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Chen Ding, Guo Biao and Zhou Guofu

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Firm fundamentals and the cross-section of implied volatility shapes☆

Ding Chen [a], Biao Guo [b,*], Guofu Zhou [c]

[a] Department of Accounting and Finance, University of Sussex, United Kingdom
[b] School of Finance & International Monetary Institute, Renmin University of China, China
[c] Olin School of Business, Washington University in St. Louis, United States of America

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ABSTRACT

With machine learning tools, we document that firm fundamentals have explanatory power on the shape of the option implied volatility (IV) curve that is both economically and statistically significant. We also find that, after accounting for fundamentals, the associated IV process can generate overreaction in the long-term IV with respect to change in the short-term IV, and can allow a positive profit from at-the-money straddle writing, explaining puzzling patterns in the literature. We also provide a simple model linking the IV to firm fundamentals, which permits realistic IV curves and is consistent with the empirical findings.

1. Introduction

The option implied volatility (IV) is one of the central objects of research in derivatives since it reveals the market price of risk of the underlying asset. It also captures investors' demand of compensation for taking the associated higher moment risks, such as volatility (Bakshi and Kapadia, 2003; Carr and Wu, 2009), skewness (Xing et al., 2010; Chang et al., 2013), and variance-of-variance risk premiums (Kaeck, 2018). The IV curve is flat under the assumptions of constant volatility and without jump risk in the underlying asset's stochastic process in Black and Scholes' (1973) framework. Cox and Ross (1976) and Hull and White (1987), among others, relax the constant volatility assumption by allowing the volatility to be a deterministic function or a stochastic process over time. Bakshi et al. (1997) and Pan (2002) introduce an additional jump factor into the IV process. Duan and Wei (2009) show that a systematic risk ratio can provide an understanding of the level and slope of the IV curve. The generalized autoregressive conditional heteroskedasticity option pricing model developed by Duan (1995), along with its subsequent refinements, such as that in Christoffersen et al. (2013), explain more stylized facts about the IV. However, none of these studies explain the IV curve using the information of firm fundamentals. On the other hand, main stream asset pricing models, such as the well known (Fama and

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* Corresponding author.
E-mail addresses: ding.chen@sussex.ac.uk (D. Chen), biao.guo@ruc.edu.cn (B. Guo), zhou@wustl.edu (G. Zhou).

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French, 1993) three-factor model, rely on firm fundamentals to explain the expected stock returns. Green et al. (2017) and Han et al. (2022), among others, use more firm characteristics to discern expected returns across equities. Intuitively, those fundamentals that can explain stochastic volatility or jump risk may have an influence on the IV curve; however, there is a lack of research on whether firm fundamentals affect options.

In this paper, we examine how firm fundamentals help us to understand the shapes of the IV curve in the cross-section. We focus on three null hypotheses. The first hypothesis is that the IV level, smile, smirk, or skew have no relation with firm fundamentals; the second is that the inclusion of firm fundamentals does not provide any economic benefit to the IV level or shape portfolios; and the third is that firm fundamentals are unrelated to the option expectations and overreaction puzzle. Following Green et al. (2017) and the influential machine learning paper of Gu et al. (2020), we start from the large and representative set of 94 firm fundamentals. Since simultaneously including 94 variables in a regression has the problem of overfitting, in the spirit of Gu et al. (2020) and others, we apply the Tibshirani (1996) least absolute shrinkage and selection operator (LASSO) to select the most important subset of fundamentals that show their relevance in the training period.

Based on a cross-section of monthly options' data on 3345 U.S. firms for the 1996–2019 period, the LASSO approach selects those firm fundamentals that matter in predicting the IV curve changes. On average, the selection frequency is 27.44% for the IV levels, captured by the at-the-money (ATM) IV; is 9.62% for the IV slopes, measured by the difference between the out-of-the-money (OTM) and ATM IVs; and is 9.32% for the IV curvatures, gauged by the average of OTM and in-the-money (ITM) IVs minus the ATM IV, respectively. This provides us with 13 fundamentals representing the top ten variables for the IV levels, slopes, and curvatures: illiquidity, beta, size, dividend-price ratio, cash flow volatility, unexpected quarterly earnings, R&D to sales, scaled earnings forecast, idiosyncratic return volatility, earnings volatility, depreciation, number of analysts covering stock, and volatility of share turnover. We then use them to test the three hypotheses.[1]

The data strongly reject the first two hypotheses after we control for the risk-neutral skewness and kurtosis (Bakshi et al., 2003), and the systematic risk ratio (Duan and Wei, 2009). Our finding indicates that firm fundamentals play an important role in determining the shape of the IV curve and option prices. Using 30-day put options, 11 out of the 13 firm fundamentals show statistical significance in explaining the IV level; 9 of the fundamentals are significant in explaining the IV slope; and 11 explain significantly the IV curvature. Those fundamentals determine differently the shape of the IV curve and the inclusion of firm fundamentals yields more improved modeling for the IV slope and curvature. The inclusion of fundamentals also makes the long–short hedging portfolio, based on the estimated IV curve, generate significant returns. These empirical findings are robust to the choice of option types and inference methods. Additional empirical tests conducted using call options, 60-day time-to-maturity, and a Bayesian shrinkage regression that addresses possible nonlinearity and multicollinearity among firm fundamentals yield qualitatively the same results.

We then investigate whether firm fundamentals can provide any insight on two major puzzles in option pricing. The first is the expectations puzzle, which is the observation that the option IV tends to exceed the realized volatility of the underlying asset (Bakshi and Kapadia, 2003). The second is the overreaction puzzle, which is the fact that the long-term implied variance overreacts to the changes in the short-term variance (Stein, 1989; Poteshman, 2001). To investigate whether firm fundamentals can help explain these puzzles, we test the third hypothesis that firm fundamentals have no relation with them. Empirically, we find strong evidence, from the significant volatility term-structure regression coefficients and the substantial rewarding short straddle returns, that the null hypothesis is rejected.

Next, we provide a simple expository theoretical model to understand the relation between firm fundamentals and option prices. In this model, we extend the Geske (1977) structural model by treating both the equity and equity options, whose pricing functions depend on firm fundamentals, as contingent claims on the firm value, and by assuming that the time to default is a function of the firm's credit-sensitive fundamental variables. In addition to the leverage effect considered in Geske et al. (2016), our theoretical model shows that dividend payout, time to default, and book-to-market ratio can also influence the IV function, providing channels through which fundamental variables can affect the IV curve. It allows us to identify the relationship between a fundamental variable and the IV curve through its relation with leverage, dividend policy, time to default, or book-to-market ratio, while keeping the other fundamentals fixed. Numerical simulations show that this model generates realistic IV smirks, and the implied theoretical impact of firm fundamentals on the IV function supports our empirical findings.

Overall, our study demonstrates that firm fundamentals are important in option pricing and they have substantial explanatory power on the cross-sectional variation in the IV curve. Our study complements other explanations of the changes in the IV curve, such as leverage (Toft and Prucyk, 1997), transaction costs (Pena et al., 1999), size and trading volume (Dennis and Mayhew, 2002), net buying pressure (Bollen and Whaley, 2004), investor sentiment (Han, 2007), and demand-pressure effects (Garleanu et al., 2009). We also provide evidence that firm fundamentals help predict equity option returns, in line with the studies by Vasquez and Xiao (2019) and Zhan et al. (2022). However, these studies focus on the delta-hedged ATM options while we investigate the whole IV curve of individual firms.

Our paper adds to the literature of growing applications of machine learning to finance. Rapach et al. (2013), Chinco et al. (2019), DeMiguel et al. (2020), Feng et al. (2020), Freyberger et al. (2020), Gu et al. (2020), Kozak et al. (2020), Avramov et al. (2022a), Bryzgalova et al. (2021, 2022), Cong et al. (2022) and Han et al. (2022), among others, focus on equities based on firm characteristics; Avramov et al. (2022b) and Guo et al. (2022) study bonds; and Filippou et al. (2022) analyze foreign exchanges.

[1] We also ad hoc select nine representative fundamental measures that are widely used in the stock return literature: a firm's financial leverage, interest coverage, liquidity, profitability, investment, size, equity market momentum, dividend-price ratio, and book-to-market ratio (e.g., Welch and Goyal, 2007; Rapach et al., 2010; Zhang, 2013; Bai and Wu, 2016), our conclusion remains similar.

By contrast, we provide the first study on the volatility shapes. Bali et al. (2021) study how firm fundamentals predict option returns. Conversely, Neuhierl et al. (2022) examine how both firm characteristics and option information impact stock returns. Recently, Nagel (2021), Giglio et al. (2022), and Zaffaroni and Zhou (2022) provide a review of some of the machine learning studies in finance.

It is important to point out that there is a lack of theoretical guidance on what exactly the firm fundamentals are that determine the IV surface. In this paper, we attempt a partial solution by using machine learning tools to identify the variables from a large set of firm fundamentals. Our solution is empirical and the research design is still limited in possible variables and is subject to estimation errors. Future research is needed for both theoretical modeling and empirical testing to identify additional variables to provide an improved understanding of the problem.

The rest of the paper is organized as follows. In Section 2, we explain the data and empirical procedures. In Section 3, we report the empirical results. In Section 4, we provide a compound option model for equity option pricing, and discuss the channels through which firm fundamentals affect the option IV. We discuss robustness tests in Section 5. We conclude in Section 6.

2. Data and methodology

2.1. Data

In this section, we describe the data we use to examine the explanatory power of firm fundamentals on the shape of the IV function. We analyze all firms in the U.S. market with options traded between January 1996 and December 2019. We follow Green et al. (2017) and Han et al. (2022) to construct firm fundamentals. The data are from CRSP, Compustat, and I/B/E/S.[2] Following Green et al. (2017), we winsorize the fundamentals at the 1st and 99th percentiles of their monthly observations. We use the cross-sectional mean and standard deviation to standardize the observations for each fundamental for each month, and we replace the missing values with the fundamental's cross-sectional standardized mean value of zero. Table 1 lists the definitions of the 94 firm fundamentals. The Appendix in Green et al. (2017) provides their detailed information.

The data on stock option IV and stock price historical volatility are from the Ivy DB Option Metrics. During our sample period, a firm is included if its 30-day IV curve is available and its stock has been traded for at least 365 calendar days. To align the IV curve characteristics with firm fundamentals, we assume that annual firm characteristics are available in month $t-1$ if the firm's fiscal year ended at least six months before month $t-1$, and that quarterly accounting data are available in month $t-1$ if the fiscal quarter ended at least four months before month $t-1$. Subsequently, we match all the fundamentals with the corresponding option and stock data on the last Wednesday of each month. The sample contains 288 months, 3345 firms, and 417,752 observations.

To capture the shape of the IV curve, we analyze the 30-day IV curve produced by put options by considering only option IVs with a delta equal to −0.8, −0.5, or −0.2.[3] We use the IV levels at an option delta equal to −0.5, the slopes of the IV curve approximated by the IVs of a put spread option strategy simultaneously buying and selling of put options at different delta levels, and the curvature of the IV curve approximated by a butterfly spread options strategy buying the OTM and ITM options and selling the ATM options. Denoting the IV at a delta equal to −0.8, −0.5, or −0.2 by IV^+, IV°, or IV^-, the slope of the IV function is captured by the following[4]:

$$IV^s = IV^- - IV^\circ, \tag{1}$$

and the curvature of the IV function is captured by:

$$IV^c = \frac{IV^+ + IV^-}{2} - IV^\circ. \tag{2}$$

The main advantage of the measures defined by Eqs. (1)–(2) is their simplicity. There are sophisticated procedures for capturing the shape of the IV function. For example, one may wish to specify a particular functional form on the IV curve and surface such that, once fitted to the market observed IV, the set of parameters obtained can serve as a storage for the information on the IV shape. Dumas et al. (1998), Goncalves and Guidolin (2006), and Bedendo and Hodges (2009) estimate the shape of the IV function using a polynomial fitting to market observed IVs with respect to the time-to-maturity and strike price/option moneyness to various degrees. Alternatively, one may specify diffusive processes for the underlying stock return and its instantaneous volatility complemented by jump processes of various types. Subsequently, the IV can be approximated either by direct calibration from market observed Black–Scholes IV, as in Jacquier and Lorig (2015) and Aït-Sahalia et al. (2021), or via the calibration from option prices, as in Bates (2000) and Christoffersen et al. (2009). The non-parametric models, such as principal component analysis and Karhunen–Loéve decomposition, are also widely used for constructing the measures for the shape of the IV curve. Alexander (2001), Cont et al. (2002), and Christoffersen et al. (2018) take this approach.

[2] We thank Jeremiah Green for providing the SAS code for extraction of firm fundamentals: https://sites.google.com/site/jeremiahrgreenacctg/.

[3] Options seldom trade exactly on these deltas on each date; hence, we use the interpolated IV curve provided by the Ivy DB Option Metrics, which estimates the IV for options that have an American-style exercise feature based on the industry-standard Cox–Ross–Rubinstein (CRR) binomial tree model.

[4] The IV slope is sometimes measured as the difference between the OTM and ITM IVs in the literature; our results remain robust with this alternative definition.

Table 1
Fundamental variable descriptions.

