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Delayed retirement policy and unemployment rates

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ABSTRACT

This paper examines the impact of retirement policy on the unemployment rates for both young and old workers. It employs a labor search framework with a constant elasticity of substitution production function and cross-market matching to investigate the channels through which the delayed retirement policy has impacts. The findings show that through the cross-market matching channel, retirement policy increases the unemployment of young workers (it is ambiguous for old workers) and has a negative effect on the wages of cross-market matched workers. The latter effect is negative for young workers (positive for old workers) through the capital-skill complementarity. The paper calibrates the model to the U.S. data and quantifies the effects of retirement policy during the first decade of this century. Counterfactual experiments highlight the contribution of each channel.

1. Introduction

Over the past few decades, there were two important eras of retirement policy in many developed countries, featuring contrasting policy targets. The first era arrived after the Great Depression and lasted until the oil shocks of the 1970s. The high unemployment rate, especially among the young, forced European and North American countries to introduce early retirement policies to create more vacancies for the young. This policy catered to the desire for longer retirement lives by the elderly at that time (see, for example Gruber and Wise, 1999; Mulligan and Sala-i Martin, 2004). Later, with the aging of baby boomers and declining fertility, the financial stability of social security was becoming a central concern. According to World Population Prospects 2017, Europe is the region with the most aging population globally. Europe's rate of population growth of people over 60 is 24%, followed by 21% in North America and 16.5% in the Oceania region. The average for Asia and the world' is around 12%. As a result, countries started to switch their retirement policies back and entered the second era of retirement policy, namely delayed retirement.[1]

In the literature, researchers have examined the impact of delayed retirement on both the youth and elderly labor markets. On the one hand, we want to know the effectiveness of such policy. Gruber and Wise (2004) use Organisation for Economic Co-operation and Development (OECD) data and find that a delay of three years in eligibility for receiving retirement benefits would reduce the proportion of men ages 56 to 65 out of the labor force by 23% to 36% on average. Rust and Phelan (1997) and Panis et al. (2002) also find that the delayed retirement policy can significantly increase the labor force participation of older workers. Mastrobuoni (2009) finds that an increase in the normal retirement age by two months delays effective retirement by around one month. Staubli

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[1] NBER published a volume titled "Social Security Programs and Retirement around the World", which examines the evolution of retirement policies in different countries. Other papers and related works include but are not limited to: Diamond and Gruber (1999), Baker et al. (2010), Banks et al. (2010) and Krueger and Pischke (1992).

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and Zweimuller (2013) empirically estimate the impact of early retirement age policy on elderly labor participation in Austria, where employment increases are accompanied by a substantial increase in registered unemployment for both genders.[2]

On the other hand, since this is a completely contrasting policy target from the one in the first era, we want to know whether the delayed retirement policy squeezes the labor market for young workers. If the answer is yes, through what channels? According to OECD data, the average unemployment rates of young workers in European and OECD countries were 19.8% and 14.5% in 2005–2016, respectively. However, Carnevale et al. (2013) believe that delayed retirement does not squeeze the young labor market. Although many people retired in the past, a large number of people will retire in the future, and especially those born during the baby boom will eventually leave the labor market. Although the authors do not mention the impact of baby boomers on the young labor force, Munnell and Wu (2012) respond to this question by showing that the baby boomer labor force does not worsen the employment level of the young labor force, but rather prompts it, regardless of gender, education, and the period of the Great Recession. A popular explanation for these positive findings is that there is a certain degree of substitution between young and old workers. For example, Böheim (2014) uses OECD data to show that young and older workers are complements for each other rather than substitutes.

Most of the above papers are empirical and focus on changes in labor participation but not the unemployment rate. In this paper, we theoretically investigate the channels through which the delayed retirement policy has an impact on the unemployment rates of both young and old workers. In particular, we extend Shimer's (2005) labor search model to study the interaction among firms as well as young and old workers. Our model has two important features. First, we consider a nested constant elasticity substitution (CES) production function following Jaimovich et al. (2013), in which old workers use capital to produce intermediate goods, and young workers use intermediate goods to produce final output. This assumption is used to capture the degree of complementarity between young and old workers. Second, we allow the old workers to do cross-market matching but not the reverse. This assumption is based on Chassamboulli and Palivos (2014) and is consistent with the reality that in some industries, old workers can do junior jobs if they want to, whereas young workers cannot. Furthermore, we do not restrict the old workers' productivity to be smaller relative to the young. In the sensitivity test, we show that the unemployment rate of the old may reverse if the relative productivity is large enough.

