our *De Facto* continuous exchange rate bandwidth. Since we have daily data, we try different rolling windows, for example, 3 months, 6 months, 12 months, and 24 months. They are depicted in Figure A.7 in Appendix A.2.

Columns (3), (4), (5), and (6) in Table 5 report regression results using 3-month and 6-month rolling windows for continuous *De Facto* bands. In general, the coefficients on uncertainty are all negative and statistically significant. The bandwidth has strong inertia in all samples. The change of RMB central parity does not have significant effects on the bandwidth. For the interaction terms, we also get robust results: if we use the exchange rate in the Hong Kong market, the coefficient on the interaction term becomes negative and statistically significant. This shows that uncertainty will have a larger effect on the bandwidth when there is currency depreciation.

We then examine whether the negative correlation between RMB exchange rate bandwidth and uncertainty measures relies on two special episodes: (1) the global financial crisis period from 2008:Q4 to 2009:Q1 that has been mostly used in the literature and (2) the RMB reform in August of 2015 and the large RMB depreciation together with PBOC intervention afterwards (from August 11, 2015 until the end of 2015). We drop the two special episodes and conduct the same empirical tests as Table 5. The results are presented in Table A.2 in Appendix A.2. The key empirical findings remain robust: in most cases, there is negative correlation between exchange rate bandwidth and uncertainty measures, and this negative correlation is a robust feature of RMB policies. However, when uncertainty is measured by the RMB Option Implied Volatility, the relationship becomes insignificant. Our results are robust because in our sample period we can observe that the RMB exchange rate was maintained in a narrow band after the 2008–2009 global financial crisis and was expanded slowly as the uncertainty further faded in the second half of 2010. As another example, the bandwidth was actually narrowed down before 2015 RMB exchange rate reform, especially in 2014 when the Federal Reserve started its "taper tantrum" and uncertainty began to rise again.[31]

As another robustness test, we use two different uncertainty measures: (1) the economic policy uncertainty as Baker et al. (2016) and (2) the uncertainty measure in Jurado et al. (2015). Our main empirical results remain robust to these alternative uncertainty measures, as shown in the Table A.3 in Appendix A.2. The effect of uncertainty on the RMB exchange rate bandwidth is negative and statistically significant.

Table 6. One standard deviation increase in uncertainty measures

	Bandwidth (%)	Band (3M) (%)	Band (6M) (%)
Option price implied volatility	− 9.76	− 9.07	−33.63
Terms of trade uncertainty	−11.58	−15.64	−13.57
CNH uncertainty	−13.07	− 7.51	− 5.60
VIX	− 7.12	− 4.07	− 4.72
China CDS volatility	− 5.79	− 2.50	− 4.13

Finally, since the exchange rate bandwidth is highly persistent, standard linear regression techniques might not be the optimal way for such examination. In the main empirical specifications, the 12-month moving average of the bandwidth is included in the control variables. Moreover, we use the block-bootstrap method to deal with autocorrelation. The empirical results from the block-bootstrap are presented in Table A.4 in Appendix A.2. As evident from the results, in most of the cases, the negative correlation between bandwidth and uncertainty is robust.

6.4. Economic Significance

We also calculate the economic significance of uncertainty measures' effect on the *De Facto* bandwidth. Since our measures of uncertainty have different units, we calculate the one standard deviation increase in these measures to see how that change will affect the Central Bank's bandwidth choice. The results are shown in Table 6. It reports the percentage change of the bandwidth, which are calculated using the discrete bandwidth, as well as the continuous bands from 3-month and 6-month rolling windows. We put them in the order of economic significance. Option price implied RMB exchange rate volatility and terms of trade volatility seem to the be most important uncertainty measures. For example, one standard deviation increase in terms of trade shock is associated with about 15.64% and 13.57% decrease in the bandwidth for the 3M and 6M measure of *De Facto* continuous bands, respectively. On the other hand, China's CDS volatility seems to have a smaller effect on the bandwidth choices.

6.5. Comments on Currency Basket

This paper focuses on the daily trading band imposed by the Central Bank. In China's case, it is the trading band of RMB/USD. The US dollar is the single most important currency for China in terms of international trade and finance. Ilzetzki et al. (2017) recently finds China still uses the US dollar as the anchoring currency. However, it is interesting to check the band for a currency basket. Frankel and Wei (2007) have the detailed discussion where they estimated both the currency basket and flexibility of Chinese exchange rate regime. We follow Frankel and Wei (2007)'s method to get the floating band to the major currency basket

https://doi.org/10.1017/S1365100520000383 Published online by Cambridge University Press

(see Appendix A.3) and the negative relation between currency basket bands and uncertainty measures remains robust.

7. CONCLUSION

This paper studies how uncertainty influences optimal exchange rate target zones. In recent years, understanding China's exchange rate policy is a key global monetary issue since China has stepped up its efforts to internationalize the RMB.[32] We first document the stylized facts for China's daily exchange rate target zone. We show that increased uncertainty causes the PBOC to narrow the daily RMB/USD exchange rate band. Using real options theory, we find that two opposing effects influence the choice of bandwidth: (1) a permanent increase in the level of uncertainty increases the value of the "wait and see" component, which widens the target zone band and (2) when volatility is stochastic, temporary increases in uncertainty cause the Central Bank to *narrow* the band. Our calibration results show that a stochastic volatility target zone model matches the dynamics of China's exchange rate trading band well, except during the initial experimental period when the PBOC just started liberalizing its exchange rate regime. We provide empirical evidence that terms of trade shocks correlate negatively with the size of daily target zone choice. We also systematically test the relationship between various uncertainty measures and the bandwidth.

We would like to highlight that our analysis leaves scope for future research along two important dimensions. First, it is important to study how the dynamics of the central parity, studied by Jermann et al. (2019), interact with the dynamics of the bandwidth. Specifically, the asymmetric interaction between bandwidth and exchange rate appreciation/depreciation should be carefully examined. This could be done by introducing dynamics into the mean of the uncontrolled exchange rate process. Second, while we have documented robust empirical evidence for a strong negative relationship between RMB exchange rate bandwidth and various uncertainty measures, a creative empirical identification strategy is still needed in future empirical work to distinguish the "wait and see" and "fear of floating" components and formally establish how these two channels work separately in the data.

NOTES

1. See, for example, Ilzetzki et al. (2017) and Han and Wei (2018).

2. We are not the first to study the effect of stochastic volatility in a small open economy setting. Fernández-Villaverde et al. (2011) show that in emerging markets, stochastic volatility of real interest rates significantly contributes to fluctuations in these countries. Mumtaz and Theodoridis (2015) study the international transmission of volatility shocks.

3. Marconi (2018) finds that the width of RMB/USD band is negatively related to volatility of the RMB/USD exchange rate, but no formal theory is provided to explain it. A target zone is also consistent with the PBOC's official mandate, which emphasizes that one aim of its monetary policy is to keep the RMB exchange rate stable within a reasonable band around a gradually evolving central parity.

4. The bandwidth in Panel B of Figure 1 in most of the sample period is as the same as Jermann et al. (2019). Two notable differences are: (1) from the end of 2008 to June 2010, China repegged to the dollar with a very narrow band of less than 0.1%, as announced by former Governor Zhou Xiaochuan and (2) after August 11 2015, and the reform of the central parity, the PBOC implemented an effective band of 0.8% based on our estimation, similar in magnitude to Jermann et al. (2019)'s band of 0.5%. Appendix A.1 provides further institutional details.

5. Although the central parity plays an important role in Chinese exchange rate policy, we show in the empirical part of the paper that the central parity does not have a statistically significant influence on the width of the exchange rate band, but uncertainty does.

6. As Frankel and Wei (2007) argues, "The issue of the regime governing the Chinese exchange rate, and specifically whether the currency is moving away from the *De Facto* peg that for 10 years had tied it to the US dollar, is much more than just another application, to a particular country, of the long-time question of fixed versus floating exchange rates. It is a key global monetary issue."

7. Galí (2015) provides a textbook review.

8. Obviously, China is *not* a small open economy. For example, Eickmeier and Kühnlenz (2018) documents the importance of China in the global inflation. However, relaxing this assumption raises substantial complications. For example, in the presence of other large economies (e.g., the USA), it would require analysis of strategic interaction. Still, the domestic economy remains a monopolistic supplier of its own domestic good, which has non-negligible weight in domestic utility. Hence, the domestic Central Bank has an incentive to manipulate its own terms of trade, as in Costinot et al. (2014).

9. A large literature attempts to measure the "cost of foreign exchange intervention," and the PBOC officially acknowledges the costs associated with its exchange rate intervention. However, this literature is based on UIP deviations. (Chang et al. (2015), Fanelli and Straub (2016), and Adler and Mano (2016) provide recent examples). Here, there are no UIP deviations, and interventions take the form of (high frequency) unsterilized changes in domestic monetary policy. Hence, the costs here are more accurately interpreted as microstructural transaction costs, like bid/ask spreads. Not surprisingly then, our calibrated costs turn out to be much smaller than UIP-based intervention costs.