Variable	Description	Variable	Description
absacc	Absolute accruals	*mom1m*	1-month momentum
acc	Working capital accruals	*mom36m*	36-month momentum
aeavol	Abnormal earning announcement volume	*ms*	Financial statement score
age	Number of years since first Compustat coverage	*mve*	Size
agr	Asset growth	*mve_ia*	Industry-adjusted size
baspread	Bid–ask spread	*nanalyst*	Number of analysts covering stock
beta	Beta	*nincr*	Number of earnings increases
bm	Book-to-market	*operprof*	Operating profitability
bm_ia	Industry-adjusted book to market	*orgcap*	Organizational capital
cash	Cash holdings	*pchcapx_ia*	Industry-adjusted % change in capital expenditure
cashdebt	Cash flow to debt	*pchcurrat*	% change in current ratio
cashpr	Cash productivity	*pchdepr*	% change in depreciation
cfp	Cash-flow-to-price ratio	*pchgm_pchsale*	% change in gross margin - % in sales
cfp_ia	Industry-adjusted cash-flow-to-price ratio	*pchsale_pchinvt*	% change in sales - % change in inventory
chatoia	Industry-adjusted change in asset turnover	*pchsale_pchrect*	% change in sales - % change in A/R
chcsho	Change in shares outstanding	*pchsale_pchxsga*	% change in sales - % change in SG&A
chempia	Industry-adjusted change in employees	*pchsaleinv*	% change in sales-to-inventory
chfeps	Change in forecasted EPS	*pctacc*	Percent accruals
chinv	Change in inventory	*pricedelay*	Price delay
chmom	Changed in 6-month momentum	*ps*	Financial statements score
chnanalyst	Change in number of analysts	*rd*	R&D inrease
chpmia	Industry-adjusted change in profit margin	*rd_mve*	R&D to market capitalization
chtx	Change in tax expense	*rd_sale*	R&D to sales
cinvest	Corporate investment	*realestate*	Real estate holdings
convind	Convertible debt indicator	*retvol*	Return volatility
currat	Current ratio	*roaq*	Returns on assets
depr	Depreciation/PP&E	*roavol*	Earnings volatility
disp	Dispersion in forecasted EPS	*roeq*	Return on equity
divi	Dividend initiation	*roic*	Return on invested capital
divo	Dividend omission	*rsup*	Revenues surprise
dy	Dividend-price ratio	*salecash*	Sales to cash
ear	Earnings announcement return	*saleinv*	Sales to inventory
egr	Growth in common shareholder equity	*salerec*	Sales to receivables
ep	Earnings to price	*secured*	Secured debt
fgr5yr	Forecasted growth in 5-year EPS	*securedind*	Secured debt indicator
gma	Gross profitability	*sfe*	Scaled earnings forecast
grcapx	Growth in capital expenditures	*sgr*	Sales growth
grltnoa	Growth in long-term net operating assets	*sin*	Sin stocks
herf	Industry sales concentration	*sp*	Sales to price
hire	Employee growth rate	*std_dolvol*	Volatility of liquidity (dollar trading volume)
idiovol	Idiosyncratic return volatility	*std_turn*	Volatility of liquidity (share turnover)
ill	Illiquidity	*stdcf*	Cash flow volatility
indmom	Industry momentum	*sue*	Unexpected quarterly earnings
invest	Capital expenditures and inventory	*tang*	Debt capacity/firm tangibility
IPO	New equity issue	*tb*	Tax income to book income
lev	Leverage	*turn*	Share turnover
mom12m	12-month momentum	*zerotrade*	Zero trading days

This table reports the descriptions of 94 firm fundamentals from Green et al. (2017).

2.2. The LASSO approach: a machine learning tool

To test whether and what firm fundamentals drive individual equity IV curves, we investigate the role of firm fundamentals in predicting the changes of the cross-section of IV characteristics of individual equity options. We apply the LASSO approach since simultaneously including 94 variables in a regression has a problem of overfitting.[5]

The first step is to generate a forecast of the month $t + 1$ IV characteristics based on fundamental information available in month t. We estimate a series of cross-sectional univariate regressions for each fundamental and for IV levels, slopes, and curvatures, respectively, as follows:

$$\Delta IV_{i,t} = \alpha_{j,t} + \beta_{j,t} z_{i,j,t-1} + \epsilon_{i,t}, \quad i = 1, \dots, I_t; \quad j = 1, \dots, J_{t-1}, \tag{3}$$

where ΔIV is the change of IV levels, slopes, or curvatures, $z_{i,j,t-1}$ is the jth firm fundamental for firm i in month $t-1$, I_t is number of firms with options available in month t, and J_{t-1} is the number of fundamentals available at the end of month $t-1$. After estimating

[5] We also test the Zou and Hastie (2005) elastic net (ENET) approach and obtain similar results.

the above OLS regression, we are able to construct month $t+1$ IV characteristics forecasts for each fundamental,

$$\Delta \hat{IV}^{(j)}_{i,t+1|t} = \hat{\alpha}_{j,t} + \hat{\beta}_{j,t} z_{i,j,t}, \quad i = 1, \ldots, I_{t+1}; \quad j = 1, \ldots, J_t, \tag{4}$$

where $\hat{\alpha}_{j,t}$ and $\hat{\beta}_{j,t}$ are the OLS estimate of $\alpha_{j,t}$ and $\beta_{j,t}$ in Eq. (3).

The LASSO approach allows us to select the most relevant individual forecasts to include in the combination in high-dimensional settings, and is therefore especially suitable for our analysis with 94 fundamentals and facilitates our tracking of the importance of firm fundamentals in predicting cross-sectional equity IV curves over time.

Specifically, considering the following multiple regression of realized cross-sectional IV characteristics to the forecasts based on the individual fundamentals in Eq. (3):

$$\Delta IV_{i,t} = \alpha^{MR}_t + \sum_{j=1}^{J_{t-1}} \beta^{MR}_{j,t} \Delta \hat{IV}^{(j)}_{i,t|t-1} + \epsilon_{i,t}, \quad i = 1, \ldots, I_t, \tag{5}$$

we estimate Eq. (5) with the weighted LASSO:

$$\arg\min\left(\frac{1}{2I_t} \sum_{i=1}^{I_t} \omega_{i,t} \tilde{\epsilon}^2_{i,t} + \lambda_t \|\tilde{B}^{MR}_t\|_1\right), \tag{6}$$

where:

$$\begin{aligned} \tilde{B}^{MR}_t &= [\tilde{\beta}^{MR}_{1,t} \ldots \tilde{\beta}^{MR}_{J_{t-1},t}]', \\ \tilde{\epsilon}_{i,t} &= \Delta IV_{i,t} - \left(\tilde{\alpha}^{MR}_t + \sum_{j=1}^{J_{t-1}} \tilde{\beta}^{MR}_{j,t} \Delta \hat{IV}^{(j)}_{i,t|t-1}\right), \\ \|\tilde{B}^{MR}_t\|_1 &= \sum_{j=1}^{J_{t-1}} |\tilde{\beta}^{MR}_{j,t}|, \end{aligned} \tag{7}$$

where λ is a regularization parameter that shrinks the estimates, and Eq. (6) reduces to the familiar WLS and OLS (when $\omega = 1$) regression when $\lambda = 0$. The LASSO selects variables by the second penalty term with a sufficiently large λ and allows for shrinkage to zero. We choose λ in Eq. (6) to shrink the slope estimates by the Hurvich and Tsai (1989) corrected version of the Akaike information criterion (AIC). The LASSO combination forecast is therefore the average of the individual forecasts selected by the LASSO in Eq. (6).

2.3. Hypotheses

Using the subsets of fundamentals we chose using the LASSO approach, we can test whether the firm fundamentals influence the IV curve of the firm's equity options by examining the following three null hypotheses.

- Hypothesis 1: The IV level, smile, smirk, or skew are unrelated to the firm's fundamentals.
- Hypothesis 2: The inclusion of firm fundamentals does not provide any economic benefit to the IV level or shape portfolios.
- Hypothesis 3: Firm fundamentals are unrelated to the option expectations or overreaction puzzles.

Duan and Wei (2009) and others find that the risk-neutral skewness and kurtosis impact a firm's equity option IV curve. Given this, we first establish a benchmark model by controlling for the historical volatility (HV), risk-neutral skewness (NS), and kurtosis (NK). The latter two are calculated using the Bakshi, Kapadia, and Madan (BKM) framework (Bakshi et al., 2003). Denoting $M = (HV, NS, NK)$, we consider the following benchmark model:

$$IV^i_{jt} = \alpha_t + \beta_{0,t} M_{jt} + \epsilon^i_{jt}, \quad i = \circ, s, c, \tag{8}$$

where j denotes the corresponding variables for firm j, i represents the IV level, slope, and curvature, respectively.

Duan and Wei (2009) find that systematic risk, defined as the proportion of systematic variance in the total variance, influences the shape of the IV function of individual equity options. We denote the systematic risk ratio Sys, measured as the explanatory power R^2 of the CAPM model $r_{jt} = \alpha_j + \beta_j r_{mt} + \epsilon_{jt}$, where r_{jt} and r_{mt} are returns for stock j and the market. With the firm fundamental measure vector F, we analyze the following cross-sectional regression models for the IV shape on each date t:

$$IV^i_{jt} = \alpha_t + \beta_{1,t} F_{jt} + \epsilon^i_{jt}, \tag{9}$$

$$IV^i_{jt} = \alpha_t + \beta_{0,t} M_{jt} + \beta_{1,t} F_{jt} + \epsilon^i_{jt}, \tag{10}$$

$$IV^i_{jt} = \alpha_t + \beta_{0,t} M_{jt} + \beta_{1,t} F_{jt} + \beta_{2,t} Sys_{jt} + \epsilon^i_{jt}. \tag{11}$$

To test Hypothesis 1, we run the above regressions and examine whether the coefficient $\beta_{1,t}$ is significantly different from zero. To test Hypothesis 2, we explore whether these fundamentals are economically useful in predicting the change in the IV curve by using a set of delta-neutral option trading strategies based on the regression results from Eqs. (8)–(11). To test Hypothesis 3, we examine whether the volatility estimates obtained from Eqs. (8)–(11) exceed the physical volatility for the expectation puzzle and whether the long-term IV overreacts to changes in the short-term IV for the overreaction puzzle, following Stein (1989) and Christoffersen et al. (2013).

Table 2
Summary statistics of IV curve and firm fundamentals.

Characteristics	Mean	Sd	Min	Max	25%	75%
Panel A: IV characteristics and benchmark variables						
Level	0.43	0.25	0.01	2.99	0.26	0.52
Slope	0.08	0.15	−2.50	2.66	0.02	0.10
Curvature	0.06	0.12	−2.28	1.96	0.01	0.09
HV	0.40	0.22	0.05	5.38	0.25	0.50
NS	−0.47	0.74	−5.92	8.88	−0.82	−0.12
NK	5.11	2.33	0.38	55.63	3.45	6.17
Sys	0.25	0.16	0.00	0.86	0.12	0.35
Panel B: Representative firm fundamentals						
ill	0.00	0.92	−0.48	21.60	−0.30	0.00
beta	0.00	1.00	−2.68	4.70	−0.71	0.58
mve	0.00	0.92	−2.69	2.36	−0.59	0.51
dy	0.00	0.91	−0.89	7.16	−0.66	0.19
stdcf	0.00	0.82	−0.29	20.93	−0.16	0.00
sue	0.00	0.91	−15.25	12.02	−0.01	0.11
rd_sale	0.00	0.68	−0.26	17.43	−0.18	0.00
sfe	0.00	0.89	−17.12	5.84	0.00	0.23
idiovol	0.00	1.00	−1.87	6.69	−0.73	0.46
roavol	0.00	0.89	−0.69	9.58	−0.45	0.00
depr	0.00	0.90	−1.37	9.78	−0.49	0.00
nanalyst	0.00	0.92	−1.64	4.26	−0.68	0.37
std_turn	0.00	0.92	−1.21	8.09	−0.50	0.00

This table reports the sample statistics of the firm fundamental and IV curve characteristics from monthly put options of U.S. firms that have options actively traded between January 1996 and December 2019. Statistics includes pooled average (Mean), pooled standard deviation (Sd), min value (Min), max value (Max), 25th percentile value (25%) and 75th percentile value (75%). Panel A reports the statistics for the IV characteristics and the historical volatility (HV), risk-neutral skewness (NS), kurtosis (NK), and systematic ratio (Sys). Panel B reports the statistics for the representative firm fundamentals chosen using the LASSO approach. We use the cross-sectional mean and standard deviation to standardize the observations for each fundamental for each month following Green et al. (2017).

Panel A of Table 2 provides the summary statistics of put options' IV curve characteristics. Statistics are calculated on the pooled data of all observations. The ATM put option IV has a pooled average of 43%; the pooled averaged IV slope and IV curvature are positive, indicating that the put option IV curve indeed smiles.

3. Empirical results

In this section, we report the results for Hypotheses 1–3. First, we report the number of relevant fundamentals selected using the LASSO approach, then we demonstrate the relation between the cross-sectional variation in the IV shapes and firm fundamentals. Subsequently, we construct an option trading strategy to compare the economic significance with and without fundamentals. Finally, we show whether the inclusion of fundamentals improves the understanding of option puzzles.

3.1. Number of relevant fundamentals

Table 3 reports the Fama–MacBeth (FM) regression results for cross-sectional put option IV characteristics forecasts of the 94 individual firm fundamentals. We first run a univariate regression of realized IV characteristics on forecasted IV characteristics cross-sectionally, then we estimate the time series averages of the slope coefficients and R^2, with t-statistics adjusted by Newey and West (1987) standard errors with 12 lags; 14, 15, and 9 out of 94 fundamentals are significant at the 10% level for predicting the changes of IV levels, slopes, and curvatures, respectively.

The LASSO approach allows us to gauge how the number and nature of relevant firm fundamentals evolve over time. Fig. 1 presents the ten-year moving average of the number of fundamentals selected by the LASSO approach. On average, the number of selected fundamentals is stable over time, the average number of selected fundamentals is 26 for the IV levels, and is 9 for both the IV slopes and curvatures. We also notice the number has gradually increased in recent years, suggesting that fundamentals have started to play more important roles in predicting the changes of IV characteristics out-of-sample.

Table 4 reports the selection frequencies by the LASSO. On average, the selection frequency is 27.44%, 9.62%, and 9.32% for the IV levels, slopes, and curvatures, respectively. Focusing on the IV levels, the largest (smallest) selection frequency is 55.94% (12.24%), and the median frequency is 25.70%, indicating that most of the firm fundamentals matter over time, although their behavior varies. The top ten fundamentals in terms of selection frequencies all exceed 39%, they are beta (beta), 56%; illiquidity (ill), 50%; share turnover (turn), 49%; R&D to sales (rd_sale), 46%; bid–ask spread (baspread), 43%; size (mve), 43%; cash flow volatility (stdcf), 43%; scaled earnings forecast (sfe), 42%; zero trading days (zerotrade), 41%; and volatility of share turnover (std_turn), 40%. Five of them are related to the stock market liquidity (ill, turn, baspread, zerotrade, std_turn), three of them are

Table 3
FM regression results for cross-sectional IV characteristics forecasts of firm fundamentals.