We find that, in general, the effects of the delayed retirement policy are ambiguous on the unemployment rates for both young and old workers. We then provide three special cases to study each channel separately. First, we single out that through the cross-market matching channel, the delayed retirement policy has a negative effect on young workers' market tightness and wages and a positive effect on their unemployment rate. Capital-skill complementarity works in the opposite way, through which the delayed retirement policy has a positive effect on young workers' market tightness and a negative effect on their unemployment rate. Moreover, it has a direct negative effect on old workers' market tightness and increases their unemployment rate. If we shut down the above two channels and keep the transition from the young to the old, the delayed retirement policy would have a negative effect on old workers' market tightness and their wages by raising firms' flow of value in hiring old workers. In particular, we find that there is a negative effect of the old workers' market tightness on the young workers' market tightness, which is due to the changes in firms' flow of values. However, the overall effect on both unemployment rates is ambiguous through this channel, which suggests that allowing the transition from young to old itself might complicate the results.

In the recent literature, some papers attempt to study retirement policy using the labor search framework. Bhattacharya et al. (2004) study the optimal public policy using a one-period extension of the standard Mortensen (1982) and Pissarides (2000) model. They find that the optimal policy discourages retirement, because old workers receive much higher payments than the theory predicted. Bhattacharya and Reed (2006) extend the model to a dynamic Overlapping generations model to evaluate the effects of public pension policy on labor participation. The public pension program redistributes bargaining strength from young to old workers, raising the wage of the old and decreasing that of the young. The public pension program also induces unemployed old workers to retire and creates more vacancies for the young. Hairault et al. (2015) study the impacts of searching on retirement decisions. They find that search friction has an impact on the retirement decision of unemployed workers, but not on that of employed workers. As a result, they propose an unfair pension adjustment as an optimal policy. The above models focus on optimal public policies and are either partial equilibrium models or only consider one type of job. In contrast, we endogenize the price of final goods by modeling a CES production function. By considering the demand side, the price of final goods would be influenced by labor market tightness, which in turn affects wages and unemployment rates. The CES production function also provides a capital-skill complementarity channel through which the delayed retirement policy has a negative effect on the unemployment rate of the young. By adding two labor market interactions, the influx of old workers would not have an unambiguously positive effect on the unemployment rate of the young as what we would get in the case without the capital-skill complementarity channel.

Keuschnigg and Keuschnigg (2004) apply the labor search model to study the Austrian pension system. García-Pérez and Sánchez-Martín (2015) discuss the unemployment insurance system in the Spanish economy. Hairault et al. (2010) find that the search model with overlapping generations better explains the empirical evidence that the closer to mandatory retirement age an individual is, the less likely they are to be employed. Fisher and Keuschnigg (2011) extend the individual homogeneity setting to analyze the impact of pension reform on the unemployment rates of old or young workers. Lefébvre (2012) uses a similar framework to study

[2] Although most studies show positive effects of the policy on elderly labor participation, Sánchez-Martin et al. (2014) estimate the welfare effects of Spain's delayed retirement policy and find that the welfare of unemployed seniors is undermined by weakening the enthusiasm of the unemployed to find work and increasing the unemployment time of those who desired early retirement. Gustman and Steinmeier (1986) estimate the effects of the U.S. social security reforms in 1983 and find a large negative impact on the welfare level of the workforce. This is because delaying retirement jeopardizes the effectiveness of workers' rest, especially for those who are unhealthy.

the impact of the delayed retirement policy on youth unemployment. In his model, the unit output of the old labor force is the key to affecting the unemployment rate of the young labor force. Michaelis and Debus (2011) consider monopoly unions to show that more older workers have no effect on young and old unemployment, whereas, the unemployment rate of the old (young) will increase (decrease) if the union puts more weight on the old.[3]

We calibrate our model to U.S. data to simulate the effects of the increase in retirement age during 2000–2009. We find that during the first decade of this century, the probability of retirement decreased 23.65%. We find significant positive overall effects on the number of employed old workers in both markers. However, the old earn more in the senior market and less in the junior market. Young workers earn more in the face of the aging population. The unemployment rates for both young and old decreases 2.42% and 3.91%, respectively, during this period. To highlight the contribution of each channel, we conduct three counterfactual experiments by removing the three channels one by one. We find that the aging population has positive effects on the unemployment rate of the young through the aging transition channel, and the cross-market matching channel and has a negative effect through the capital-skill complementarity channel.