10. The explicit expressions are: $a_{11} = \frac{1-\beta}{\kappa}$, $b_{11} = -\frac{1}{\kappa}$, $a_{22} = 1$, $b_{22} = 0$.

11. A Dickey–Fuller test cannot reject the null hypothesis that China's monthly inflation rate has a unit root, which is consistent with the findings of Zhang (2017), who estimates a nonlinear new Keynesian using Chinese data.

12. In practice, there are many tools that the PBOC uses to change the exchange rate. Although the interest rate is perhaps the most prominent one, the PBOC also uses tools such as capital controls and sterilized intervention. In fact, capital controls have historically been a major tool of exchange rate intervention in China, but these too have costs. Chang et al. (2015) argue that the main challenge the PBOC faced after the financial crisis is the trade-off between price stability and costly sterilization. Due to the safe haven status of the US dollar, interest rates on US dollar reserves have been much lower than interest rates on domestic asset, which imposes a financing cost on the PBOC. He and Luk (2017) provide a model of China's capital account liberalization, and Montecino (2018) discusses how capital controls can influence the exchange rate, both in the short term and long term. Kim and Pyun (2018) and Han and Wei (2018) provide empirical evidence showing that capital account openness plays an important role in the international transmission of monetary policy shocks.

13. Given the recursive structure of the problem, henceforth time subscripts are eliminated, unless necessary.

14. Note, this problem is identical to an inventory or cash management problem. For a detailed derivation of the optimality conditions, see Harrison and Taksar (1983).

15. The smooth-pasting and higher-order contact conditions for the infinitesimal control problem are derived in Dumas (1991).

16. The idea of embedding CIR stochastic volatility into asset pricing models comes from Heston (1993).

17. Where $a_i = \beta_i + \kappa q_i - \xi(\frac{1}{4\beta_i^2} - \frac{1}{2\beta})$ and $b_i = \frac{\kappa}{2}p_i + \xi(\frac{1}{4\beta_i} - \frac{1}{2})$.

18. This means we lose one year of data, so the starting month is January 2001.

19. Following Gali and Monacelli (2005), we consider the case of log utility ($\sigma = 1$), unit elasticity of substitution between domestic and foreign goods ($\eta = 1$), and unit elasticity of substitution between domestic and foreign goods ($\gamma = 1$).

20. The same methodology of computing a *De Facto* band is used in Reinhart and Rogoff (2004).

21. Klein and Shambaugh (2012) survey this literature.

22. The Appendix contains the same plots for the 3-month and 1-year bands.

23. See Obstfeld (2006) and Prasad et al. (2005) for more detailed discussion of the PBOC's policy stance in the early-reform period, and Brunnermeier et al. (2017) for a more general introduction to Chinese policy makers' usual "crossing the river by touching the stone" practice.

24. It would be interesting to expand our model to accommodate this asymmetric pattern in future research.

25. See Schmitt-Grohé and Uribe (2018) and the references therein.

26. For instance, there are neither daily trading bands nor central parities in the CNH market. Therefore, one can therefore regard it as a hypothetical RMB exchange rate if the PBOC's band restrictions were lifted.

27. Sovereign default risk is an important measure related to macroeconomic and financial uncertainty. See, for example, Arellano (2008) and Qian et al. (2017).

28. We use daily data for most regressions but use monthly data for terms of trade uncertainty.

29. Note that this seems to be a robust feature of the PBOC's exchange rate policy. However, the empirical pattern is about correlation rather than causality. In the model, we discuss the "fear of floating" versus "wait and see" motives, and the empirical results claim that the negative correlation between bandwidth and uncertainty demonstrates that "fear of floating" concerns denominate "wait and see" concerns, at least in China's case.

30. One possible explanation is that while the central parity is tightly controlled by the PBOC, the exchange rate in the Hong Kong offshore market is less manipulated and can therefore serve as a better measure of currency depreciation/appreciation. This asymmetric interaction between bandwidth and exchange rate level in the offshore market merits further investigation.

31. For institutional details and in-depth analysis of the evolution of China's exchange rate regimes, see the recent work by Das (2019).

32. For recent studies on RMB internationalization, see Lu and Liu (forthcoming) and Qian et al. (2019).

33. Variables denoted in small letters below all implies percentage deviation from the steady state, except interest.

34. We derive proof on the upper barrier, but the argument applies to the lower barrier too.

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APPENDIX A

A.1. INSTITUTIONAL BACKGROUND OF CHINA'S FX REGIMES AND ITS EVOLUTION

China's government has a long tradition of inward-looking and believes in pragmatism. Thus, China only gradually added market force into the determination of its exchange rate and changed the width of the RMB exchange rate trading band. Over the past two decades, China took several reforms and experiments on its exchange rate policy. In this section, we lay out the brief history of RMB exchange rate policy, with the focus on the justification of the bandwidth we use in Figure 1. Figure A.1 plots the times series of bilateral RMB/USD exchange rate with the major reforms. For more details of China's foreign exchange regime, see Sun (2016) and Yu et al. (2017).

Period 1: July 2005 to End of 2008

The RMB had been effectively pegged to the US dollar at the rate of 8.28 from 1997 until 2005. The exchange rate reform on July 21, 2005 aimed to increase the flexibility of RMB exchange rate but at a tightly controlled pace. The PBOC set the daily *De Jure* target band of ±0.3%. In the official announcement, PBOC said "...the daily trading price of the US dollar against the RMB in the interbank foreign exchange market will be allowed to float within a band of ±0.3% around the central parity published by PBOC". And the PBOC "will make adjustment of the RMB exchange rate band when necessary according to market development as well as the economic and financial situation" and "maintain the RMB exchange rate basically stable at an adaptive and equilibrium level so as to promote

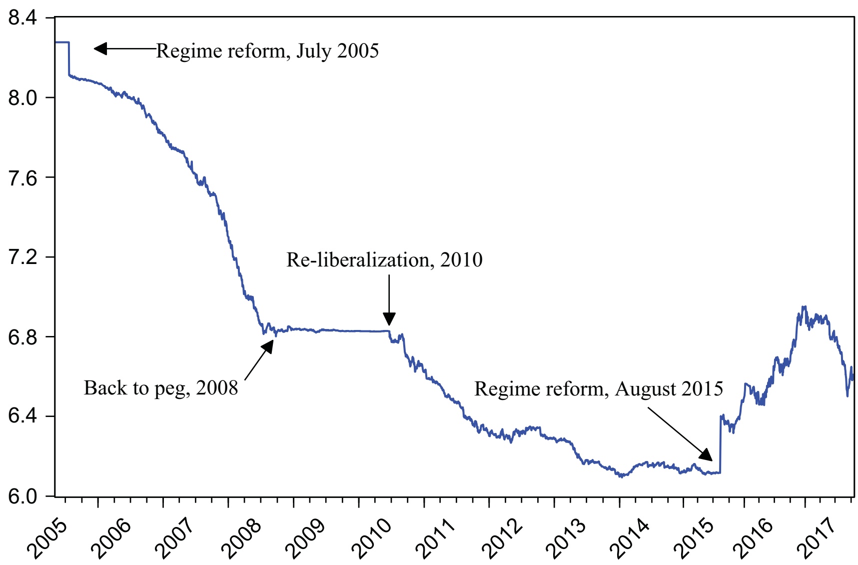

FIGURE A.1. China's exchange rate regime reforms.

https://doi.org/10.1017/S1365100520000383 Published online by Cambridge University Press

the basic equilibrium of balance of payment and safeguard macroeconomic and financial stability." On May 21 2007, PBOC further expanded the daily *De Jure* target band from ±0.3% to ±0.5%.

Period 2: End of 2008 to June 2010

As the global financial crisis peaked in the summer of 2008, China became worried about the volatile exchange rate and its potential negative effects on the economy. At the end of 2008, China started to *De Facto* repeg to the US dollar at 6.82 within a very narrow band less than ±0.1%. Based on Hu and Zhang (2012) and Shuo et al. (2016), governor Zhou Xiaochuan said in public in early 2009 that "the choice of RMB's repeg to US dollar is a response to the financial crisis which generated large increase in the uncertainty". Therefore, the effective trading band set by PBOC is much narrower than the *De Jure* band of ±0.5%. Many studies have found that RMB pegged to US dollar at that period, including Klein and Shambaugh (2012) and Marconi (2018). In a recent academic paper by Jermann et al. (2019), they study the formation of RMB central parity and argue on Page 4 that "...width of the trading band has been much smaller than the announced width. For example, during the financial crisis between mid 2008 and mid 2010, the RMB was essentially re-pegged to the US dollar." Note that in some periods the PBOC did not seem to make the formal statement about trading band changes but it is admitted by PBOC governors/senior officers that they actually use the narrower band. This is just one of these periods.