Fundamental	Level			Slope			Curvature		
	Coeff	*t*-stat	R^2	Coeff	*t*-stat	R^2	Coeff	*t*-stat	R^2
absacc	−2.38*	−1.64	0.23%	−0.38	−0.58	0.13%	−1.55	−1.56	0.12%
acc	0.79	0.36	0.17%	0.74	1.31	0.10%	−5.42	−1.36	0.09%
aeavol	−0.44	−0.54	0.21%	1.64	0.79	0.09%	−2.09	−1.15	0.11%
age	1.25*	1.82	0.39%	0.68	0.81	0.13%	−2.09	−1.38	0.13%
agr	3.40	0.71	0.25%	0.33	0.35	0.09%	0.93	0.92	0.12%
baspread	0.00	0.00	1.14%	0.31	0.23	0.21%	0.86	1.60	0.20%
beta	−1.90	−1.37	1.29%	0.62	1.10	0.21%	3.18	0.76	0.20%
bm	0.07	0.10	0.27%	5.18	0.97	0.12%	−0.39	−0.56	0.14%
bm_ia	0.66	1.18	0.15%	−0.15	−0.09	0.11%	0.22	0.24	0.12%
cash	−0.38	−0.53	0.54%	9.68	0.65	0.14%	2.88	0.56	0.15%
cashdebt	−5.58	−1.10	0.30%	−1.90*	−1.70	0.11%	−1.78	−1.32	0.10%
cashpr	−3.74	−0.79	0.10%	1.08	0.93	0.08%	−0.57	−0.85	0.07%
cfp	−0.43	−0.74	0.29%	−0.71	−0.59	0.15%	1.11	0.51	0.13%
cfp_ia	1.24	0.71	0.21%	−1.98*	−1.70	0.15%	0.50	0.85	0.12%
chatoia	−0.92	−1.07	0.10%	−0.81***	−2.76	0.08%	−25.12	−1.10	0.10%
chcsho	3.94**	2.07	0.20%	13.81	1.05	0.08%	−0.56	−1.04	0.08%
chempia	4.89	1.21	0.13%	2.30	0.99	0.10%	−1.10	−0.35	0.09%
chfeps	−2.43*	−1.82	0.14%	0.68	0.49	0.10%	−40.00	−1.08	0.10%
chinv	−3.67	−1.10	0.12%	−5.77	−0.89	0.09%	−0.45	−1.32	0.08%
chmom	−1.04	−0.64	0.34%	−0.49	−0.80	0.12%	−0.48	−1.26	0.13%
chnanalyst	0.29	0.45	0.10%	−0.53	−1.45	0.06%	3.51	1.00	0.07%
chpmia	3.51	0.88	0.15%	−1.54	−1.38	0.08%	−0.74	−0.43	0.10%
chtx	0.01	0.01	0.19%	−1.02	−1.40	0.08%	−0.61***	−2.47	0.10%
cinvest	7.06	0.90	0.18%	−1.59	−0.84	0.11%	2.24	0.78	0.09%
convind	0.82	0.64	0.16%	2.22	1.04	0.07%	0.97	0.46	0.09%
currat	1.49	0.75	0.18%	−1.14	−1.47	0.13%	−26.34	−1.04	0.11%
depr	−1.05	−1.11	0.26%	−1.01***	−2.45	0.14%	34.85	1.09	0.15%
disp	−2.88	−1.47	0.24%	−3.37	−0.73	0.12%	−1.17	−1.25	0.11%
divi	0.19	0.19	0.09%	−0.33	−0.52	0.10%	−0.88	−0.39	0.11%
divo	−2.01*	−1.83	0.15%	−3.84**	−2.23	0.08%	−0.82	−0.70	0.09%
dy	−0.01	0.00	0.31%	−1.13**	−2.22	0.18%	−0.54	−1.12	0.15%
ear	−0.20	−0.11	0.13%	−1.65	−1.52	0.09%	−8.41	−0.82	0.09%
egr	−0.47	−1.17	0.23%	−2.83***	−2.43	0.09%	4.65	0.95	0.11%
ep	0.69	1.05	0.33%	−6.14	−1.12	0.14%	2.41	0.56	0.13%
fgr5yr	−26.59	−0.77	0.54%	−0.49	−0.37	0.14%	−0.07	−0.12	0.14%
gma	−2.10	−0.86	0.22%	−0.82	−0.48	0.10%	0.48	0.68	0.09%
grcapx	−2.33	−1.01	0.18%	0.68	0.33	0.07%	−0.28	−0.66	0.10%
grltnoa	1.41	0.61	0.10%	−0.54	−0.29	0.08%	−0.42	−0.45	0.10%
herf	−0.21	−0.20	0.11%	0.25	0.45	0.07%	−2.81	−1.48	0.07%
hire	4.56	1.18	0.22%	0.48	0.32	0.10%	−0.43	−0.76	0.11%
idiovol	0.17	0.28	1.17%	−0.01	−0.03	0.27%	−0.54	−1.10	0.22%
ill	−0.42	−0.31	0.50%	−3.05	−0.91	0.22%	−0.66	−1.08	0.23%
indmom	−0.26	−0.24	0.29%	−1.30**	−2.06	0.10%	−0.16	−0.30	0.12%
invest	−0.96**	−2.14	0.16%	−1.42*	−1.79	0.08%	1.69	0.63	0.09%
IPO	1.21	1.32	0.15%	−0.85	−0.42	0.11%	−5.99	−1.17	0.10%
lev	−1.13*	−1.85	0.32%	7.81	1.20	0.10%	3.16	1.03	0.10%
mom12m	−5.51	−1.51	0.37%	0.43	1.24	0.12%	0.16	0.31	0.15%
mom1m	−0.63	−0.33	0.43%	0.63	1.53	0.12%	0.66	1.06	0.16%
mom36m	−3.13	−0.99	0.38%	−2.06*	−1.69	0.10%	−2.77	−1.18	0.12%
ms	−1.45	−0.72	0.18%	−1.14	−1.26	0.10%	−0.70	−0.31	0.09%
mve	−1.11	−0.73	0.53%	57.38	1.02	0.24%	5.08	1.53	0.17%
mve_ia	−1.72	−1.55	0.32%	0.07	0.19	0.16%	1.09	1.28	0.12%
nanalyst	6.61	1.21	0.36%	0.76	0.89	0.21%	−0.87	−1.08	0.15%
nincr	0.46	0.49	0.13%	−0.60	−0.72	0.09%	14.80	1.34	0.08%
operprof	61.49	0.69	0.11%	−0.05	−0.04	0.11%	−1.02	−0.75	0.10%
orgcap	−0.64	−0.42	0.13%	0.29	0.24	0.08%	−6.42	−1.23	0.08%
pchcapx_ia	−0.55	−0.82	0.16%	−0.51	−1.28	0.07%	7.13	0.73	0.09%
pchcurrat	−5.78	−0.92	0.13%	−0.52	−1.04	0.09%	0.35	0.34	0.11%
pchdepr	−0.25	−0.53	0.15%	−0.06	−0.17	0.10%	−0.89	−0.69	0.08%
pchgm_pchsale	−0.69	−0.79	0.15%	−0.51	−1.11	0.08%	1.31	1.25	0.11%
pchsale_pchinvt	2.68	1.21	0.11%	0.33	0.26	0.09%	3.25	0.89	0.10%
pchsale_pchrect	29.18	1.02	0.17%	0.23	0.39	0.11%	109.71	1.02	0.10%
pchsale_pchxsga	0.08	0.19	0.14%	1.23	0.73	0.08%	−3.92*	−1.65	0.09%
pchsaleinv	0.26	0.39	0.14%	−0.96	−0.56	0.10%	−0.30	−0.35	0.09%

(*continued on next page*)

Table 3 (*continued*).

Fundamental	Level			Slope			Curvature		
	Coeff	*t*-stat	R^2	Coeff	*t*-stat	R^2	Coeff	*t*-stat	R^2
pctacc	−1.37	−0.89	0.09%	−2.16	−0.89	0.12%	−46.86	−1.07	0.10%
pricedelay	1.04	0.75	0.12%	−0.81**	−2.07	0.10%	−0.46	−1.63	0.10%
ps	0.20	0.30	0.14%	−8.11	−1.35	0.10%	0.65	0.40	0.09%
rd	3.30	0.92	0.15%	−2.21	−1.27	0.10%	0.25	0.33	0.10%
rd_mve	1.41	0.38	0.29%	−0.36	−0.45	0.13%	−0.79	−1.10	0.13%
rd_sale	0.12	0.15	0.39%	0.42	0.58	0.09%	−1.82*	−1.78	0.10%
realestate	−3.42	−0.87	0.14%	−0.28	−0.68	0.08%	0.27	0.32	0.11%
retvol	−0.51	−0.50	1.27%	−0.38	−1.14	0.27%	0.19	0.09	0.21%
roaq	0.02	0.03	0.28%	−0.22	−0.35	0.12%	4.14*	1.72	0.13%
roavol	−0.30	−0.40	0.43%	1.34	0.73	0.16%	1.27	1.16	0.15%
roeq	8.62	1.05	0.23%	−0.93***	−3.31	0.11%	−0.93	−0.65	0.10%
roic	0.63*	1.65	0.34%	−3.39**	−2.28	0.11%	−4.66***	−2.34	0.11%
rsup	−3.06*	−1.70	0.15%	−1.28***	−2.88	0.09%	0.55	1.03	0.09%
salecash	−0.94*	−1.67	0.13%	−0.53	−0.44	0.10%	−1.29**	−2.25	0.10%
saleinv	−0.97	−1.14	0.09%	0.91	0.60	0.10%	1.27	0.86	0.11%
salerec	−1.06***	−2.52	0.13%	0.79	1.09	0.09%	−10.49	−0.99	0.10%
secured	0.38	0.81	0.23%	−1.32	−0.30	0.09%	−0.81	−0.40	0.09%
securedind	−0.91	−0.96	0.14%	−0.53	−1.17	0.10%	0.55	0.26	0.10%
sfe	−0.24	−0.72	0.33%	−0.14	−0.11	0.14%	−0.41	−0.21	0.14%
sgr	0.56	1.00	0.24%	−1.09	−1.04	0.09%	−9.15	−1.31	0.11%
sin	−0.38	−1.08	0.10%	−0.78	−1.25	0.06%	0.27	0.35	0.07%
sp	1.63	0.67	0.18%	0.33	0.31	0.10%	−0.70*	−1.75	0.08%
std_dolvol	0.14	0.46	0.35%	−0.04	−0.06	0.14%	−1.24**	−2.27	0.11%
std_turn	5.20	1.49	0.66%	−0.22	−0.47	0.12%	−0.71	−0.94	0.13%
stdcf	−0.88***	−2.48	0.41%	−254.98	−1.06	0.11%	−1.08***	−2.59	0.12%
sue	−5.12**	−2.01	0.23%	−13.06	−0.98	0.14%	0.38	0.65	0.11%
tang	−0.21	−0.18	0.34%	0.54	0.60	0.09%	−0.56	−0.52	0.11%
tb	6.04	1.54	0.09%	−0.29	−0.92	0.10%	0.67	1.02	0.09%
turn	−2.46*	−1.79	0.99%	−0.55	−0.89	0.17%	−10.76	−1.35	0.18%
zerotrade	0.29	0.43	0.61%	−1.68***	−2.36	0.22%	−1.37	−1.10	0.18%

This table reports FM regression results for cross-sectional IV curve characteristics forecasts of 94 individual firm fundamentals. We first run univariate regression of realized on forecasted IV characteristics cross-sectionally, then we estimate the time-series averages of the slope coefficients and R^2, with *t*-statistics adjusted by Newey and West (1987) standard errors with 12 lags, respectively for the changes of IV levels, slopes and curvatures. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

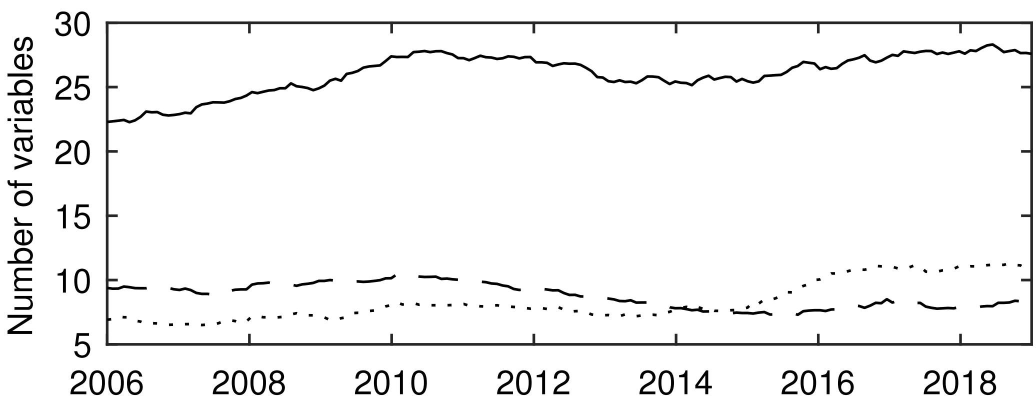

Fig. 1. Ten-year rolling averages of number of fundamentals selected by the LASSO. This figure shows the ten-year moving average of the number of fundamentals selected by the LASSO approach. The solid-, dashed- and dotted-line is for the changes of IV levels, slopes and curvature, respectively.

directly associated with a firm's activity (rd_sale, stdcf, sfe), and the other two are about valuation (beta, mve), indicating the crucial roles of fundamentals in determining the magnitude of a firm's option IV.