The remainder of the paper is organized as follows. The general model is described in Section 2. The special cases are analyzed in Section 3. Calibration, counterfactual experiments, and sensitivity tests are detailed in Section 4. Section 5 concludes.

2. The model

Time is continuous. The economy is populated by a continuum of workers and a continuum of jobs. Workers are either young (y) or old (o). Young workers become old with probability λ_y and old workers retire with probability λ_o. We denote P as the total number of workers, P_o as the total number of old workers, and P_y as the total number of young workers. At the steady-state, the following equations should hold: $\dot{P} = \lambda_a P_y - \lambda_o P_o = 0$, $\dot{P}_y = \lambda_a P_y - \lambda_y P_y = 0$, and $\dot{P}_o = \lambda_y P_y - \lambda_o P_o$, where λ_a is the arrival rate of young workers. Therefore, we have $\lambda_a P_y = \lambda_y P_y = \lambda_o P_o$. By normalizing the total number of young workers $P_y = 1$, we have $P_o = \frac{\lambda_y}{\lambda_o}$. The mass of jobs is determined endogenously as part of the equilibrium. All agents are risk-neutral and discount the future at the common interest rate $r > 0$.

2.1. Production

There are two types of intermediate goods produced in two intermediate sectors: j (for junior) and s (for senior). Firms operate in either sector and use linear technologies. They employ old and young workers to produce inputs Y_j and inputs Y_s one for one, respectively. Intermediate inputs are non-storable. Once produced, they are sold in competitive markets and immediately used for final goods production (Y).

For the final goods sector, following Jaimovich et al. (2013), we assume that age is equated with labor market experience and old workers are relatively more complementary to capital than young workers. The production function has a nested CES form:

$$Y = [\eta(Y_j)^\sigma + (1-\eta)Q^\sigma]^{1/\sigma}, \sigma \le 1 \tag{1}$$

$$Q = [\tau K^\gamma + (1-\tau)(Y_s)^\gamma]^{1/\gamma}, \gamma \le 1 \tag{2}$$

where K denotes capital and η and τ are income shares between 0 and 1. Capital (K) and inputs (Y_s) are nested together in sub-aggregate inputs Q. Firms use Q and Y_j to produce the final good. $(1-\sigma)^{-1}$ is the elasticity of substitution between Q and Y_j. $(1-\gamma)^{-1}$ is the elasticity of substitution between K and Y_s. Capital-experience complementarity is defined as $\sigma > \gamma$, which implies that an increase in the capital stock raises the skill premium. If either σ or γ equals zero, it becomes a Cobb–Douglas production function. Since the two intermediate inputs are sold in competitive markets, their prices, p_s and p_j, will be equal to their marginal products, that is,

$$p_j = \eta Y_j^{\sigma-1} Y^{1-\sigma} \tag{3}$$

$$p_s = (1-\eta)(1-\tau) Y_s^{\gamma-1} Q^{\sigma-\gamma} Y^{1-\sigma} \tag{4}$$

Furthermore, we assume that the capital market is also competitive and firms can buy and sell capital without delay. Thus, the rental price (p_K) is equal to the marginal product of capital, which in turn is equal to the interest rate (r) plus its depreciation rate (δ). Therefore, we have the following relationship:

$$\begin{aligned} p_K &= r + \delta \\ &= (1-\eta)\tau K^{\gamma-1} Q^{\sigma-\gamma} Y^{1-\sigma} \end{aligned} \tag{5}$$

[3] See also Hairault et al. (2006) who extend McCall's (1970) labor search model to include life cycle and retirement decisions, to study the tax effect on continued activity. Sánchez-Martin et al. (2014) study the moral hazard problem due to the compensation of unemployment benefits affecting the level of labor supply. Croix et al. (2013) use an overlapping generations model with search friction to argue that neglecting labor market friction and unemployment dynamics might bias the results of pension reform studies. Volker (2009) finds that the model is better for demonstrating that the employment rate locus with age is an inverted U-shaped curve.