Period 3: June 2010 to August 2015

The global financial crisis was over in 2009, but the fixed exchange rate was maintained until the June of 2010 as more information have arrived showing that the uncertainty has faded. On June 19, 2010, China let RMB exchange rate to regain some flexibility with the daily target band 0.5%, back to the precrisis level. But during that time, European sovereign debt crisis was still looming and China did not further expand the band when they still face significant uncertainty. In 2012, market forces of demand and supply were more balanced and the RMB exchange rate started to present two-way fluctuations, and therefore, the PBOC expanded the target band to 1%. In 2014, it further widened the target zone from 1% to 2%.

Period 4: August 2015 to Current

On August 11, 2015, PBOC implemented a new reform to improve the mechanism to set the central parity. Around the same period, Fed started to quit quantitative easing and raised the interest rate. The PBOC started to implement an effective narrower band in face of large RMB depreciation pressure. Again, even though PBOC did not make the official statement about trading band changes, it is admitted by the quarterly monetary policy report by PBOC and some researchers. In the third and fourth quarter of monetary policy report in 2015 PBOC admit that they provide dollar liquidity to restrict RMB depreciation and maintain RMB exchange rate in a relatively stable zone. Jermann et al. (2019) argue on Page 4 that "...since August 11, 2015, the band around the central parity has effectively been limited to 0.5%." We get the high-frequency trading data from China Foreign Exchange Trading System (CFETS) and find that for this period the trading band set and maintained by PBOC is 0.8%.

A.2. DATA, VARIABLES, TABLES, AND FIGURES

The section shows the description of data and their sources, as well as extra tables and figures. Table A.1 summarizes the variable construction and data sources. Table A.2, Table A.3, and Table A.4 provide extra empirical results. Figure A.2 shows the terms of trade fluctuations in China calculated as YoY changes. Figure A.3 (Figure A.4) illustrates the relation between bandwidth and terms of trade uncertainty (exchange rate uncertainty). Figure A.5 depicts the relationship between China CDS spreads and the bandwidth in which the negative correlation is apparent. We calculate the option price implied volatility using the 3-month at-the-money options and illustrate it in Figure A.6. Notice that the option price time series are shorter, starting from the year of 2011. Figure A.7 shows the De Facto continuous bands using 3-month, 6-month, 1-year, and 2-year rolling windows, respectively.

A.3. THE CURRENCY BASKET BAND

In the main text, we consider the RMB/USD exchange rate. While it makes a lot of sense to focus on the RMB/USD exchange rate, it is also important and interesting to look at the basket of currencies that RMB is pegged to. As the officially announced in PBOC's statement, it attempted to stabilize the RMB exchange rate against a basket of currencies. In this section, we calculate the band of RMB exchange rate to the basket of currencies.

The calculation consists of two steps: in the first step, since the weight of China's currency basket is opaque, we need to estimate the weights. In the second step, we calculate the distance between RMB exchange rate relative to this currency basket.

We estimate the weight of currency basket following Frankel and Wei (2007) and Frankel (2009). The four currencies are US dollar, Euro, Japanese Yen, and Korea Won. We estimate the following equation:

$$dlog(CNY) = \alpha_1 dlog(USD) + \alpha_2 dlog(EUR) + \alpha_3 dlog(JPY) + \alpha_4 dlog(KRW). \qquad \textbf{(A1)}$$

We focus on the data period from September 2003 to August 2015. In this period, China announced that its exchange rate will be kept stable relative to a basket of currencies. For the baseline currency, we follow the literature to use the Special Drawing Right. Based on the institutional background, we divide the full sample into four subperiods: September 2003 to July 2005 when RMB is strictly pegged to USD, so we are interested in the horizontal band; July 2005 to October 2008 when PBOC started to use a wider band and let the central parity to gradually appreciated, so we need to calculate the crawling bands; October 2008 to June 2010 when China narrow the band and got back to pegging to USD; June 2010 to August 2015 when China enlarge the trading band and let the central parity to crawl. We get different weights in different subperiods. We also detrend the central parity's change.

We find that the floating band around the major currency basket is similar to Sun (2010). By stabilizing RMB/USD exchange rate, the PBOC indirectly stabilizes the RMB exchange relative to the currency basket. Obstfeld (2006) points out this policy objective of China's monetary authority. We also confirm the negative correlation between currency basket bands and uncertainty measures.

A.4. AN OPEN ECONOMY NEW KEYNESIAN SETTING

Let us consider a small open economy model with money, imperfect competition, and nominal price rigidity. Consumption goods are consumed and traded across countries as

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TABLE A.1. Variable construction and data source

Name	Description	Source
RMB/USD central parity rate	Central parity announced by PBOC	PBOC
RMB/USD closing rate	The last traded price for RMB/USD in Shanghai	China Foreign Exchange Trading System (CFETS)
Discrete band	The upper and lower band set and maintained by PBOC	PBOC and author's calculation
De Facto continuous band	Use different rolling windows as Reinhart and Rogoff (2004)	Author's calculation
Terms of trade	Ratio of export vs Import prices	China National Bureau of Statistics
VIX	Implied volatility from option price of SP500	FRED St.Louis Federal Reserve
RMB Hongkong exchange rate	1 year RMB NDF prices	Thomson Reuters
China CDS spread	5-year credit default swap spreads	Bloomberg
RMB option prices	3 month at the money option	Bloomberg

TABLE A.2. Regression of RMB exchange rate bandwidth on various uncertainty measures (excluding two special episodes: 2008–2009 global financial crisis period and RMB reform in late 2015)

Dependent variables	(1) Bandwidth	(2) Bandwidth	(3) Band (3M)	(4) Band (3M)	(5) Band (6M)	(6) Band (6M)
(a) Term of trade uncertainty						
Term of trade volatility	−0.701***	−0.588**	−0.841***	−0.537***	−0.913***	−0.561***
BandMA	1.100***	1.036***	0.818***	0.723***	0.886***	0.774***
CentralParity	−0.472	0.385	0.220	0.266	0.855*	1.132
Uncertainty*CNH exchange rate change	−22.691***		−33.659***		−39.891***	
Uncertainty*Centra parity change		−12.072		−1.445		−2.422
Observations	158	158	155	155	152	152
R-squared	0.882	0.877	0.732	0.712	0.822	0.793
(b) Exchange rate uncertainty						
CNH volatility	−128.090***	−86.373***	−112.233***	−71.576***	−206.971***	−141.075***
BandMA	1.072***	1.043***	1.061***	1.022***	1.002***	0.936***
CentralParity	0.008	0.046	−0.042	−0.049	−0.023	0.015
Uncertainty*CNH exchange rate change	−14.372***		−14.531***		−24.381***	
Uncertainty*Central parity change		−78.925		−9.983		−84.674
Observations	2822	2822	2822	2822	2798	2798
R-squared	0.911	0.907	0.798	0.792	0.817	0.801
(c) Global uncertainty						
VIX	−0.004***	−0.004***	−0.004***	−0.003***	−0.005***	−0.004***
BandMA	1.065***	1.044***	1.064***	1.026***	0.981***	0.944***

TABLE A.2. Continued

Dependent variables	(1) Bandwidth	(2) Bandwidth	(3) Band (3M)	(4) Band (3M)	(5) Band (6M)	(6) Band (6M)
CentralParity	0.014	0.057	−0.044	0.012	−0.024	−0.022
Uncertainty*CNH exchange rate change	−0.000***		−0.000***		−0.000***	
Uncertainty*Central parity change		−0.003		−0.004		−0.001
Observations	2791	2791	2746	2746	2717	2717
R-squared	0.908	0.907	0.793	0.790	0.792	0.789
(d) China macroeconomic uncertainty						
China CDS volatility	−0.112***	−0.013***	−0.066***	−0.005***	−0.063***	−0.005***
BandMA	1.056***	1.053***	1.040***	1.036***	0.959***	0.956***
CentralParity	−0.005	−0.003	−0.054	−0.050	−0.038	−0.034
Uncertainty*CNH exchange rate change	−0.013***		−0.008***		−0.008***	
Uncertainty*Central parity change		−0.014		−0.017		−0.018
Observations	2872	2872	2827	2827	2798	2798
R-squared	0.907	0.906	0.789	0.788	0.787	0.786
(e) Option price implied RMB volatiliy						
RMB option implied volatility	−0.010	−0.056***	0.009	−0.066***	−0.068***	−0.104***
BandMA	1.094***	1.058***	1.053***	0.966***	0.943***	0.907***
CentralParity	0.128**	−0.047	0.142	−0.147	0.046	−0.292
Uncertainty*CNH exchange rate change	−0.011***		−0.017***		−0.009**	
Uncertainty*Central parity change		0.025		0.041		0.060
Observations	247	247	247	247	247	247
R-squared	0.861	0.853	0.732	0.712	0.782	0.777

Note: *, **, *** mean the 10%, 5%, 1% level of significance, respectively.