In predicting the changes of IV slopes and IV curvatures, the top ten fundamentals in terms of selection frequencies all exceed 13%. The top fundamentals for the IV slopes are zero trading days, 24%; size, 21%; illiquidity, 21%; dividend-price ratio (dy), 19%; unexpected quarterly earnings (sue), 16%; Beta, 15%; number of analysts covering stock (nanalyst), 15%; earnings volatility (roavol), 15%; depreciation/PP&E (depr), 14%; and idiosyncratic return volatility (idiovol), 14%. The top fundamentals for the IV curvatures are illiquidity, 19%; zero trading days, 19%; size, 18%; Beta, 17%; industry-adjusted size: (mve_ia), 16%; idiosyncratic return volatility, 15%; unexpected quarterly earnings, 15%; dividend-price ratio, 14%; number of analysts covering stock, 14%; and cash flow volatility, 13%.

Table 4
LASSO selection frequencies for IV characteristics.

Fundamental	Level	Slope	Curvature	Fundamental	Level	Slope	Curvature
absacc	28.32%	13.64%	11.89%	*mom1m*	38.11%	8.39%	12.59%
acc	25.17%	9.79%	8.04%	*mom36m*	31.82%	8.74%	6.99%
aeavol	33.22%	10.49%	10.14%	*ms*	26.92%	7.69%	6.64%
age	35.66%	11.19%	9.09%	*mve*	43.36%	21.33%	18.18%
agr	22.03%	4.90%	3.50%	*mve_ia*	29.37%	12.24%	15.73%
baspread	43.36%	10.14%	11.19%	*nanalyst*	28.67%	14.69%	13.64%
beta	55.94%	14.69%	17.13%	*nincr*	17.13%	9.09%	8.04%
bm	24.13%	6.99%	9.09%	*operprof*	18.88%	10.84%	8.39%
bm_ia	15.73%	7.69%	9.44%	*orgcap*	20.63%	6.99%	6.29%
cash	33.92%	7.69%	7.69%	*pchcapx_ia*	24.48%	3.85%	7.34%
cashdebt	31.12%	9.44%	9.44%	*pchcurrat*	20.28%	7.69%	7.69%
cashpr	13.64%	6.64%	5.94%	*pchdepr*	27.27%	9.44%	9.79%
cfp	19.93%	9.09%	9.09%	*pchgm_pchsale*	23.08%	9.44%	10.14%
cfp_ia	19.58%	9.44%	10.49%	*pchsale_pchinvt*	14.69%	7.69%	11.89%
chatoia	18.18%	9.44%	6.99%	*pchsale_pchrect*	33.57%	12.59%	11.19%
chcsho	18.53%	7.34%	4.20%	*pchsale_pchxsga*	22.03%	6.64%	7.69%
chempia	18.53%	8.04%	7.69%	*pchsaleinv*	20.63%	8.04%	6.99%
chfeps	23.08%	6.64%	7.34%	*pctacc*	22.03%	9.09%	9.44%
chinv	15.38%	7.34%	8.04%	*pricedelay*	24.48%	10.49%	11.54%
chmom	32.87%	11.19%	12.24%	*ps*	18.53%	6.99%	8.39%
chnanalyst	12.24%	2.45%	3.50%	*rd*	19.93%	6.64%	7.34%
chpmia	27.97%	7.69%	10.49%	*rd_mve*	38.46%	11.89%	12.24%
chtx	25.52%	4.90%	6.64%	*rd_sale*	46.15%	11.89%	8.74%
cinvest	32.17%	11.54%	6.99%	*realestate*	21.33%	8.39%	8.74%
convind	23.08%	5.24%	5.94%	*retvol*	31.12%	10.14%	4.55%
currat	20.98%	13.64%	11.19%	*roaq*	26.57%	8.74%	8.04%
depr	34.97%	14.34%	9.79%	*roavol*	34.97%	14.69%	10.49%
disp	30.77%	10.84%	12.24%	*roeq*	32.87%	8.74%	7.69%
divi	19.93%	11.89%	12.24%	*roic*	38.81%	8.04%	7.69%
divo	24.48%	10.14%	7.69%	*rsup*	28.32%	10.14%	8.74%
dy	37.41%	19.23%	13.99%	*salecash*	12.24%	10.14%	8.39%
ear	21.68%	9.44%	8.04%	*saleinv*	16.08%	9.44%	11.19%
egr	20.98%	6.29%	6.29%	*salerec*	27.97%	9.44%	8.04%
ep	32.52%	8.74%	9.09%	*secured*	22.73%	6.99%	5.59%
fgr5yr	31.12%	9.09%	8.74%	*securedind*	23.43%	9.44%	6.99%
gma	26.22%	5.94%	7.34%	*sfe*	41.96%	12.59%	11.89%
grcapx	25.87%	5.59%	7.69%	*sgr*	28.67%	6.64%	7.34%
grltnoa	12.24%	5.24%	7.34%	*sin*	18.88%	6.29%	5.94%
herf	15.38%	5.94%	4.55%	*sp*	21.33%	9.09%	6.29%
hire	24.13%	6.29%	5.24%	*std_dolvol*	37.41%	11.54%	10.84%
idiovol	33.92%	13.99%	15.38%	*std_turn*	39.86%	7.69%	9.09%
ill	50.35%	20.63%	18.53%	*stdcf*	43.01%	13.64%	13.29%
indmom	33.57%	7.69%	8.04%	*sue*	36.36%	16.08%	14.69%
invest	19.23%	1.40%	4.20%	*tang*	19.58%	4.55%	7.34%
IPO	25.17%	13.99%	12.94%	*tb*	13.64%	9.09%	8.39%
lev	34.62%	8.74%	11.19%	*turn*	49.30%	12.59%	12.94%
mom12m	36.36%	10.49%	12.24%	*zerotrade*	41.26%	24.13%	18.53%

This table reports the LASSO selection frequencies for firm fundamentals in cross-sectional regressions. We first estimate a cross-sectional univariate regression of IV curve characteristics in month t on each of 94 firm fundamentals in month $t-1$, then we generate forecasts for month $t+1$ with the fitted OLS coefficients and the fundamentals in month t. We estimate a cross-sectional multiple regression with the LASSO to select out those individual fundamentals to be included in combination forecasts, respectively, for the changes of IV levels, slopes, and curvatures.

Interestingly, the common fundamentals that drive the IV curves are related to the market liquidity (ill, zerotrade) and valuation (beta, mve), while those fundamentals that affect the IV skew or smirk are associated with a firm's activity (dy, sue), idiosyncratic volatility (idiovol), and attention from analysts (nanalyst). Overall, Fig. 1 and Table 4 indicate that firm fundamentals matter in predicting the cross-sectional equity IV curves over time. Therefore, we use 13 fundamentals chosen by the LASSO approach representing the top ten variables for the IV levels, slopes, and curvatures for our subsequent analysis: illiquidity, beta, size, dividend-price ratio, cash flow volatility, unexpected quarterly earnings, R&D to sales, scaled earnings forecast, idiosyncratic return volatility, earnings volatility, depreciation, number of analysts covering stock, and volatility of share turnover.[6] Panel B of Table 2 provides the summary statistics of those firm fundamentals. Statistics are calculated on the pooled data of all observations, which are standardized by the cross-sectional mean and standard deviation.

[6] The variables ill, zerotrade, turn, and baspread all measure a firm's stock liquidity, so we use ill to represent liquidity; similarly, we use mve to represent size as both mve and mve_ia measure a firm's size. In our robustness tests, we use a Bayesian shrinkage regression to address the concern about possible multicollinearity among firm fundamentals.

3.2. IV shapes and firm fundamentals

We assess the explanatory power of firm fundamentals on its equity option IV curve shapes by four sets of regressions using Eqs. (8)–(11). First, we run the cross-sectional regressions on each month and obtain the intercept and other coefficients. Then, we average the coefficients and calculate the corresponding t-statistics using Newey–West standard errors with 12 lags. We report the results for IV_{ATM}, $IV_{OTM} - IV_{ATM}$, and $(IV_{ITM} + IV_{OTM})/2 - IV_{ATM}$ to represent the level, slope, and curvature, respectively, of the IV shape.

Table 5 provides the regression results, together with the estimates for:

$$IV^{i}_{jt} = \alpha_t + \beta_{0,t} M_{jt} + \beta_{2,t} Sys_{jt} + \epsilon^{i}_{jt}, \tag{12}$$

as in the study of Duan and Wei (2009). For the volatility level, our benchmark model performs well; at the 1% significance level, the historical volatility and the skewness are positively significant, and the kurtosis is negatively significant. Among the firm fundamentals, the illiquidity, beta, cash flow volatility, R&D to sales, idiosyncratic return volatility, earnings volatility, depreciation, number of analysts covering stock, and volatility of share turnover are positively significant, while the size and scaled earnings forecast are negatively significant. The findings suggest that a firm with a poor liquidity, larger beta, more attention from analysts, smaller predicated earnings, smaller size, and so on is associated with a larger IV. The sign and significance of these firm fundamentals remain similar after controlling for the risk-neutral skewness and kurtosis in Bakshi et al. (2003) and with the inclusion of the systematic risk ratio in Duan and Wei (2009). The systematic risk ratio is negatively related to the volatility level, indicating that a lower systematic risk ratio leads to a higher level of IV. The adjusted R^2 shows that the addition of firm fundamentals contributes to a robust explanation of the cross-sectional differences in the level of IVs. It increases from 61.87% to 69.20% after the fundamentals are included in the regression, while the addition of the systematic risk ratio to the benchmark leads to a much smaller magnitude.

For the volatility slope, all the historical volatility, skewness, and kurtosis are strongly significant in explaining the volatility difference between the OTM and ATM options. Nevertheless, both the significance and sign of the historical volatility change after the inclusion of firm fundamentals, suggesting that its role may be subsumed by fundamentals. The coefficients of the dividend-price ratio and scaled earnings forecast are positively significant, while the coefficient estimates are negative for the illiquidity, size, unexpected quarterly earnings, earnings volatility, and volatility of share turnover. The findings suggest that these fundamentals perform differently when determining the volatility shape. For instance, a smaller firm suffers different downside and upside risks, resulting in a steeper volatility slope. Controlling for the systematic risk ratio does not change the performance of fundamentals, and a smaller amount of systematic risk leads to a steeper IV, consistent with the findings of Duan and Wei (2009). Again, the adjusted R^2 significantly increases from 18.85% to 21.46% as a result of incorporating firm fundamentals to the benchmark model.

A significantly larger volatility curvature is observed for firms with a smaller beta, size, unexpected quarterly earnings, and for firms with a larger dividend-price ratio, R&D to sales, and earnings forecast. The systematic risk ratio is significant in explaining the curvature. The firm fundamentals improve the adjusted R^2 from 10.98% to 12.73%. These findings reveal the relationships between firm fundamentals and the level, slope, and curvature of the IV. Generally, firms with poorer liquidity, smaller earning forecast, larger earnings volatility, or more analysts covering stock have a significantly higher volatility level, flatter volatility slope, and less pronounced curvature; smaller size firms have a higher volatility level, steeper volatility slope, and more pronounced curvature. These findings are largely consistent after controlling for the risk-neutral skewness, kurtosis, and systematic risk ratio. Thus, these findings reject Hypothesis 1 and provide evidence that the level, smile, smirk, or skew of IV are associated with the firm fundamentals. Particularly, the inclusion of fundamentals improves the explanation of the volatility slope and curvatures better than that for the volatility level, suggesting the importance of firm fundamentals in understanding the IV shape.

3.3. Economic value

Equity options are not quoted and traded in terms of standardized deltas and time to maturities appearing in the Option Metrics IV surface file. Therefore, we construct the option trading strategies using the actual market observed option price data from the Option Metrics Option Prices file. Following Goyal and Saretto (2009), we apply several data filters to remove the following: (i) options violating the no-arbitrage conditions; (ii) options with a bid price higher than the ask price; (iii) options with a zero bid price; (iv) options with a bid–ask spread lower than the minimum tick size ($0.05 for options traded below $3 and $0.1 otherwise); and (v) options with a zero open interest.

Then, we reformat the option price data to approximate the IV curve discussed in the previous section. On the last Wednesday of each month, we select firms with put options maturing in six to seven weeks and having at least one option traded in each delta range of −0.125 to −0.275, −0.425 to −0.575, or −0.725 to −0.875. We select one option from each of these delta ranges with the highest liquidity (in terms of trading volume and open interest). If multiple options have the same liquidity level, we choose the option having the lowest delta difference to −0.2, −0.5, or −0.8. To negate model fitting errors, an additional filter is applied to remove months when less than 10 firms satisfy the sample selection criterion.

As we primarily examine the profitability owing to changes in the IV, we want to remove the changes in option prices due to changes in the underlying price. We perform delta-hedging to the options by taking the delta amount of the underlying stock. At time t, the delta-neutral long position of these put options is as follows:

$$\pi^{(+,\circ,-)}_{t} = P^{(+,\circ,-)}_{t} - \delta^{(+,\circ,-)}_{t} S_t, \tag{13}$$

Table 5
Regression results for put options.