2.2. Labor market

There are two labor markets with search friction in which a representative firm posts a vacancy to hire an old worker for the senior sector production or a young worker for junior sector production. In addition, we allow the unemployed old workers to search for jobs in the junior sector. However, the young cannot search for jobs in the senior sector due to their lack of work experience. Firms cannot anticipate which type of worker they would be matched with when they open a vacancy in the junior sector. In each market, unemployed workers and unfilled vacancies are brought together via a stochastic matching technology $M(u_i; v_i) = M_0 u_i^{\epsilon} v_i^{1-\epsilon}$, where $i = j; s$. M_0 is an efficiency parameter. u_i and v_i denote the number of unemployed workers and vacancies in labor market i, respectively. The function $M(\cdot)$ exhibits standard properties: it is at least twice continuously differentiable, increasing in its arguments, exhibits constant returns to scale, and satisfies the familiar Inada conditions. Using the property of constant returns to scale, we can write the flow rate of a match for a worker as $M(u_i; v_i)/u_i = m(\theta_i)$ and the flow rate of a match for a vacancy as $M(u_i; v_i)/v_i = q(\theta_i)$, where $\theta_i = v_i/u_i = m(\theta_i)/q(\theta_i)$ is the tightness prevailing in labor market i. We have the usual properties for $m(\theta_i)$ and $q(\theta_i)$:

$$m'(\theta_i) > 0 \quad \lim_{\theta_i \to 0} m(\theta_i) = 0, \quad \lim_{\theta_i \to \infty} m(\theta_i) = \infty, \tag{6}$$

$$q'(\theta_i) < 0 \quad \lim_{\theta_i \to 0} q(\theta_i) = \infty, \quad \lim_{\theta_i \to \infty} q(\theta_i) = 0, \tag{7}$$

At any point in time, a worker is either employed (E_i^{κ}) or unemployed (U^{κ}); and a vacancy is either filled (J_i^{κ}) or not (V_i). Here, E_i^{κ}, U^{κ}, J_i^{κ}, and V_i denote the present discounted value associated with each state, where $i = j; s$ and $\kappa = y; o$. Since unemployed old workers can do cross-market searching in the junior labor market, we further assume that the productivity of old workers is μp_j, where μ is the relative productivity. We do not put any restrictions on this parameter. The value functions of firms and agents are as follows:

$$rJ_s^o = p_s - w_s^o - s_s(J_s^o - V_s) - \lambda_o J_s^o, \tag{8}$$

$$rJ_j^y = p_j - w_j^y - s_j(J_j^y - V_j) - \lambda_y(J_j^y - J_s^o), \tag{9}$$

$$rJ_j^o = \mu p_j - w_j^o - (s_j + m(\theta_s))(J_j^o - V_j) - \lambda_o J_j^o, \tag{10}$$

$$rV_j = -c_j + q(\theta_j)[\phi J_j^y + (1-\phi) J_j^o - V_j], \tag{11}$$

$$rV_s = -c_s + q(\theta_s)(J_s^o - V_s), \tag{12}$$

$$rE_s^o = w_s^o - s_s(E_s^o - U^o) - \lambda_o E_s^o, \tag{13}$$

$$rE_j^y = w_j^y - s_j(E_j^y - U^y) - \lambda_y(E_j^y - E_s^o), \tag{14}$$

$$rE_j^o = w_j^o - s_j(E_j^o - U^o) + m(\theta_s)(E_s^o - E_j^o) - \lambda_o E_j^o, \tag{15}$$

$$rU^o = b^o + m(\theta_s)(E_s^o - U^o) + m(\theta_j)(E_j^o - U^o) - \lambda_o U^o, \tag{16}$$

$$rU^y = b^y + m(\theta_j)(E_j^y - U^y) - \lambda_y(U^y - U^o), \tag{17}$$

where $\phi = \frac{u^y}{u^y + u^o}$ is the endogenous fraction of unemployed young workers. w_i^{κ} is the wage rate; c_i is the vacancy cost; s_i is the separation rate; and b^{κ} is the unemployment benefit, where $i = j; s$ and $\kappa = y; o$. The first two equations are the flow values of firms with vacancies filled by old and young workers, respectively, which depend on worker productivity and the wages offered. For both sectors, with probability s_i, workers separate from their current jobs. In addition, the last terms in the first two equations indicate that with probability λ_o, old workers leave their jobs and retire, and with probability λ_y, young workers become old workers and provide J_s^o to firms instead. Here we assume that the employment status does not change when a young worker gets old, and that the firm switches to intermediate goods production as workers' experience accumulates.