TABLE A.3. Regression of RMB exchange rate bandwidth on alternative uncertainty measures

Dependent variables	(1) Bandwidth	(2) Bandwidth	(3) Band (3M)	(4) Band (3M)	(5) Band (6M)	(6) Band (6M)
(a) Alternative uncertainty measure from Baker et al. (2016)						
Economic policy uncertainty	−0.000	−0.001**	−0.001***	−0.002***	−0.002***	−0.002***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
BandMA	1.110***	1.078***	0.823***	0.797***	0.883***	0.853***
	(0.044)	(0.047)	(0.058)	(0.059)	(0.037)	(0.038)
CentralParity	−0.437	−1.613	0.029	1.350	0.603	1.275
	(0.431)	(1.080)	(0.512)	(1.611)	(0.489)	(1.464)
Uncertainty*CNH exchange rate change	−0.012*		−0.009		−0.009	
	(0.006)		(0.007)		(0.006)	
Uncertainty*Central parity change		0.008		−0.011		−0.004
		(0.008)		(0.012)		(0.012)
Observations	159	159	156	156	153	153
R-squared	0.880	0.878	0.737	0.736	0.827	0.819
Adjusted R2	0.877	0.875	0.730	0.729	0.822	0.814

Table A.3. Continued

Dependent variables	(1) Bandwidth	(2) Bandwidth	(3) Band (3M)	(4) Band (3M)	(5) Band (6M)	(6) Band (6M)
(b) Alternative uncertainty measure from Jurado et al. (2015)						
Macro uncertainty	−1.046***	−0.802***	−0.806***	−0.480**	−0.716***	−0.347**
	(0.252)	(0.188)	(0.196)	(0.198)	(0.165)	(0.169)
BandMA	1.094***	1.014***	0.778***	0.709***	0.842***	0.766***
	(0.041)	(0.039)	(0.066)	(0.062)	(0.047)	(0.044)
CentralParity	−0.370	2.873	0.146	−3.876	0.775	−3.763
	(0.419)	(3.316)	(0.547)	(4.402)	(0.533)	(3.812)
Uncertainty*CNH exchange rate change	−3.386***		−2.891**		−3.235***	
	(0.963)		(1.146)		(1.084)	
Uncertainty*Central parity change		−5.605		6.023		7.051
		(4.941)		(6.546)		(5.658)
Observations	159	159	156	156	153	153
R-squared	0.891	0.884	0.724	0.714	0.804	0.791
Adjusted R2	0.889	0.881	0.716	0.707	0.799	0.785

Note: *, **, *** mean the 10%, 5%, 1% level of significance, respectively.

TABLE A.4. Regression of RMB exchange rate bandwidth on various uncertainty measures: Block-bootstrap method

Dependent variables	(1) Bandwidth	(2) Bandwidth	(3) Band (3M)	(4) Band (3M)	(5) Band (6M)	(6) Band (6M)
(a) Exchange rate uncertainty						
CNH volatility	−128.090***	−86.373***	−112.233*	−71.576	−206.971***	−141.075**
	(30.648)	(28.283)	(62.334)	(45.010)	(65.184)	(64.216)
BandMA	1.072***	1.043***	1.061***	1.022***	1.002***	0.936***
	(0.034)	(0.029)	(0.081)	(0.104)	(0.071)	(0.082)
CentralParity	0.008	0.046	−0.042	−0.049	−0.023	0.015
	(0.035)	(0.075)	(0.037)	(0.066)	(0.029)	(0.076)
Uncertainty*CNH exchange rate change	−14.372***		−14.531*		−24.381***	
	(3.881)		(8.328)		(7.305)	
Uncertainty*Central parity change		−78.925		−9.983		−84.674
		(83.331)		(90.767)		(122.826)
Observations	2822	2822	2822	2822	2798	2798
R-squared	0.911	0.907	0.798	0.792	0.817	0.801
(b) Global uncertainty						
VIX	−0.004***	−0.004**	−0.004	−0.003	−0.005***	−0.004**
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
BandMA	1.065***	1.044***	1.064***	1.026***	0.981***	0.944***
	(0.033)	(0.035)	(0.060)	(0.073)	(0.073)	(0.090)
CentralParity	0.014	0.057	−0.044	0.012	−0.024	−0.022
	(0.044)	(0.075)	(0.033)	(0.087)	(0.035)	(0.048)
Uncertainty*CNH exchange rate change	−0.000*		−0.000		−0.000**	
	(0.000)		(0.000)		(0.000)	
Uncertainty*Central parity change		−0.003		−0.004		−0.001
		(0.004)		(0.006)		(0.003)
Observations	2791	2791	2746	2746	2717	2717
R-squared	0.908	0.907	0.793	0.790	0.792	0.789

TABLE A.4. Continued

Dependent variables	(1) Bandwidth	(2) Bandwidth	(3) Band (3M)	(4) Band (3M)	(5) Band (6M)	(6) Band (6M)
(c) China macroeconomic uncertainty						
China CDS volatility	−0.112***	−0.013	−0.066	−0.005	−0.063**	−0.005
	(0.042)	(0.008)	(0.042)	(0.005)	(0.028)	(0.006)
BandMA	1.056***	1.053***	1.040***	1.036***	0.959***	0.956***
	(0.039)	(0.038)	(0.082)	(0.082)	(0.086)	(0.089)
CentralParity	−0.005	−0.003	−0.054	−0.050	−0.038	−0.034
	(0.044)	(0.043)	(0.035)	(0.036)	(0.030)	(0.034)
Uncertainty*CNH exchange rate change	−0.013**		−0.008		−0.008**	
	(0.005)		(0.007)		(0.004)	
Uncertainty*Central parity change		−0.014		−0.017		−0.018
		(0.026)		(0.021)		(0.035)
Observations	2872	2872	2827	2827	2798	2798
R-squared	0.907	0.906	0.789	0.788	0.787	0.786
(d) Option price implied RMB volatiliy						
RMB option implied volatility	−0.010	−0.056***	0.009	−0.066***	−0.068**	−0.104***
	(0.018)	(0.012)	(0.029)	(0.024)	(0.028)	(0.027)
BandMA	1.094***	1.058***	1.053***	0.966***	0.943***	0.907***
	(0.055)	(0.056)	(0.103)	(0.097)	(0.086)	(0.089)
CentralParity	0.128*	−0.047	0.142	−0.147	0.046	−0.292
	(0.074)	(0.433)	(0.124)	(0.450)	(0.156)	(0.501)
Uncertainty*CNH exchange rate change	−0.011**		−0.017**		−0.009**	
	(0.005)		(0.007)		(0.004)	
Uncertainty*Central parity change		0.025		0.041		0.060
		(0.081)		(0.089)		(0.120)
Observations	247	247	247	247	247	247
R-squared	0.861	0.853	0.732	0.712	0.782	0.777

Note: *, **, *** mean the 10%, 5%, 1% level of significance, respectively.

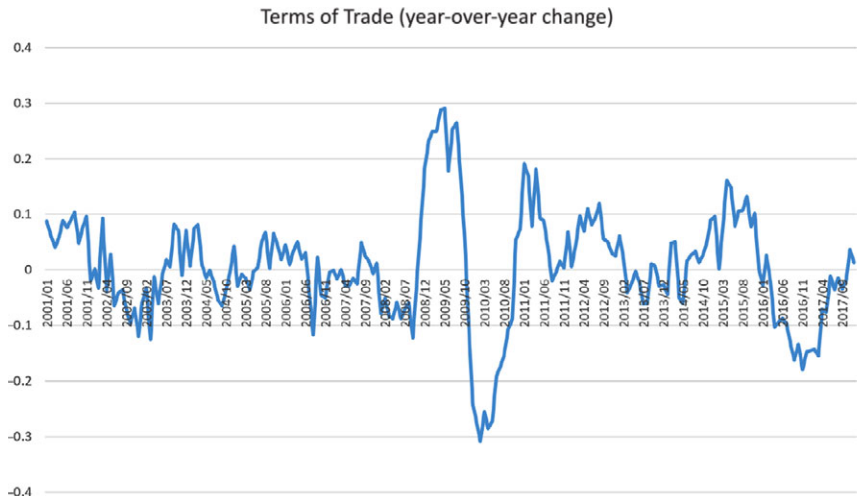

FIGURE A.2. China's terms of trades.