	Level					Slope					Curvature				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
HV (%)	75.91***		21.97***	21.63***	73.72***	3.65***		−2.23***	−1.24	3.24***	3.23***		−0.05	0.75	2.90***
	32.77		5.54	5.28	30.14	5.98		−2.90	−1.59	4.37	12.02		−0.08	1.37	9.04
NS (%)	1.28***		−0.01	0.01	1.07**	−5.48***		−5.74***	−5.76***	−5.50***	−1.83***		−2.02***	−2.03***	−1.87***
	4.48		−0.03	0.03	4.31	−13.73		−13.25	−13.26	−13.84	−7.12		−7.63	−7.64	−7.31
NK (%)	−2.58***		−2.56**	−2.56**	−2.56***	1.42***		1.28***	1.29***	1.42***	1.38***		1.29***	1.29***	1.37***
	−7.52		−10.27	−10.36	−7.82	13.08		9.73	9.82	13.24	18.07		15.27	15.48	18.65
ill (%)		1.37***	1.02***	1.01***			−0.75***	−0.39***	−0.36***			−0.66***	−0.42***	−0.39***	
		8.47	7.75	7.93			−4.43	−3.46	−3.25			−4.43	−3.89	−3.81	
beta (%)		2.78***	1.38***	1.17***			0.05	0.34***	0.73***			−0.19***	0.07	0.28***	
		6.85	4.00	3.76			0.56	3.41	5.98			−2.74	1.00	3.20	
mve (%)		−3.47***	−3.86***	−3.90***			−1.78***	−2.20***	−2.13***			−1.30***	−1.33***	−1.28***	
		−9.60	−12.02	−11.66			−5.69	−5.35	−5.34			−7.94	−6.14	−6.16	
dy (%)		0.02	0.22***	0.21***			0.15***	0.17***	0.18**			0.35***	0.28***	0.28***	
		0.26	3.04	2.92			2.82	4.61	4.80			11.61	8.61	8.59	
stdcf (%)		0.58***	0.59***	0.58***			0.06	0.04	0.05			0.04	0.03	0.04	
		6.60	6.50	6.46			0.89	0.80	1.00			1.24	0.93	1.19	
sue (%)		−0.08	−0.07	−0.07			−0.05**	−0.06***	−0.05***			−0.06***	−0.05***	−0.05***	
		−1.13	−1.08	−1.05			−2.42	−2.86	−2.80			−3.14	−3.00	−2.90	
rd_sale (%)		1.07***	1.12***	1.12***			0.03	0.00	0.01			0.10**	0.08**	0.08**	
		8.72	9.20	9.28			0.49	0.06	0.20			2.39	1.97	2.13	
sfe (%)		−1.30***	−1.23***	−1.23***			0.35***	0.33***	0.31***			0.19***	0.18***	0.17***	
		−7.11	−6.61	−6.65			10.55	8.42	8.17			6.20	5.49	5.21	
idiovol (%)		11.74***	6.72***	6.98***			−1.42***	0.20	−0.38*			−0.80***	0.14	−0.21*	
		20.38	9.88	9.53			−5.64	1.01	−1.87			−5.06	1.29	−1.82	
roavol (%)		1.17***	1.03***	1.03***			−0.43***	−0.23***	−0.24***			−0.14***	−0.01	−0.02	
		9.55	9.37	9.42			−5.00	−3.63	−3.78			−4.22	−0.39	−0.73	
depr (%)		0.25***	0.23***	0.24***			−0.03	0.01	0.01			−0.02	0.00	0.01	
		5.02	4.82	4.91			−0.71	0.16	0.32			−1.03	0.15	0.35	
nanalyst (%)		0.41***	−0.07	−0.05			−0.44***	0.06	0.03			−0.27***	0.07*	0.05	
		3.64	−0.68	−0.51			−2.73	0.77	0.40			−2.63	1.85	1.34	
std_turn (%)		0.31***	0.21**	0.22**			−0.13**	−0.16***	−0.17***			0.06*	0.05*	0.05	
		3.28	2.30	2.33			−2.48	−3.67	−3.79			1.89	1.82	1.55	
Sys (%)				0.82	−9.05***				−4.32***	−1.88**				−2.53***	−2.34***
				0.45	−2.79				−5.39	−2.07				−3.94	−4.41
Adj R^2 (%)	61.87	66.33	69.20	69.24	62.46	18.85	4.22	21.46	21.54	19.05	10.98	3.47	12.73	12.78	11.17

This table reports the cross-sectional regressions results, regressing IV function characteristics on realized volatility, implied moments, fundamental measures, and systematic risk ratio:

$$IV^i_{jt} = \alpha_t + \beta_{0,t} M_{jt} + \beta_{1,t} F_{jt} + \beta_{2,t} Sys_{jt} + \epsilon^i_{jt},$$

where IV^i_{jt} represents one of the level, slope and curvature of the IV function captured by IV_{ATM}, $IV_{OTM} - IV_{ATM}$, and $(IV_{ITM} + IV_{OTM})/2 - IV_{ATM}$ respectively. ITM, ATM, and OTM options are with delta of −0.8, −0.5, and −0.2, respectively. Specification (1) is the benchmark model considering only M_{jt}, the vector containing historical volatility, risk-neutral skewness and kurtosis, whilst specification (2) only considers firm fundamental vector F_{jt}. Specification (4) considers all the market and fundamental measures, as well as the systematic risk ratio Sys_{jt}. Specification (3) and (5) are cases without the systematic risk ratio and the firm fundamental vector, respectively. On each month we first obtain the intercept and other coefficients, these coefficients are then averaged and the corresponding t-statistic is calculated using the Newey–West standard errors with 12 lags. The sample period is from 1996 to 2019.

where $P^{(+,\circ,-)}_t$ and $\delta^{(+,\circ,-)}_t$ denote the put option prices and their corresponding deltas. We use these delta-hedged put positions as building blocks to construct other delta-neutral option strategies approximating the IV curve:

$$\pi^s_t = \pi^-_t - \pi^\circ_t, \tag{14}$$

$$\pi^c_t = \left(\frac{\pi^+_t + \pi^-_t}{2}\right) - \pi^\circ_t. \tag{15}$$

The option strategy (14) corresponds to an OTM-minus-ATM put spread, while the option strategy (15) corresponds to an ATM butterfly.

We build portfolios of option strategies based on the difference between model estimated and market observed IV curve characteristics. For any particular IV curve characteristic, if its model estimated value is greater (smaller) than the market observed value, we invest in a long (short) position in its corresponding option strategy:

$$\Pi^i_t = \sum_{j=1}^{J} w^i_{j,t} \pi^i_{j,t}, \quad i = \circ, s, c. \tag{16}$$

On each trading date t, we normalize the weights $w^i_{j,t}$ for J firms by the distance between the model estimated value $\widehat{IV}^i_{jt}$ and the market observed value IV^i_{jt}:

$$w^i_{j,t} = \frac{\widehat{IV}^i_{jt} - IV^i_{jt}}{\sum_{j=1}^{J} \left| \widehat{IV}^i_{jt} - IV^i_{jt} \right|}. \tag{17}$$

We also construct equal-weighted portfolios for robustness, taking an equal amount of long (short) positions in the relevant option strategies when the model estimated IV curve characteristics are higher (smaller) than their market observed counterparts.

We hold these portfolios for one month and analyze the portfolio performance by calculating the portfolio return as the weighted average of returns from each firm:

$$\Delta\Pi^i_t = \sum_{j=1}^{J} w^i_{j,t-1} \frac{\pi^i_{j,t} - \pi^i_{j,t-1}}{\pi^i_{j,t-1}}, \quad i = \circ, s, c. \tag{18}$$

Table 6
Put option portfolios returns.

Panel A: Distance weighted	Level					Slope					Curvature				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
Mean (%)	19.00***	22.94***	21.22***	21.30***	19.09***	8.87***	10.69***	9.35***	9.21***	8.83***	3.69***	4.08***	3.95***	3.91***	3.68***
	22.26	24.58	23.46	23.59	22.35	3.93	5.90	4.77	4.56	3.98	8.85	10.73	9.65	9.60	8.96
ΔMean (%)		3.94***	2.22***	2.29***	0.08		1.82*	0.49	0.34	−0.03**		0.39**	0.26*	0.22	−0.01
		8.71	5.93	6.06	0.64		1.88	0.60	0.38	−0.34		2.24	1.87	1.55	−0.16
IR	4.55	5.03	4.80	4.82	4.57	0.80	1.21	0.98	0.93	0.81	1.81	2.19	1.97	1.96	1.83
ΔIR (%)		10.43	5.43	5.99	0.43		50.16	21.51	16.04	1.37		21.19	8.94	8.49	1.25
Panel B: Equal weighted	Level					Slope					Curvature				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
Mean (%)	3.00***	5.55***	4.53***	4.65***	3.11***	0.26	1.26	1.63**	1.66**	0.44	0.47*	0.90***	0.75***	0.79***	0.44*
	6.09	12.02	10.02	10.42	6.38	0.29	1.58	2.15	2.18	0.48	1.69	3.56	3.02	3.10	1.65
ΔMean (%)		2.56***	1.53***	1.65***	0.11		1.00	1.37**	1.40**	0.18		0.43**	0.28**	0.32***	−0.03
		6.42	4.55	5.05	0.76		1.58	2.00	2.06	1.14		2.57	2.40	2.65	−0.46
IR	1.25	2.46	2.05	2.13	1.30	0.06	0.32	0.44	0.45	0.10	0.34	0.73	0.62	0.63	0.34
ΔIR (%)		97.41	64.57	71.19	4.74		445.76	640.03	652.31	65.41		111.18	79.03	83.89	−2.19

This table reports the annualized mean returns generated from the delta-neutral put option portfolios, distance weighted in Panel A, and equal weighted in Panel B. ΔMean reports the mean of improvements over the benchmark model. Information ratio (IR) and IR improvement over the benchmark portfolio are also reported. Specification (1) is the benchmark model considering only the vector containing historical volatility, risk-neutral skewness and kurtosis, whilst specification (2) only considers the firm fundamental vector. Specification (4) considers all the market and fundamental measures, as well as the systematic risk ratio. Specification (3) and (5) are cases without the systematic risk ratio and the firm fundamental vector, respectively.

Table 6 reports the investment performance of portfolios constructed for ATM IV level, IV slope, and IV curvature. The economic benefit of incorporating firm fundamentals is significant in most of our investment exercises. For the IV level, the benchmark portfolio performs well, providing an annualized return of 19.00% and information ratios (IR), a risk-adjusted return measured by annualized return over annualized standard deviation, of 4.55. While the systematic risk ratio can modestly improve the portfolio performance, portfolios constructed using fundamentals perform significantly superior, increasing the IR ratio by 10.43%.

The IV slope and curvature portfolios incorporating firm fundamentals outperform those of the benchmark model in Eq. (8) and of the model with only the systematic risk ratio, especially for the distance-weighted portfolio. For example, the slope and curvature portfolios of the benchmark model generate annualized returns of 8.87% and 3.69% and IR ratios of 0.80 and 1.81, and the portfolios with fundamentals increase significantly the returns to 10.69% and 4.08% and the IR ratios to 1.21 and 2.19, respectively.

Overall, the investment analysis using the quoted price of the options consolidates our finding in Section 3.2, providing consistent evidence rejecting Hypothesis 2.

3.4. Reconciliation with option puzzles

In this subsection, we investigate the role of firm fundamentals in improving the understanding of two robust stylized facts and option puzzles: the overreaction and expectations puzzles.

Stein (1989) documents a puzzle in options markets that the longer-term IV overreacts to changes in the shorter-term IV. We follow Christoffersen et al. (2013) by using the 2-month and 1-month IVs with the following regression:

$$(IV_{t+1}^{1M} - IV_t^{1M}) - 2(IV_t^{2M} - IV_t^{1M}) = \alpha_0 + \alpha_1 IV_t^{1M} + \epsilon_{t+1}, \tag{19}$$

where IV^{1M} and IV^{2M} denote the 1- and 2-month ATM IVs. If the overreaction exists, the regression coefficient α_1 should be significantly negative. We run the above regression for each firm, estimate the average of α_1 and the corresponding t-statistics using the Newey–West standard errors with 12 lags, and test the null hypothesis that $\alpha_1 = 0$ for the market observed IVs, and IVs generated by the models with and without firm fundamentals, respectively.

Panel A of Table 7 reports the coefficient estimates, and Panel B reports the coefficient difference. We notice an overreaction; all models generate a significantly negative α_1. Panel B reveals that relative to the model without fundamentals, the model with fundamentals generates a significantly larger α_1, indicating that the inclusion of fundamentals moderates the overreaction. In other words, the incorporation of fundamentals holds significance and relates to the overreaction puzzle.

The other stylized fact is that, on average, the risk-neutral volatility exceeds the physical volatility owing to the negative price of variance risk, examined by Bakshi and Kapadia (2003); hence, the short-sell straddles are profitable (Coval and Shumway, 2001; Christoffersen et al., 2013). To test this puzzle, we hypothetically construct a short straddle strategy for each firm by using 30-day ATM call and put options and hold them for one month. The market observed and model estimated volatilities are used to compute the option premium by the Black–Scholes formula, and the final return of the short straddle portfolio is the difference between the sum of call and put option premiums and $|S_T - K|$. We restrain our investment of one dollar on each straddle position and estimate the equal-weighted average returns.

Panel A of Table 8 reports the straddle returns, and Panel B reports their difference. The straddle returns using market observed and model estimated volatilities are significantly larger than zero. This finding indicates that the expectations puzzle widely acknowledged in index options also holds for individual options. The volatilities estimated with fundamentals generate significantly smaller straddle returns than those market observed and by the benchmark model.

Overall, our results demonstrate that the two stylized facts for index options are also true for individual options. The firm fundamentals help to understand both the overreaction and expectations puzzles and Hypothesis 3 is rejected.

Table 7
Overreaction facts.

Panel A: Coefficient					
Obs	(1)	(2)	(3)	(4)	(5)
−0.290***	−0.412***	−0.386***	−0.391***	−0.389***	−0.409***
−38.94	−45.75	−45.91	−46.65	−47.03	−43.50
Panel B: Coefficient difference					
(1)-obs	(2)-obs	(3)-obs	(4)-obs	(5)-obs	
−0.121***	−0.096***	−0.100***	−0.098***	−0.116***	
−15.91	−12.93	−14.39	−14.11	−15.08	
(2)–(1)	(3)–(1)	(4)–(5)			
0.025***	0.020***	0.017***			
7.63	8.09	6.55			

This table reports the result for:

$$(IV_{t+1}^{1M} - IV_t^{1M}) - 2(IV_t^{2M} - IV_t^{1M}) = \alpha_0 + \alpha_1 IV_t^{1M} + \epsilon_{t+1},$$

where IV^{1M} and IV^{2M} denote the one- and two-month ATM IVs. We run the above regression for each firm, take the average of α_1, and then test the null hypothesis that $\alpha_1 = 0$ in Panel A. Panel B reports the coefficient difference between models and market observations. Obs is for observed option data, specification (1) is the benchmark model considering only the vector containing historical volatility, risk-neutral skewness and kurtosis, whilst specification (2) only considers the firm fundamental vector. Specification (4) considers all the market and fundamental measures, as well as the systematic risk ratio. Specification (3) and (5) are cases without the systematic risk ratio and the firm fundamental vector, respectively.

Table 8
Straddle returns using market and model volatilities.