The third equation is the flow value of hiring an old worker in the junior market. This kind of worker might retire or separate from the current job or be matched to the senior market. The next two equations represent the flow value of a firm holding a vacancy in either market.

Eqs. (13)–(15) are the flow values of employed young workers in the junior market and old workers in both markets. Eq. (13) states that an old worker earns wages and might separate from the current job and retire with probabilities s_s and λ_o. Eq. (14) has a similar interpretation, except that the worker gets old with probability λ_y. Instead of separating from the current job and retiring, cross-market matched old workers might be matched back to the senior market with probability $m(\theta_s)$. The final two equations are the flow values of unemployed workers. Eq. (16) indicates that old unemployed workers earn unemployment benefits, with the probability of finding a job in either market or retiring directly. Eq. (17) shows that with probability $m(\theta_j)$, unemployed young workers might match with firms, and with probability λ_y, they might become old and remain unemployed. Finally, we assume free entry in each labor market. Thus, the expected net payoff of posting a vacancy is zero:

$$V_s = V_j = 0 \tag{18}$$

2.3. Nash bargaining

Once a firm and an unemployed worker are matched, they bargain over the total surplus according to Nash bargaining, with β being the worker's bargaining power. They solve the bargaining problem by maximizing the following equation:

$$Max_{w_i^\kappa}(E_i^\kappa - U^\kappa)^\beta (J_i^\kappa - V_i)^{1-\beta} \tag{19}$$

The solution must satisfy the following:

$$(1-\beta)(E_i^\kappa - U^\kappa) = \beta(J_i^\kappa - V_i). \tag{20}$$

Using Eqs. (8)–(18) and (20), we can derive the expression for w_i^κ

$$w_j^o = \frac{((1-\beta)b^o + \beta\mu p_j)(r + s_j + m(\theta_s) + \lambda_o) + \beta\mu p_j m(\theta_j)}{r + s_j + m(\theta_s) + \beta m(\theta_j) + \lambda_o} \tag{21}$$

$$w_s^o = \frac{\chi_1}{(r + s_s + \beta m(\theta_s) + \lambda_o)(r + s_j + m(\theta_s) + \beta m(\theta_j) + \lambda_o)} \tag{22}$$

$$w_j^y = \frac{\chi_2}{r + s_j + \beta m(\theta_j) + \lambda_y} \tag{23}$$

$$\begin{aligned}
\chi_1 &= (1-\beta)b^o(r + s_s + \lambda_o)(r + s_j + m(\theta_s) + \lambda_o) \\
&+ \beta\{\lambda_o^2 p_s + m(\theta_j)\mu p_j(r + s_s) + p_s(r + s_j + m(\theta_s)) + \beta m(\theta_j)[p_s(m(\theta_s) + r + s_s) - \mu p_j(r + s_s)] \\
&+ \lambda_o[m(\theta_j)(\mu p_j - \beta\mu p_j + \beta p_s) + p_s(2m(\theta_s + 2r + s_j + s_s))]\} \\
\chi_2 &= (1-\beta)b^y(r + s_j + \lambda_y) + \beta p_j(r + s_j + m(\theta_j) + \lambda_y) \\
&- \frac{(1-\beta)\beta\lambda_y m(\theta_j)[\beta m(\theta_j)\mu p_j + b^o(r + s_j + m(\theta_s) + \lambda_o) - p_s(r + s_j + m(\theta_s) + \beta m(\theta_j) + \lambda_o)]}{(r + s_j + m(\theta_s) + \beta m(\theta_j) + \lambda_o)(r + s_s + \beta m(\theta_s) + \lambda_o)}
\end{aligned}$$

where w_i^κ denotes the wage rate for skill type i for generation κ. The above equations show that wages depend on workers' productivity, unemployment benefits, probability of retirement, probability of getting old, and separation rates.