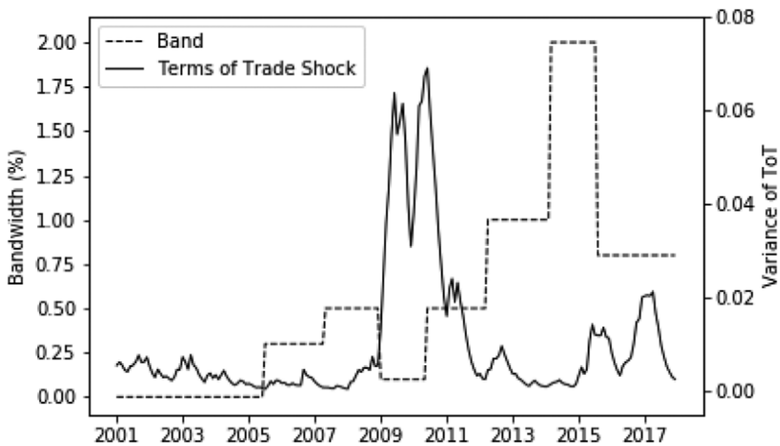

FIGURE A.3. Terms of trade volatility and China's exchange rate band.

in Galí (2015) and Clarida et al. (2001). Households consume domestic and foreign goods that are imperfect substitutes. The consumption goods are a composite of a continuum of differentiated goods, each produced by an associated monopolistically competitive firms. The home economy is small, in the sense that it does not affect world output, world price, and world interest rate. There is perfect risk sharing in consumption risk internationally.

The log linear form of consumption is denoted by.[35]

$$c_t = (1 - \hat{\alpha})c_t^h + \hat{\alpha}c_t^f, \tag{A2}$$

where the parameter $\hat{\alpha} \in [0, 1]$ is (inversely) related to the degree of home bias of consumption goods preference. Higher α implies a higher degree of openness. Now define the

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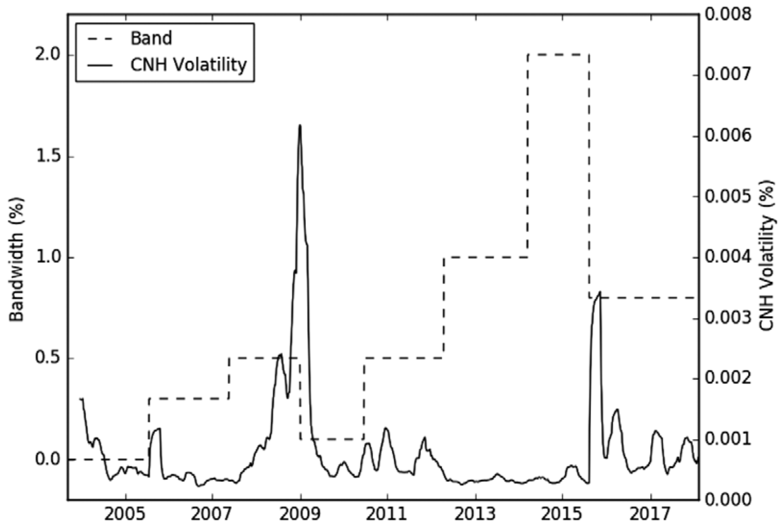

FIGURE A.4. CNH volatility and China's exchange rate band.

FIGURE A.5. Bandwidth and China's CDS spreads.

effective terms of trade S_t as a composite of a continuum of import goods to export goods price ratio. Let i be the index of countries, that is,

$$S_t = \frac{P_{F,t}}{P_{H,t}} = \left(\int_0^1 S_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}, \tag{A3}$$

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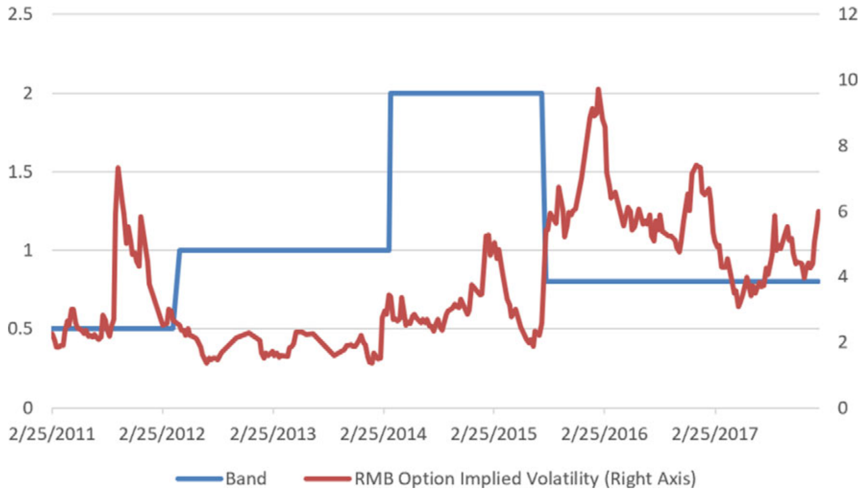

FIGURE A.6. Bandwidth and RMB option implied volatility.

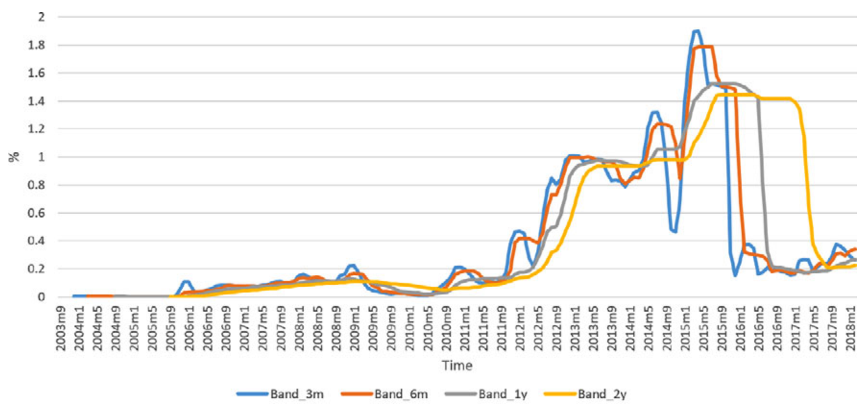

FIGURE A.7. *De Facto* continuous band using different rolling windows.

where γ governs the degree of substitution across goods from different countries, which can be approximated up to first order by the log-linear expression

$$s_t = \int_0^1 s_{i,t} di = p_{F,t} - p_{H,t}, \tag{A4}$$

where s_t becomes the log of the effective terms of trade, where $p_{F,t} = \log P_{F,t}, p_{H,t} = \log P_{H,t}$. With law of one price, we have

$$s_t = e_t + p_t^* - p_{H,t}, \tag{A5}$$

where e_t is the log of a composite of bilateral nominal exchange rate, and p_t^* denotes the log of foreign price index. Note that in our fixed price economy, assuming that domestic and foreign prices start at the same level, there is then no distinction between domestic, foreign,

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or world price index. That is, the overall inflation level in the home country equals domestic inflation, adjusted by "imported inflation." This allows to write down the consumption Euler equation as follows:

$$c_t = \mathbb{E}_t c_{t+1} - \frac{1}{\sigma}[i_t - \mathbb{E}_t(\pi_{H,t} + \alpha \mathbb{E}_t \Delta s_{t+1})], \quad \textbf{(A6)}$$

where σ is the coefficient of relative risk aversion, $\pi_{H,t} = p_{H,t} - p_{H,t-1}$. With imports and exports, we need to adjust the relation between aggregate domestic output with aggregate domestic consumption. As in Galí (2015), we have $x_t = c_t + \frac{\hat{\alpha} w}{\sigma} s_t$, where x_t denotes log output, $w = \sigma\gamma + (1 - \hat{\alpha})(\sigma\eta - 1)$. Using the same parameters as in Galí (2015), we have $w = 1$.

With complete international financial markets, UIP holds, that is:

$$i_t - i_t^* = \mathbb{E}_t \Delta(e_{t+1}). \quad \textbf{(A7)}$$

Expanding this with the definition of nominal exchange rate and terms of trade, one get

$$s_t = (i_t^* - \mathbb{E}_t \pi_{t+1}^*) - (i_t - \mathbb{E}_t \pi_{H,t+1}) + \mathbb{E}_t(s_{t+1}). \quad \textbf{(A8)}$$

Combining this with the consumption euler equation, we get essentially equation (1).

Under complete markets, one can get the relation between domestic and world consumption, that is,

$$c_t = c_t^* + (\frac{1 - \hat{\alpha}}{\sigma}) s_t. \quad \textbf{(A9)}$$

Again, combining this with $x_t = c_t + \frac{\hat{\alpha} w}{\sigma} s_t$, we get $s_t = \sigma_\gamma x_t$ as in equation (3), where $\sigma_\gamma = \frac{\sigma}{1+\gamma(w-1)}$. Finally, the Phillips curve takes the usual form

$$\pi_{H,t} = \beta \mathbb{E}_t(\pi_{H,t+1}) + \kappa_\gamma x_t + u_t, \quad \textbf{(A10)}$$

where β is the discrete-time discount factor, $\lambda = \frac{(1-\beta\theta)(1-\theta)}{\theta}$ is related to the frequency of price adjustment, and ψ denotes the inverse of Fischer elasticity. With $\theta = 1$ (fixed price), we have $\lambda = \kappa_\gamma = 0$. Finally, u_t represents an exogenous cost push shock to the equilibrium system.