Panel A: Average return					
Obs	(1)	(2)	(3)	(4)	(5)
0.419***	0.419***	0.421***	0.416***	0.413***	0.422***
29.67	28.74	30.71	27.76	28.17	28.09
Panel B: Average return difference (%)					
(1)-obs	(2)-obs	(3)-obs	(4)-obs	(5)-obs	
0.043	0.148	−0.337*	−0.410**	0.229	
0.18	0.88	−1.72	−2.38	0.92	
(2)–(1)	(3)–(1)	(4)–(5)			
0.158	−0.444**	−0.746***			
0.82	−2.05	−4.52			

This table reports the results for short straddles. We estimate the straddle returns for each firm, take the equal-weighted average and then test its significance in Panel A. Panel B reports the coefficient difference between models and market observations. Obs is for observed option data, specification (1) is the benchmark model considering only the vector containing historical volatility, risk-neutral skewness and kurtosis, whilst specification (2) only considers the firm fundamental vector. Specification (4) considers all the market and fundamental measures, as well as the systematic risk ratio. Specification (3) and (5) are cases without the systematic risk ratio and the firm fundamental vector, respectively.

4. Model

In this section, we use a compound option model to demonstrate the mechanism of how a firm's fundamental information influences the price of its equity options.

4.1. Firm fundamentals and the IV curve

As a firm's equity and debt can be viewed as contingent claims on firm value (Black and Scholes, 1973; Merton, 1974), the individual equity options can be viewed as compound options on the underlying firm value, as in the study of Geske et al. (2016). If firm fundamentals reflect the financial status of the firm and, ultimately, the evolution of the firm value process, these fundamentals and the uncertainties associated with them should be considered important determinants of the characteristics of the firm's equity options. We demonstrate this relation by extending the Geske (1977) model.[7]

We start with the Merton model assuming that the value of the firm, V, satisfies the following stochastic differential equation (SDE):

$$dV = \alpha V dt + \sigma V dW - cV dt, \tag{20}$$

[7] In addition to the Geske (1977) model, Galai and Masulis (1976) also introduce a joint model combining capital asset and option pricing models. They show the influence of unanticipated changes in capital and asset structures on the firm's debt and equity.

where α is the expected rate of return of the firm per unit of time, c is the firm's financing cost per unit of time, σ is the volatility of the firm value process, and W is a standard Brownian motion. The financing cost comprises two components: dividend payments to its shareholders c_e and coupon/interest payments to the debt holders c_d,

$$cV = c_e + c_d. \tag{21}$$

Eqs. (20) and (21) relate the evolution of the firm value to its stability, expected profitability, dividend policy, and interest obligations.

If a marketable security, F, whose value is a function of firm value and time, $F = f(V,t)$, it must be governed by, under no arbitrage conditions,

$$dF = \alpha_F F dt + \sigma_F F dW_F - c_F dt, \tag{22}$$

where α_F is the expected rate of return of the security per unit of time, c_F is the security's payout per unit of time, and σ_F is the volatility of the security value process. Merton (1974) shows that the value of F must satisfy:

$$\frac{1}{2}\sigma^2 V^2 F_{VV} + (r-c)V F_V - rF + F_t + c_F = 0. \tag{23}$$

A firm's equity and equity options satisfy the same partial differential equation (23) but have different boundary conditions.

Now, we depart from the Merton (1974) framework and assume that firms can continue to operate their businesses under a stable capital structure. In other words, a firm cannot cease to exist at the maturity of its debt. However, it must repay its existing debt obligation by issuing new debt with a similar structure, such that a firm's equity and debt only mature when the firm can no longer secure new debt to cover its existing debt or when the firm cannot meet its interest payment obligations. On the one hand, the firm's potential debt investors will consider the firm's fundamentals when making investment decisions. Hence, the fundamental measures will influence the outcome of the firm's new debt issuance. On the other hand, fundamental measures also indicate whether the firm will face difficulties when making interest payments. Consistent with this, empirical evidence suggests that fundamental measures are important determinants of credit risk (e.g., Collin-Dufresn et al., 2001; Bai and Wu, 2016). Therefore, we assume that the time-to-maturity of the firm's equity is a function of the firm's credit-sensitive fundamental measures, **d**,

$$T_e = T(\mathbf{d}). \tag{24}$$

Conceptually, this provides a channel through which firm fundamentals (other than leverage and dividend) influence the prices of equity and equity options. In this setting, the influence to the stock and option prices are driven by firm fundamentals' influence to T_e. Bai and Wu (2016) show that better interest coverage and liquidity and more profitable firms have higher creditworthiness, and are further away from a default. A sudden change in T_e induced by the introduction of credit-sensitive information can induce jumps in equity prices, especially around earnings announcement dates (Dubinsky et al., 2019). As we focus on the cross-sectional characteristics differentiating firms with different sets of default sensitive information, we ignore the time variation of **d** within each firm to keep the model parsimonious and tractable.

Therefore, the value of the equity $S(V,t)$ is governed by:

$$\frac{1}{2}\sigma^2 V^2 S_{VV} + (r-c)V S_V - rS + S_t + c_e = 0, \tag{25}$$

subject to the boundary condition:

$$S(V,T_e) = (V_{T_e} - B)^+, \tag{26}$$

where B denotes the principal of the firm's overall debt obligation. Similarly, the value of the equity put option $P(V,t)$ maturing at T_p with strike price K satisfies:

$$\frac{1}{2}\sigma^2 V^2 P_{VV} + (r-c)V P_V - rP + P_t = 0, \tag{27}$$

subject to the boundary condition:

$$P(V,T_p) = (K - S(V,T_p))^+. \tag{28}$$

We assume that $T_e > T_p$ and c_e grows at a constant rate of g. Under this assumption, the stock price at time t can be expressed as:

$$\begin{aligned} S_t &= \int_0^{T_e-t} e^{-(r-g)u} c_e du + e^{-r(T_e-t)} \int_B^\infty (V_{T_e} - B)^+ f(V_{T_e}|t) dV_{T_e} \\ &= (1 - e^{-(r-g)(T_e-t)})\frac{c_e}{r-g} + e^{-r(T_e-t)} \int_B^\infty (V_{T_e} - B)^+ f(V_{T_e}|t) dV_{T_e}. \end{aligned} \tag{29}$$

Viewing the put option as a compound option on the firm value, its value can be determined by a modified (Geske, 1977) formula,

$$\begin{aligned} P_t &= \left(e^{-rT_p} K - (e^{-rT_p} - e^{-rT_e + g(T_e - T_p)})\frac{c_e}{r-g} \right) \int_0^{\bar{V}} f(V_{T_p}|V_0) dV_{T_p} \\ &\quad - e^{-rT_e} \int_0^{\bar{V}} \int_B^\infty (V_{T_e} - B) f(V_{T_e}|V_{T_p}) f(V_{T_p}|V_0) dV_{T_e} dV_{T_p}, \end{aligned} \tag{30}$$

where $\bar{V}$ denotes V_{T_p}, and it is the solution to:

$$(1-e^{-(r-g)(T_e-t)})\frac{c_e}{r-g}+e^{-r(T-t)}\int_B^{\infty}(V_{T_e}-B)^+ f(V_{T_e}|V_t)dV_{T_e}=K, \tag{31}$$

with $f(V_{t_2}|V_{t_1})$ denoting the transition density of V from t_1 to $t_2>t_1$,

$$f(V_{t_2}|V_{t_1})=\frac{1}{V_{t_2}\sigma\sqrt{2\pi(t_2-t_1)}}\exp\left(-\frac{1}{2}\left(\frac{\ln(V_{t_2}/V_{t_1})-(r-c-\frac{1}{2}\sigma^2)(t_2-t_1)}{\sigma\sqrt{(t_2-t_1)}}\right)^2\right). \tag{32}$$

The price of the call option can be derived in a similar fashion.

Notably, the equity price in Eq. (29) can be interpreted as the combination of the discounted dividends up to the firm's default and an option premium that gives the equity holder the right to default at time T_e. Given the solution for the equity put option in Eq. (30), one may then calculate the put option prices across different strikes K. Subsequently, these option prices can be ported back to the Black–Scholes formula for calculating the IV at each strike price.

To study the impact of fundamentals, we must determine how the IV curve changes with respect to the face value of the debt, dividend payout, and the time to default. Despite the above option formulas, the effects can only be analyzed using numerical simulations due to the need for inverting the Black–Scholes formula. Using $V_t = 100$, $B = 65$, $r = 0.01$, $\sigma = 0.2$, $T_e = 5$, $T_p = 0.25$, $c_e = 1$, $c = 0.025$, and $g = 0.002$ as the basis of the parameters, we first calculate the stock price using Eq. (29), and the option prices using Eq. (30) for an equally spaced set of option moneyness, defined as $m = \frac{K}{S_t}$, between 0.7 and 1.3. The stock price, option prices, and relevant parameters are then plugged into the Black–Scholes formula to estimate the corresponding IV curve. A new IV curve can be produced for, for example, a different dividend payout rate c_e while holding other parameters fixed.

Note that the dividend growth rate controls the book-to-market ratio of the firm in our model when holding other model parameters fixed. Therefore, it can be used to analyze the relation between the IV curve and the book-to-market ratio. For a given book-to-market ratio, we first solve g using Eq. (29) while fixing the other parameters, the IV curve can then be calculated in the same fashion as discussed in the previous paragraph.

In Fig. 2, we plot the IV curve against moneyness showing the change in the IV curve with respect to changes in the dividend payout, time to default, face value of the debt, and book-to-market ratio. We also perform numerical differentiation to produce plots for IV slope and curvature against option moneyness. Fig. 2 shows that our compound option model for option pricing can generate IV smirks, and it is scale invariant if the dividend payout is kept at a constant proportion to the current asset value.

Note that the expository compound option model has its limitations. It only considers the firm's dividend and default option embedded in the stock price, ignoring the risk premiums relating to, for example, the time-varying volatility, dividend growth, and growth options of the firm. Nevertheless, it is a useful model for understanding the relation between firm fundamentals and the IV curve.

4.2. Firm fundamental uncertainties and the volatility risk premium

The solutions to the stock and stock option prices in Eqs. (29)–(30) are derived assuming a constant dividend, dividend growth, and deterministic default time. Although they help demonstrate the relation between firm fundamentals, stock price, and stock option prices, they have no direct relation with the most important stylized fact of stock options—the volatility risk premium. To investigate the relation between firm fundamentals and the wedge between physical volatility $\sigma^{\mathbb{Q}}$ and risk-neutral volatility $\sigma^{\mathbb{P}}$, we consider alternative stock price specifications.

Specifically, we assume that the stock price is governed by the following process:

$$\frac{dS(t)}{S(t)}=(\mu-c_e(t))dt+\sigma_s dW(t)+dJ(t), \tag{33}$$

where dividend process $c_e(t)$ is driven by:

$$dc_e(t)=\delta c_e(t)dt+\sigma_c dW_c(t). \tag{34}$$

We also assume that the two Brownian motions are correlated, $dW(t)dW_c(t)=\rho dt$. The jump term is defined as $dJ(t)=Z(t)dN(t)$, where $N(t)$ is a Poisson counting process with a jump size $Z(t)$ assumed to be independently and identically distributed, $Z(t)\sim N(\mu_Z,\sigma_Z^2)$. This reduced-form specification of the stock price process is directly analogous to the structural form in Eq. (29) with two modifications. First, the dividend payout is assumed to be stochastic; the uncertainty associated with the dividend and dividend growth can, in turn, contribute to stock return volatility. Second, instead of specifying a deterministic default time of the stock, a jump term is introduced to capture any credit-related jumps in stock prices. Notably, the jump intensity and jump size distribution are related to the credit-sensitive fundamentals of the underlying firm.

In this model, explicit solutions exist for both S_t and c_t:

$$\begin{aligned} ln(S(t))-ln(S_0)=&(\mu-\frac{1}{2}\sigma_s^2)t-\frac{c_{e,0}}{\delta}(e^{\delta t}-1)+\sigma_s\int_0^t dW(u)\\ &+\frac{1}{\sigma}\int_0^t(e^{\delta(t-u)}-1)\sigma_c dW_c(u)+\int_0^t dJ(u),\\ c_e(t)=&c_e(0)+\int_0^t e^{\delta(t-u)}\sigma_c dW_c(u). \end{aligned} \tag{35}$$