A.5. DERIVATION OF BOUNDARY CONDITIONS

We prove in this section that the following two sets of boundary conditions

$$V_z(\bar{z}) = -m; V_z(\bar{z}) = m, \quad \textbf{(A11)}$$

$$V_{zz}(\bar{z}) = V_{zz}(\underline{z}) = 0, \quad \textbf{(A12)}$$

hold using discrete approximations of the continuous-time process. Recall that the two stochastic process, we have

$$dz = \sqrt{\sigma} dB \quad \textbf{(A13)}$$

$$d\sigma = \kappa(\theta - \sigma)dt + \sqrt{\xi}\sqrt{\sigma} dB_\sigma. \quad \textbf{(A14)}$$

It can be shown that the first process can be approximated using the discretization of a binomial tree with step size Δh_0, with half probability of either moving up or down, where $\Delta h_0 = \sqrt{\sigma}\sqrt{\Delta t}$. The second process of stochastic volatility can be approximated

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with probability of p_1 of moving up, and $(1-p_1)$ probability of moving down, with step size Δh_1, where

$$p_1 = \frac{\kappa(\theta-\sigma)\sqrt{\Delta t}+\sqrt{\xi}\sqrt{\sigma}}{2\sqrt{\xi}\sqrt{\sigma}}, \tag{A15}$$

and

$$\Delta h_1 = \sqrt{\xi}\sqrt{\sigma}\sqrt{\Delta t}. \tag{A16}$$

If we use $e^{-\rho t} \approx 1-\rho\Delta t$ as an approximation of time interval, we know that at the threshold of FX intervention,[36] the value of z has to go one step down. However, the volatility, which evolve exogenously, can either go up or down. Therefore, the one step ahead discrete approximation of the value function can be approximated using

$$\begin{aligned} V(\bar{z},\sigma) &= -\alpha z^2\Delta t + (1-\rho\Delta t)\left[p_1 V(\bar{z}-\Delta h_0, \sigma+\Delta h_1)\right. \\ &\quad \left. +(1-p_1)V(\bar{z}-\Delta h_0, \sigma-\Delta h_1)\right] - m\Delta h_0 \\ &= -\alpha z^2\Delta t + (1-\rho\Delta t)\left[V(\bar{z},\sigma) - \Delta h_0 V_z(\bar{z},\sigma) + (2p_1-1)\Delta h_1 V_\sigma(\bar{z},\sigma)\right] - m\Delta h_0. \end{aligned} \tag{A17}$$

Substituting the value of discrete approximation, one get

$$0 = -\alpha z^2\Delta t - \rho\Delta t V(\bar{z},\sigma) + (1-\rho\Delta t)\left[\kappa(\theta-\sigma)\Delta t V_\sigma(\bar{z},\sigma) - \sqrt{\sigma}\sqrt{\Delta t}\right] - m\sqrt{\sigma}\sqrt{\Delta t}. \tag{A18}$$

At the limit, we have $\sqrt{\Delta t} >> \Delta t >> \Delta t^{\frac{3}{2}} >> \Delta t^2$. So the order of $\sqrt{\Delta t}$ dominates. Collecting terms with $\sqrt{\Delta t}$, one get

$$V_z(\bar{z},\sigma) = -m. \tag{A19}$$

This is the smooth-pasting condition, which only uses the continuity of the value function. To pin down the solution, one needs an optimality condition. That is, for any given σ, the marginal utility before and after intervention should be the same, that,

$$V_z(\bar{z},\sigma) = V_z(\bar{z}-\Delta h_0,\sigma). \tag{A20}$$

Expand the above, one get

$$V_z(\bar{z},\sigma) = V_z(\bar{z},\sigma) - \sqrt{\sigma}\sqrt{\Delta t}V_{zz}(\bar{z},\sigma). \tag{A21}$$

So that $V_{zz}(\bar{z},\sigma)=0$ holds as the higher-order contact condition.

A.6. IMPLIED SHARE OF REGULATION COST

How quantitatively important is the regulation cost relative to the welfare cost? To answer this question, let

$$\alpha(z) = E_z[\int_0^\infty e^{-\rho t} dL] \tag{A22}$$

$$\beta(z) = E_z[\int_0^\infty e^{-\rho t} dU] \tag{A23}$$

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be the long-run average regulation costs at the lower and upper barriers, respectively. Also, let

$$f(z) = \tilde{\lambda} z^2 \tag{A24}$$

denote the flow welfare cost. Therefore, assuming that $z_0 = 0$, the discounted welfare cost can be expressed by

$$E_z\left[\int_0^\infty e^{-\rho s}\left(\rho f(z) - \frac{1}{2}\tilde{\sigma}^2 f''(z)\right) ds\right] = f'(\underline{z})E_z\left[\int_0^\infty e^{-\rho s} dL\right] - f'(\bar{z})E_z\left[\int_0^\infty e^{-\rho s} dU\right]. \tag{A25}$$

Using a well-known theorem on local times (Stokey (2009)), one can change the integration with respect to time to an integration with respect to occupancy measure on the left-hand side, that is,

$$E_z\left[\int_0^\infty e^{-\rho s}\left(\rho f(z) - \frac{1}{2}\tilde{\sigma}^2 f''(z)\right) ds\right] = \int_{\underline{z}}^{\bar{z}} \left(\rho f(z) - \frac{\sigma^2}{2} f''(z)\right) \pi(z;0) dz, \tag{A26}$$

where $\pi(z;0)$ is the expected discounted local time associated with the occupancy measure $\Pi(A; z, \underline{z}, \bar{z}; \rho) = E_z\left[\int_0^\infty e^{-\rho s} 1_A(z(s)) ds\right], A \in \mathcal{B}_{[\underline{z},\bar{z}]}$. Combining the above two equations, and plug in the definition of $\alpha(z)$, $\beta(z)$, one get

$$\int_{\underline{z}}^{\bar{z}} \left(\rho f(z) - \frac{\tilde{\sigma}^2}{2} f''(z)\right) \pi(z;0) dz = f'(\underline{z})\alpha(z) - f'(\bar{z})\beta(z). \tag{A27}$$

Plugging in the cost function, and integrating out, we have

$$2\tilde{\lambda}\underline{z}\alpha(z) - 2\tilde{\lambda}\bar{z}\beta(z) = \int_{\underline{z}}^{\bar{z}} (\tilde{\lambda}\rho z^2 - \tilde{\lambda}\sigma^2)\pi(z;0) dz. \tag{A28}$$

Due to symmetry, we know that $\alpha(z) = \beta(z)$. If we use the change of measure again $\int_{\underline{z}}^{\bar{z}} z^2 \pi(z;0) dz = \int_0^t e^{-\rho t} z^2 dt$, this reduces to

$$4\bar{z}\alpha(z) = \tilde{\sigma}^2 - \rho \int_0^t e^{-\rho t} z^2 dt, \tag{A29}$$

with the right integral, we know the value again where

$$\int_0^t e^{-\rho t} z^2 dt = f'(\underline{z})\alpha(z) - f'(\bar{z})\beta(z) = -4\alpha\bar{z}\alpha(z). \tag{A30}$$

Therefore, we get

$$\alpha(z) = \frac{\tilde{\sigma}^2}{4(1 - \tilde{\lambda}\rho)\bar{z}}. \tag{A31}$$

Finally, one can compare the regulation cost to the welfare cost by defining the ratio of long-run expected discounted value of regulation cost on the two sides to the unregulated welfare cost counterpart, that is,

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$$share = \frac{m\left(E_z[\int_0^\infty e^{-\rho t}dL] + E_z[\int_0^\infty e^{-\rho t}dU]\right)}{E_e[\int_0^\infty e^{-\rho t}z^2dt]} \quad \textbf{(A32)}$$

$$= \frac{m\frac{\tilde{\sigma}^2}{2(1-\tilde{\lambda}\rho)\bar{z}}}{\frac{\tilde{\lambda}\tilde{\sigma}^2}{\rho}} = \frac{m\rho}{4(1-\alpha\rho)\alpha(\frac{3m\tilde{\sigma}^2}{2\tilde{\lambda}})^{1/3}}. \quad \textbf{(A33)}$$

A.7. ROBUSTNESS CHECK FOR DIFFERENT PARAMETERS AND TIME WINDOWS

The first eight figures (from Figures A.8 to A.15) correspond to the bandwidth versus volatility plots by varying fixing all other parameters at its benchmark values, and change one parameter at a time in order to check if the negative correlation between the bandwidth and the volatility holds. The last two figures (Figures A.16 and A.17) show the calibration results using the 3-month and 1-year time window, respectively.

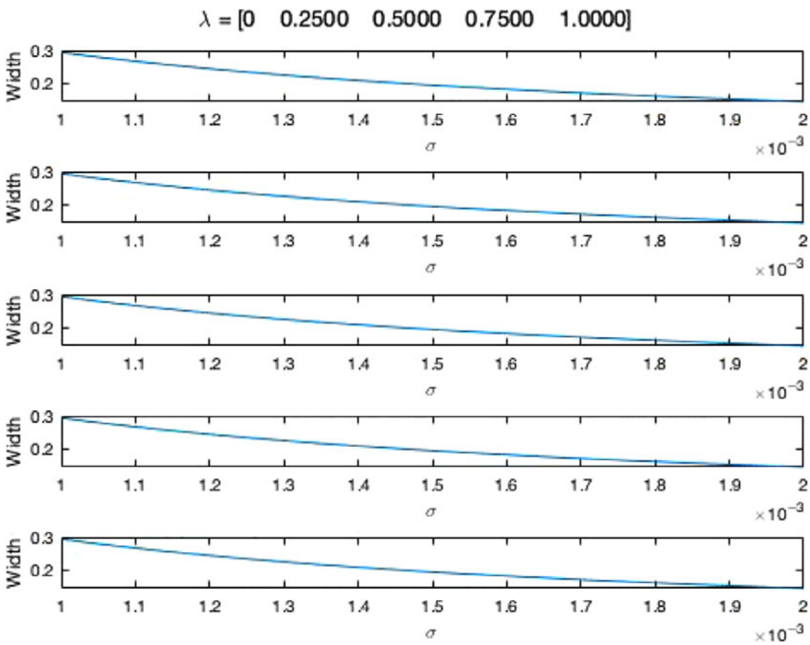

FIGURE A.8. Robustness check on λ.

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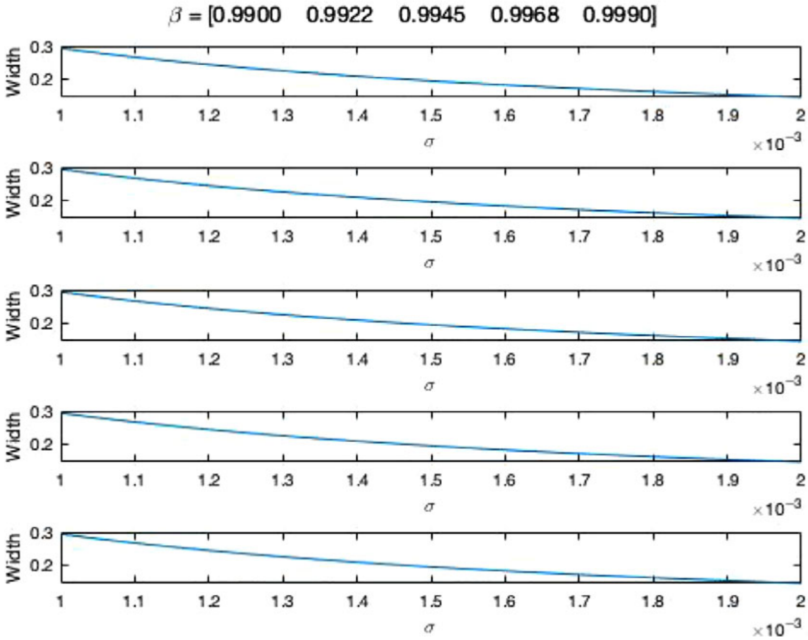

FIGURE A.9. Robustness check on β.

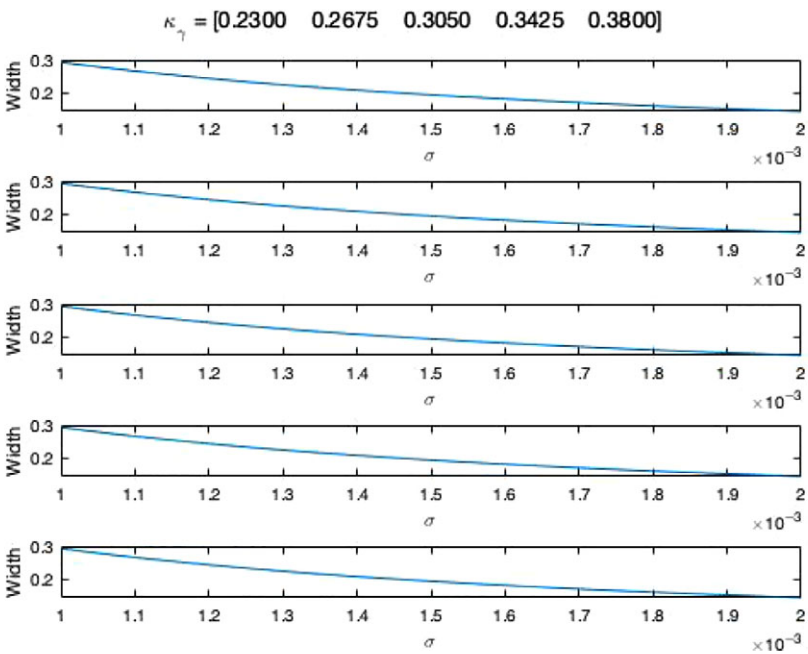

FIGURE A.10. Robustness check on κ_γ.

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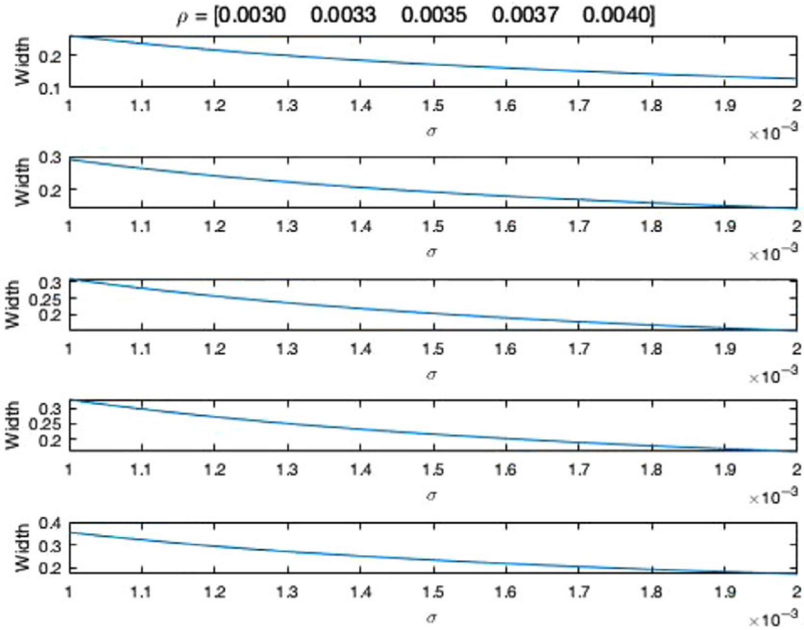

FIGURE A.11. Robustness check on ρ.

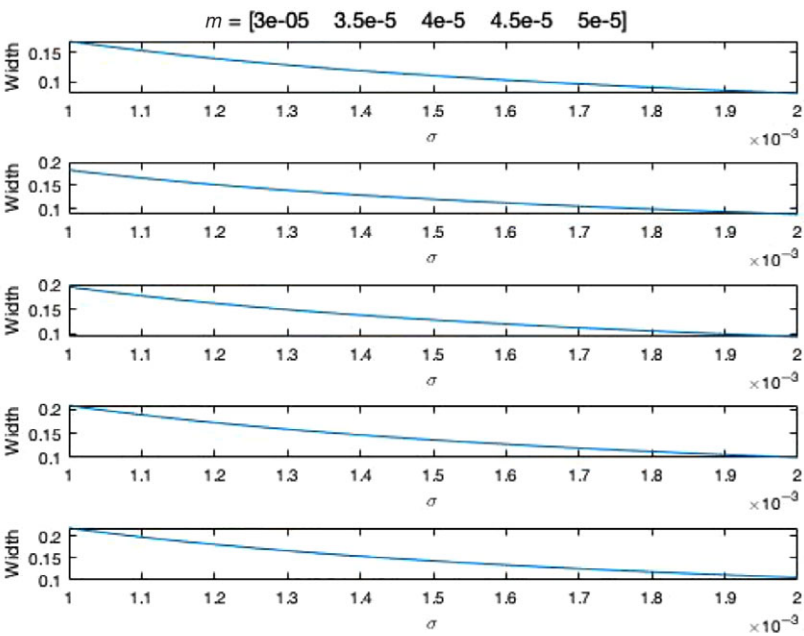

FIGURE A.12. Robustness check on m.