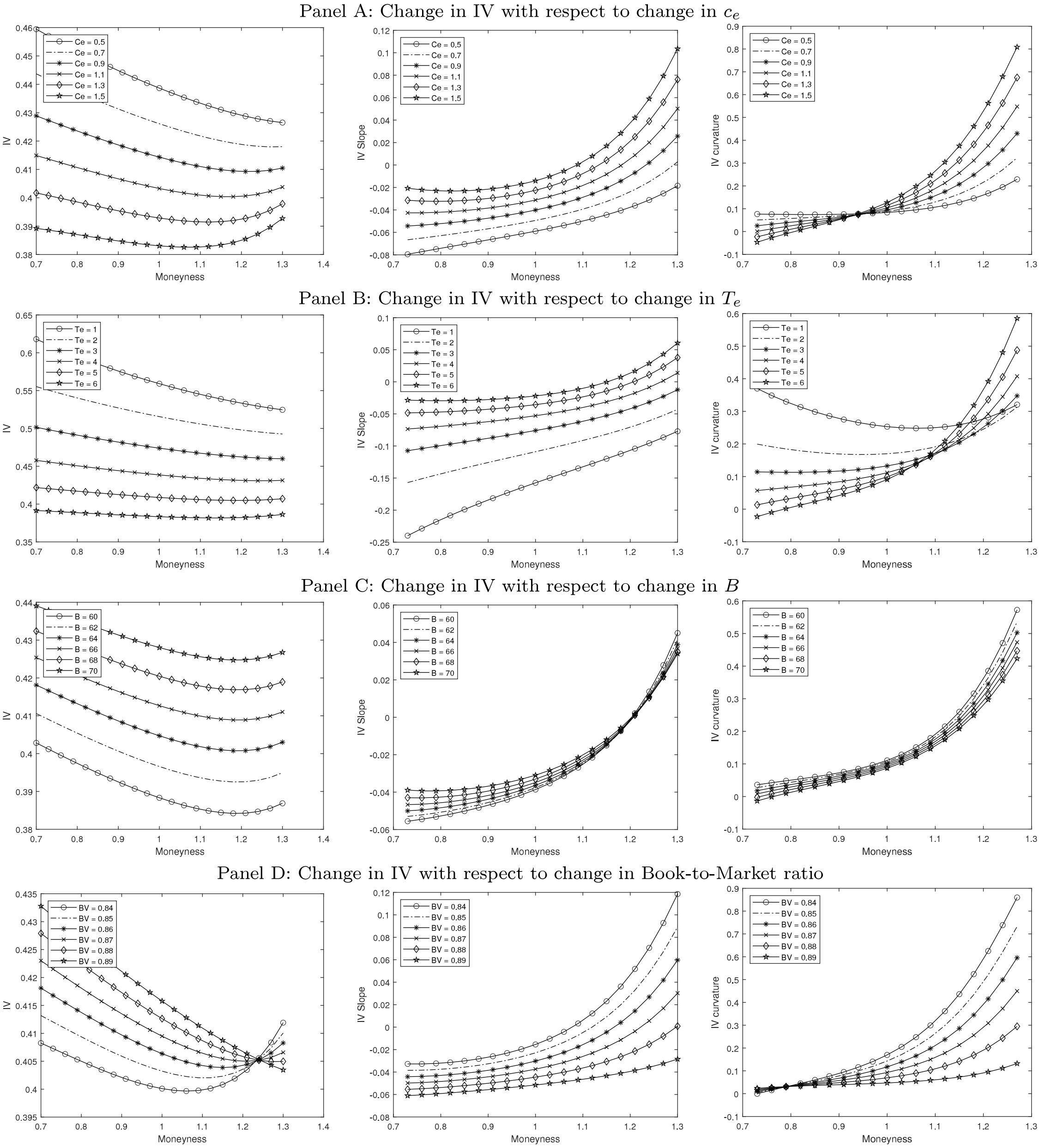

Fig. 2. Change in IV curve due to change in firm characteristics. In Fig. 2, we plot the IV level, slope, and curvature as functions of option moneyness, simulated by the compound option pricing model, with respect to change in dividend payout c_e in Panel A, time to default T_e in Panel B, face value of the debt of a firm B in Panel C, and book-to-market ratio in Panel D, respectively. We use $V_t = 100$, $B = 65$, $r = 0.01$, $\sigma = 0.2$, $T_e = 5$, $T_p = 0.25$, $c_e = 1$, $c = 0.025$, and $g = 0.002$ as the basis of the parameters used in the simulation.

We are only interested in the unconditional variance of the stock log return $r_\tau(t)$ over τ period. Under physical measure $\mathbb{P}$, the variance can be computed as follows:

$$Var^{\mathbb{P}}(r_\tau(t)) = \left(\sigma_s^2 + \frac{\sigma_c^2}{\delta^2} + \frac{\rho\sigma_s\sigma_c}{\delta}\right)\tau + \frac{\sigma_c^2}{\delta^3}\left(2e^{-\delta\tau} - \frac{1}{2}e^{-2\delta\tau} - \frac{3}{2}\right) + \frac{2\rho\sigma_s\sigma_c}{\delta^2}\left(1 - e^{-\delta\tau}\right) + \lambda\tau\sigma_j^2. \tag{36}$$

Under the risk-neutral measure, we can define the risk-neutral parameters $\delta^{\mathbb{Q}}$ and $\lambda^{\mathbb{Q}}$ owing to the uncertainty of these fundamental measures. For example, Buraschi et al. (2014) shows that dividend and dividend growth uncertainty and investor disagreement about these uncertainties produce the variance risk premium for individual stock options. Empirical evidence also supports the existence

Table 9
Regression results for call options.

	Level					Slope					Curvature				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
HV (%)	73.84***		18.66***	18.97***	71.86***	5.23***		−0.69	−0.19	4.63***	2.39***		−1.67***	−1.03	2.12***
	38.31		4.99	4.80	32.75	6.19		−1.06	−0.31	6.38	10.44		−2.57	−1.63	7.86
NS (%)	5.17***		4.00***	4.00***	5.00***	5.32***		4.85***	4.85***	5.25***	1.35***		1.17***	1.16***	1.33***
	8.41		8.18	8.20	8.62	20.39		18.71	18.67	20.08	7.20		5.57	5.50	6.93
NK (%)	−1.58***		−1.50**	−1.49**	−1.57***	2.82***		2.64***	2.65***	2.81***	1.77***		1.67***	1.68***	1.76***
	−6.94		−10.06	−10.10	−7.16	31.11		26.60	26.61	30.68	18.26		20.32	20.19	18.33
ill (%)		1.54***	1.21***	1.21***			−0.44***	−0.20*	−0.17			−0.75**	−0.52**	−0.49**	
		8.89	8.51	8.36			−2.84	−1.70	−1.54			−2.46	−2.40	−2.44	
beta (%)		2.83***	1.76***	1.68***			−0.71***	−0.05	0.22			−0.25***	0.21***	0.44***	
		6.96	5.61	6.98			−12.83	−0.77	1.63			−4.72	3.37	4.08	
mve (%)		−3.79***	−3.60***	−3.59***			−3.19***	−2.24***	−2.19***			−1.92***	−1.53***	−1.48***	
		−7.96	−9.23	−9.08			−8.25	−6.18	−6.24			−2.63	−2.55	−2.62	
dy (%)		−0.15	−0.12	−0.11			0.56***	0.24***	0.25**			0.31***	0.16***	0.16***	
		−1.47	−1.50	−1.39			9.74	6.57	6.73			9.26	5.26	5.40	
stdcf (%)		0.58***	0.58***	0.57***			0.09	0.06	0.07			0.06	0.02	0.03	
		5.38	5.20	5.20			1.65	1.41	1.55			1.06	0.60	0.75	
sue (%)		−0.09	−0.09	−0.09			−0.04	−0.03	−0.03			−0.02	−0.02	−0.02	
		−1.33	−1.29	−1.27			−1.61	−1.41	−1.41			−1.22	−1.16	−1.11	
rd_sale (%)		0.99***	1.02***	1.03***			0.18***	0.12**	0.12**			0.12**	0.074*	0.08**	
		7.47	7.80	7.85			2.95	2.32	2.33			2.93	1.88	2.01	
sfe (%)		−1.14***	−1.07***	−1.08***			0.20***	0.20***	0.19***			0.24***	0.22***	0.21***	
		−6.76	−6.53	−6.56			4.30	4.57	4.51			4.25	3.83	3.79	
idiovol (%)		11.14***	6.90***	6.85***			−0.89***	0.40***	0.05			−1.02***	0.21**	−0.14	
		21.95	10.41	8.64			−6.80	2.92	0.43			−3.21	2.02	−1.00	
roavol (%)		1.02***	0.86***	0.85***			−0.16***	−0.07*	−0.08*			−0.17***	−0.07	−0.079*	
		11.13	10.12	10.17			−3.03	−1.70	−1.85			−2.92	−1.63	−1.75	
depr (%)		0.29***	0.26***	0.26***			0.08*	0.07**	0.08**			0.02	0.02	0.02	
		5.04	5.19	5.25			1.71	2.12	2.22			0.69	0.98	1.21	
nanalyst (%)		0.60***	0.19*	0.19*			−0.30***	0.04	0.03			−0.30***	0.03	0.02	
		4.83	1.84	1.88			−3.05	0.69	0.41			−2.64	0.85	0.47	
std_turn (%)		0.12	0.10	0.09			−0.44**	−0.34***	−0.35***			−0.13**	−0.08*	−0.09**	
		1.26	1.02	0.96			−6.88	−6.37	−6.33			−2.44	−1.95	−2.03	
Sys (%)				0.18	−6.79*				−3.09***	−4.65**				−2.58***	−1.74***
				0.10	−1.95				−3.62	−7.02				−3.84	−3.11
Adj R^2 (%)	60.41	64.43	67.27	67.32	60.92	25.08	8.09	28.05	28.08	25.33	13.84	4.63	15.79	15.83	13.96

This table reports the cross-sectional regressions results, regressing IV function characteristics on realized volatility, implied moments, fundamental measures, and systematic risk ratio: $IV^i_{jt} = \alpha_t + \beta_{0,t} M_{jt} + \beta_{1,t} F_{jt} + \beta_{2,t} Sys_{jt} + \epsilon^i_{jt}$,
where IV^i_{jt} represents one of the level, slope and curvature of the IV function captured by IV_{ATM}, $IV_{OTM} - IV_{ATM}$, and $(IV_{ITM} + IV_{OTM})/2 - IV_{ATM}$ respectively. ITM, ATM, and OTM options are with delta of 0.8, 0.5, and 0.2, respectively. Specification (1) is the benchmark model considering only M_{jt}, the vector containing historical volatility, risk-neutral skewness and kurtosis, whilst specification (2) only considers firm fundamental vector F_{jt}. Specification (4) considers all the market and fundamental measures, as well as the systematic risk ratio Sys_{jt}. Specification (3) and (5) are cases without the systematic risk ratio and the firm fundamental vector, respectively. On each month we first obtain the intercept and other coefficients, these coefficients are then averaged and the corresponding t-statistic is calculated using the Newey–West standard errors with 12 lags. The sample period is from 1996 to 2019.

of the jump risk premium (Pan, 2002; Orłowski et al., 2020). Therefore, we are able to produce a wedge between the variances in $\mathbb{P}$ and $\mathbb{Q}$.

5. Robustness tests

In the sections above, we use 30-day put options for the main analysis. In the robustness tests, we obtain the results with 30-day calls and 60-day puts. We also run Bayesian shrinkage cross-sectional regressions to moderate the concerns about multicollinearity and non-linear relations, in line with Bai and Wu (2016).

5.1. Other options

We report the results of the main regression using alternative options. Table 9 shows the regression results for 30-day calls, and Table 10 shows the results for 60-day puts. Our conclusions remain unchanged. Fundamentals play a significant role in explaining the IV curve of call options and longer-dated puts; they play an even larger role in determining the IV slope of calls than that of puts.

5.2. Bayesian shrinkage regression

Concerns about our above empirical tests include possible nonlinearity, missing observations, and multicollinearity among firm fundamentals. A Bayesian shrinkage method, proposed by Bai and Wu (2016), is useful in circumventing these issues by generating a weighted average valuation. Specifically, the contributions of each firm fundamental measure to the option IV curve shapes are expressed in terms of univariate local linear regression estimates. These estimates are then stacked using weights calculated and updated by a Bayesian regression controlled by a degree of intertemporal smoothness parameter. Karolyi (1993) applies a similar approach for volatility estimation and notes an improvement in the option valuation. Details are in Appendix.

The Bayesian approach allows us to examine the explanatory powers over time. In Fig. 3, we plot the time series of the R^2 from the cross-sectional Bayesian shrinkage regression. The solid line denotes the benchmark model and the dashed line denotes the model with fundamentals. Clearly, the model with firm fundamentals outperforms the benchmark model with a larger and more stable explanatory power across different periods. Table 11 reports the summary of the R^2 estimates. On average, the benchmark model generates a 0.69, 0.29, and 0.19 R^2 for the volatility level, slope, and curvature, respectively. The model incorporating firm fundamentals significantly improves the performance to 0.76, 0.36, and 0.26, respectively.

Table 10
Regression results for 60-day put options.

	Level					Slope					Curvature				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
HV (%)	78.07***		21.56***	21.61***	75.88***	2.96***		−2.27***	−1.38*	2.68***	2.39***		0.51	1.12**	2.00***
	26.32		5.30	5.10	25.54	4.96		−2.90	−1.71	3.70	10.10		0.99	2.19	6.34
NS (%)	1.20***		0.02	0.03	0.99**	−2.87***		−3.01***	−3.03***	−2.87***	−0.79***		−0.95***	−0.96***	−0.83***
	4.79		0.13	0.18	4.80	−10.79		−10.82	−10.82	−11.13	−5.23		−6.00	−6.01	−5.52
NK (%)	−2.26***		−2.16**	−2.16**	−2.24***	0.70***		0.62***	0.63***	0.70***	0.73***		0.65***	0.66***	0.72***
	−6.72		−8.71	−8.80	−6.92	13.03		9.90	10.04	13.27	21.20		17.07	17.26	21.77
ill (%)		1.43***	1.16***	1.15***			−0.35***	−0.18*	−0.14			−0.34***	−0.22**	−0.20**	
		8.19	7.99	8.10			−2.80	−1.84	−1.53			−2.97	−2.39	−2.28	
beta (%)		2.80***	1.52***	1.34***			0.21**	0.40***	0.77***			−0.18***	−0.05	0.16**	
		6.92	4.61	4.80			2.19	3.84	5.51			−3.20	−0.97	2.33	
mve (%)		−2.77***	−3.09***	−3.11***			−1.30***	−1.56***	−1.48***			−1.06***	−1.06***	−1.01***	
		−9.83	−12.50	−12.24			−3.64	−3.91	−3.86			−5.12	−4.77	−4.81	
dy (%)		−0.04	0.13*	0.13*			0.10***	0.11***	0.12***			0.37***	0.33***	0.33***	
		−0.52	1.75	1.71			3.11	3.41	3.70			18.61	10.58	10.60	
stdcf (%)		0.58***	0.58***	0.58***			0.06	0.06	0.07			0.08**	0.07**	0.07**	
		7.04	7.07	7.02			1.13	1.19	1.43			2.13	2.06	2.23	
sue (%)		−0.09	−0.09	−0.09			−0.06***	−0.06***	−0.06***			−0.07***	−0.07***	−0.06***	
		−1.34	−1.26	−1.24			−2.70	−2.90	−2.79			−4.23	−4.07	−3.95	
rd_sale (%)		1.15***	1.20***	1.20***			0.01	−0.01	0.00			0.06*	0.05	0.05	
		8.90	9.28	9.38			0.18	−0.14	−0.02			1.89	1.49	1.62	
sfe (%)		−1.37***	−1.30***	−1.30***			0.33***	0.32***	0.31***			0.15***	0.14***	0.13***	
		−7.49	−7.09	−7.13			9.62	8.05	7.80			5.87	5.38	5.11	
idiovol (%)		12.10***	7.42***	7.56***			−0.71***	0.30	−0.26			−0.32***	0.05	−0.26**	
		20.18	11.22	10.18			−2.80	1.50	−1.10			−2.56	0.51	−2.25	
roavol (%)		1.28***	1.16***	1.16***			−0.28***	−0.18***	−0.19***			−0.05*	0.01	0.00	
		9.62	9.54	9.52			−3.86	−2.93	−3.05			−1.92	0.44	0.11	
depr (%)		0.23***	0.21***	0.21***			0.04	0.06	0.07			0.02	0.04*	0.04*	
		4.47	4.19	4.26			1.08	1.43	1.60			1.20	1.77	1.96	
nanalyst (%)		0.43***	0.07	0.08			−0.13	0.13***	0.10*			−0.03	0.14***	0.12***	
		3.65	0.76	0.91			−1.50	2.48	1.91			−0.51	3.54	3.10	
std_turn (%)		0.34***	0.25***	0.25***			−0.13***	−0.14***	−0.16***			0.03	0.03	0.02	
		4.06	3.00	3.00			−2.68	−3.30	−3.41			1.60	1.55	1.09	
Sys (%)				0.81	−8.70***				−3.91***	−1.07				−2.31***	−2.74***
				0.53	−2.64				−4.54	−1.20				−4.19	−5.21
Adj R^2 (%)	66.22	71.26	73.39	73.43	66.84	8.25	4.15	11.37	11.48	8.49	5.06	3.39	7.55	7.61	5.37