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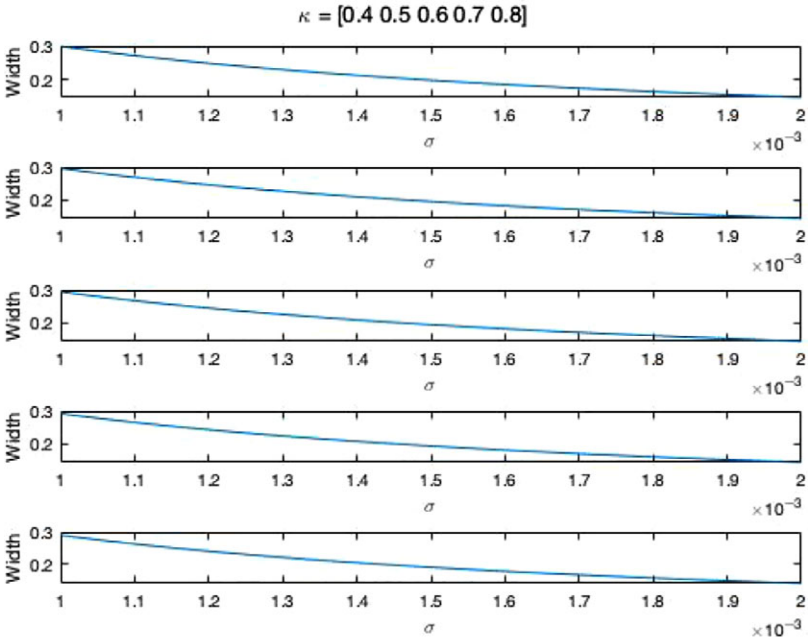

FIGURE A.13. Robustness check on κ.

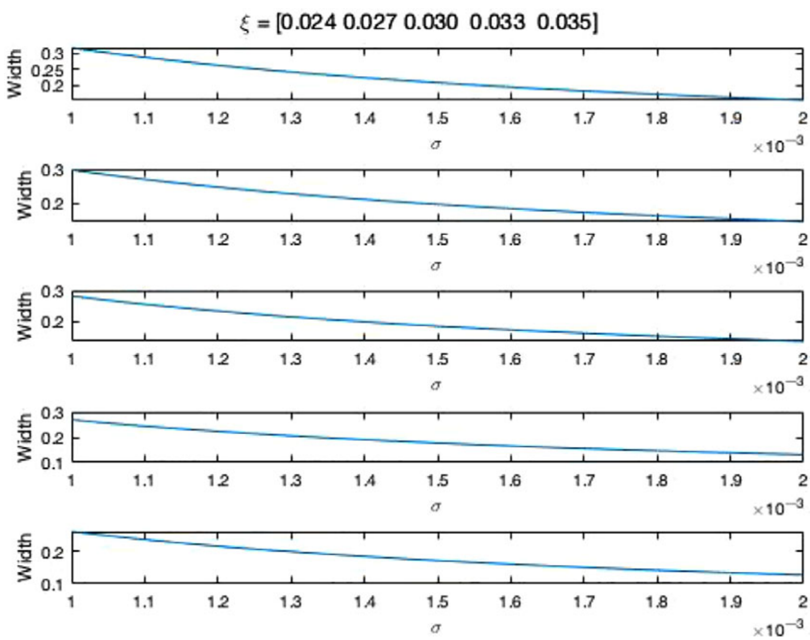

FIGURE A.14. Robustness check on ξ.

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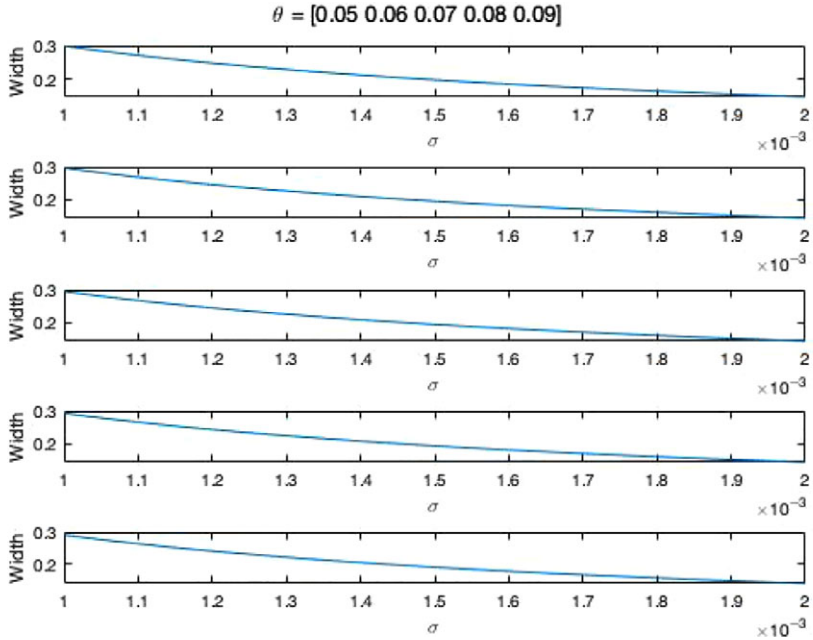

FIGURE A.15. Robustness check on θ.

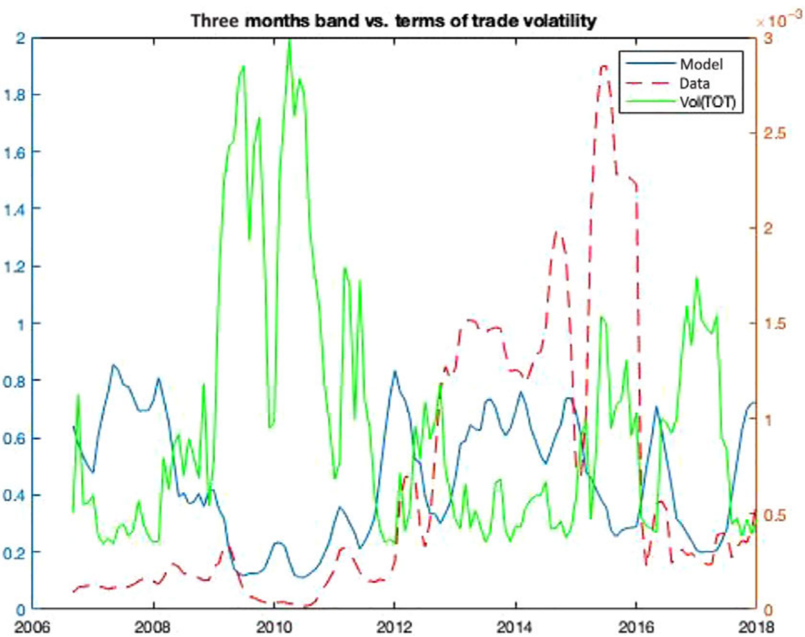

FIGURE A.16. Robustness check on θ.

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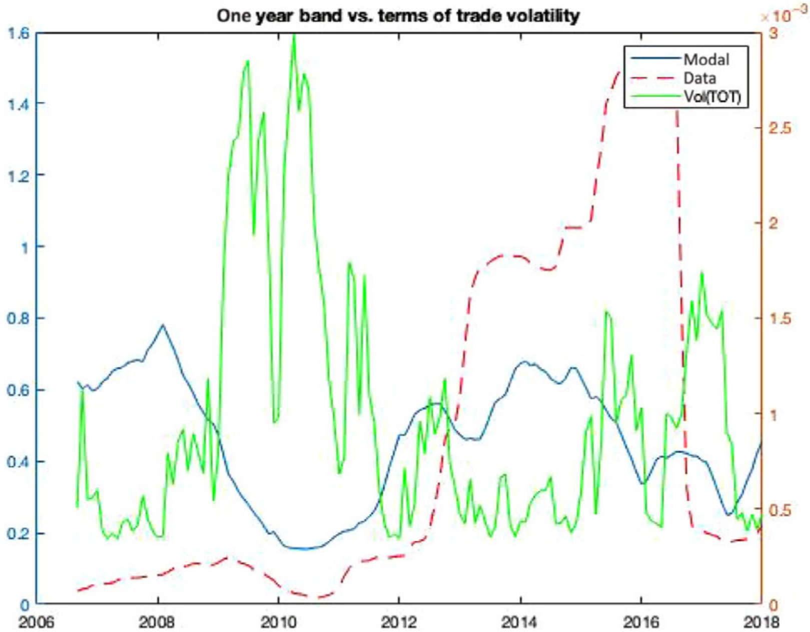

FIGURE A.17. Robustness check on θ.

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