This table reports the cross-sectional regressions results, regressing IV function characteristics on realized volatility, implied moments, fundamental measures, and systematic risk ratio:

$$IV_{jt}^{i} = \alpha_t + \beta_{0,t} M_{jt} + \beta_{1,t} F_{jt} + \beta_{2,t} Sys_{jt} + \epsilon_{jt}^{i},$$

where IV_{jt}^{i} represents one of the level, slope and curvature of the IV function captured by IV_{ATM}, $IV_{OTM} - IV_{ATM}$, and $(IV_{ITM} + IV_{OTM})/2 - IV_{ATM}$ respectively. ITM, ATM, and OTM options are with delta of −0.8, −0.5, and −0.2, respectively. Specification (1) is the benchmark model considering only M_{jt}, the vector containing historical volatility, risk-neutral skewness and kurtosis, whilst specification (2) only considers firm fundamental vector F_{jt}. Specification (4) considers all the market and fundamental measures, as well as the systematic risk ratio Sys_{jt}. Specification (3) and (5) are cases without the systematic risk ratio and the firm fundamental vector, respectively. On each month we first obtain the intercept and other coefficients, these coefficients are then averaged and the corresponding t-statistic is calculated using the Newey–West standard errors with 12 lags. The sample period is from 1996 to 2019.

Table 11
Cross-sectional explanatory powers.

Model	Level	Slope	Curvature
Panel A: Cross-sectional R^2			
HIV	0.69	0.29	0.19
FIV	0.76	0.36	0.26
Panel B: R^2 difference (%)			
FIV-HIV	7.21***	7.35***	7.04***
	18.31	20.08	12.38

This table reports the cross-sectional R^2 from the Bayesian shrinkage regression. Panel A reports the R^2 and panel B reports the difference between HIV and FIV, where HIV represents the benchmark model considering only the vector containing historical volatility, risk-neutral skewness, and kurtosis, and FIV represents the model considering both the market and fundamental measures.

The superior performance highlights the overall contribution of the fundamentals incorporated in the Bayesian shrinkage method. By construction, this method provides the time-varying marginal contribution of each fundamental by estimating the weight in Eq. (A.5). In Fig. 4, we plot the weights for the IV slope only, but the weights for the IV level and curvature are similar. The weights show the marginal contributions of each fundamental in explaining the difference between the market observed slopes and the slopes estimated by the benchmark model. The highest positive weights are reflected by the illiquidity, unexpected quarterly earnings and cash flow volatility, suggesting that these fundamentals contribute more to the variation of volatility slopes among firms. Overall, we find that the contributions of firm fundamentals increase during the 1998–1999 Asian crisis and the 2008–2009 credit crunch crisis. This finding confirms the "wake-up call" hypothesis that investors focus more on fundamentals during the crises (Bekaert et al., 2014).

In summary, the Bayesian shrinkage regression collaborates with our findings that firm fundamentals are indeed related to the IV level, slope, and curvature.

6. Conclusion

While firm fundamentals are shown to be important in mainstream asset pricing in explaining the cross-section of expected stock returns, little is known on how they influence the IV curve. Our paper fills this gap in the literature in two ways. First, we show

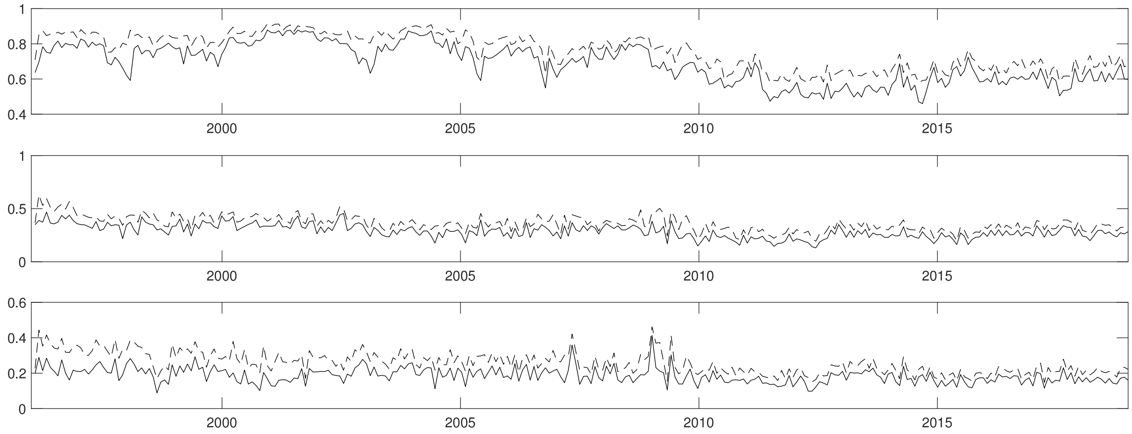

Fig. 3. Time series of R^2 from cross-sectional regressions. In Fig. 3, we plot the time series of the R^2 from the cross-sectional Bayesian shrinkage regression of 30-day put options. The plot from the top to the bottom is for the IV level, slope, and curvature, respectively. The solid line is for the benchmark model considering only the vector containing historical volatility, risk-neutral skewness, and kurtosis, and the dashed line is for the model considering both the market and fundamental measures.

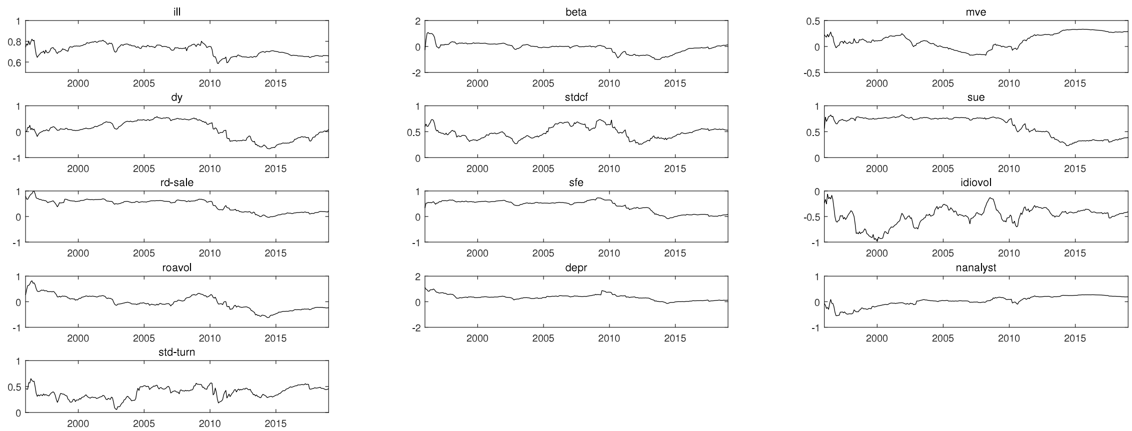

Fig. 4. Weight on each firm fundamental measure for IV slope. In Fig. 4, we plot the time series of the weights for each firm fundamental measure from the cross-sectional Bayesian shrinkage regression, in explaining the difference between the market observed slopes and the slopes estimated by the benchmark model.

empirically, via machine learning tools, that it is important to incorporate firm fundamentals in the modeling of the equity option IV curve based on data from all available U.S. listed firms. Next, we illustrate the role of firm fundamentals in a simple structural model via simulations.

Although the benchmark model with historical volatility, risk-neutral skewness, and kurtosis has excellent explanatory power on the variation of the IV shape (Bakshi et al., 2003), we find that incorporating firm fundamentals provides substantial additional explanatory power across all the IV characteristics' measures. Our results are consistent with the findings of asset pricing studies that firm fundamentals matter in the cross-section of expected returns, such as factor pricing (Fama and French, 2015), and are also consistent with the recent growing literature on demand-based asset pricing (Koijen and Yogo, 2019).

From an investment perspective, models incorporating firm fundamentals are also superior to the benchmark model. This is because the model with the firm fundamentals can yield significant economic gains vis-à-vis models ignoring them. These models also deepen our understanding of certain stylized facts and option puzzles. Future research may extend our methodology to other markets, such as foreign exchanges. Admittedly, our model is largely expository, hence future research may provide more sophisticated models that incorporate both fundamentals and more complex option features such as stochastic volatility and jumps.

Data availability

Data will be made available on request.

Appendix. Bayesian shrinkage regression model

We use the same procedure developed in Bai and Wu (2016) to run a Bayesian shrinkage regression. On each month, we start with fitting the IV level, slope, and curvature cross-sectionally by a non-parametric regression:

$$IV_t^i = f^i(HV_t, NS_t, NK_t) + \epsilon_t^i, \quad i = \circ, s, c, \tag{A.1}$$

where $f^i(.)$ denotes a local linear regression.

We then use this as the benchmark model, and examine the additional explanatory power of each firm fundamental measure to the IV shape characteristics. We denote the benchmark model by HIV_t^i and let F_t denote the $(J \times K)$ matrix of K firm fundamentals on J firms at time t. On each date, we orthogonalize the additional contribution of fundamentals to the IV characteristics from the benchmark model by applying a local linear regression on each firm's fundamentals against HIV_t^i:

$$F_t^k = f_t^{i,k}(HIV_t^i) + x_t^{i,k}, \quad i = \circ, s, c, \quad k = 1, \ldots, K. \tag{A.2}$$

Next, we regress residues of the benchmark model ϵ_t^i against these orthogonalized fundamentals $x_t^{i,k}$ by another local linear regression:

$$\epsilon_t^i = f_t^{i,k}(x_t^{i,k}) + \epsilon_t^{i,k}, \quad i = \circ, s, c, \quad k = 1, \ldots, K. \tag{A.3}$$

We then stack the predictions on the residues of the benchmark model from each orthogonalized firm fundamental into a matrix form:

$$X_t^i = \left[\hat{\epsilon}_t^{i,1} \cdots \hat{\epsilon}_t^{i,K}\right], \tag{A.4}$$

such that their weighting of contribution to the overall residue W_t^i can be estimated via

$$\epsilon_t^i = X_t^i W_t^i + \varepsilon_t^i. \tag{A.5}$$

Given that a firm's fundamentals are not always available in full, the predictions to the residues of the benchmark model from the missing firm fundamentals at certain time t is approximated by the average of the predictions from other fundamental measures weighted according to R^2 from the regression described in Eq. (A.3). For jth firm at time t, if lth firm characteristic is missing, its residue prediction on ith IV characteristic is approximated by the residue predictions from $\tilde{K}$ available firm characteristics:

$$\epsilon_t^{i,j,l} = \sum_{k=1}^{\tilde{K}} w_t^{i,k} \hat{\epsilon}_t^{i,j,k}, \tag{A.6}$$

$$w = \epsilon^\top \left(\epsilon\epsilon^\top + \mathrm{diag}\left\langle 1 - R^2 \right\rangle\right)^{-1}. \tag{A.7}$$

After completing the residue prediction matrix X_t^i, we estimate the weighting vector W_t^i at each time via a Bayesian regression update:

$$\widehat{W}_t^i = \left(X_t^{i\top} X_t^i + P_{t-1}^i\right)^{-1} \left(X_t^{i\top} \epsilon_t^i + P_{t-1}^i \widehat{W}_{t-1}^i\right), \tag{A.8}$$

$$P_t^i = \mathrm{diag}\left\langle \left(X_t^{i\top} X_t^i + P_{t-1}^i\right) \phi \right\rangle. \tag{A.9}$$

Starting with equal weights, we take the weights estimated on last period as prior, and update our beliefs on the weights according to the residue predictions of each firm fundamental. We diagonalize the precision matrix P_t^i to reduce any potential multicollinearity issue, and choose $\phi = 0.98$ to control the relative weight of the prior during the updates. Once the time weights W_t^i are estimated, we can then add the weighted average of the residue prediction in Eq. (A.5) back into Eq. (A.1) such that a new set of estimations of IV characteristics FIV is calculated:

$$FIV_t^i = RIV_t^i + X_t^i \widehat{W}_t^i. \tag{A.10}$$

The explanatory power of firm fundamentals on its equity option IV curve characteristics are assessed by two sets of cross-sectional regressions on each sample date, where we regress market observed IV curve characteristics on the model generated counterparts:

$$\mathrm{IV}_t^{i,k} = HIV_t^{i,k} + \varepsilon_t^{i,k}, \tag{A.11}$$

$$\mathrm{IV}_t^{i,k} = FIV_t^{i,k} + \varepsilon_t^{i,k}. \tag{A.12}$$

We use the regressions in Eq. (A.11) as benchmark, and check whether there are significant improvements in model fitting using Eq. (A.12).

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