2.4. Steady-state composition of the labor force

In equilibrium, the number of agents who flow in and out of each subgroup should equal the following:

$$u^y + e_j^y = 1, \tag{24}$$

$$u^o + e_j^o + e_s^o = \lambda_y/\lambda_o = P_o, \tag{25}$$

$$s_j e_j^y + \lambda_y P_y = (m(\theta_j) + \lambda_y)u^y, \tag{26}$$

$$(m(\theta_s) + s_j + \lambda_o)e_j^o = m(\theta_j)u^o, \tag{27}$$

$$(s_s + \lambda_o)e_s^o = m(\theta_s)(u^o + e_j^o) + \lambda_y e_j^y, \tag{28}$$

where $u^y(u^o)$ is the number of unemployed young (old) workers. $e_j^y(e_j^o)$ is the number of employed young (old) workers in the junior market. e_s^o is that in the senior market. The first two equations are identities. The last three equations imply that the flow in and out of unemployment or employment is the same for each type of worker. For example, Eq. (26) states that the number of young workers who separate from their current job plus the number of newly unemployed young workers should equal the number of newly matched young workers plus the number of young workers who become old. Eq. (27) states that a cross-market matched old worker might separate from the current junior job, match with a senior job, or retire. The total number of outflows should equal the number of old workers who are cross-market matched with junior jobs. The last equation is for old workers and has a similar intuition. Now we can write the equations for the steady-state unemployment rates and employment levels. The unemployment rate of young (old) workers, $UR^y(UR^o)$, is defined as the ratio of the number of unemployed young (old) workers to the total number of young (old) workers.

$$UR^y = u^y = \frac{s_j + \lambda_y}{m(\theta_j) + s_j + \lambda_y}, \tag{29}$$

$$UR^o = u^o/P_o = \frac{(\lambda_o + m(\theta_s) + s_j)(\lambda_o\lambda_y + \lambda_o s_j + \lambda_y s_s + s_s m(\theta_j) + s_j s_s)}{(\lambda_y + m(\theta_j) + s_j)(\lambda_o + m(\theta_j) + m(\theta_s) + s_j)(m(\theta_s) + s_s + \lambda_o)}, \tag{30}$$

$$e_j^y = \frac{m(\theta_j)}{m(\theta_j) + s_j + \lambda_y}, \tag{31}$$

$$e_j^o = \frac{\lambda_y m(\theta_j)(\lambda_o\lambda_y + \lambda_o s_j + \lambda_y s_s + s_s m(\theta_j) + s_j s_s)}{\lambda_o(\lambda_y + m(\theta_j) + s_j)(\lambda_o + m(\theta_j) + m(\theta_s) + s_j)(m(\theta_s) + s_s + \lambda_o)}, \tag{32}$$

$$e_s^o = \frac{\lambda_y(\lambda_o m(\theta_j) + \lambda_y m(\theta_s) + m(\theta_s)m(\theta_j) + m(\theta_s)s_j)}{\lambda_o(\lambda_y + m(\theta_j) + s_j)(m(\theta_s) + s_s + \lambda_o)}. \tag{33}$$

2.5. Equilibrium

From Eqs. (8)–(18) and (20)–(22), we can write a two-equation system with two unknowns:

$$p_s = f_s(\theta_s, \theta_j) \equiv B_s \tag{34}$$

$$p_j = f_j(\theta_s, \theta_j) \equiv B_j \tag{35}$$

The expressions for the endogenous fraction of unemployed young workers, ϕ and Y_s/Y_j, are given below

$$\begin{aligned}\phi &= \frac{u^y}{u^y + u^o} \\ &= \frac{s_j + \lambda_y}{s_j + \lambda_y + \frac{\lambda_y(\lambda_o + s_j + m(\theta_s))[\lambda_o(\lambda_y + s_j) + s_s(m(\theta_j) + s_j + \lambda_y)]}{\lambda_o(\lambda_o + s_j + m(\theta_s) + m(\theta_j))(m(\theta_s) + s_s + \lambda_o)}},\end{aligned} \tag{36}$$

$$\begin{aligned}\frac{Y_s}{Y_j} &= \frac{e_s^o}{e_j^y + e_j^o} \\ &= \frac{\lambda_y(\lambda_o + s_j + m(\theta_s) + m(\theta_j))[\lambda_o m(\theta_j) + m(\theta_s)(\lambda_y + m(\theta_j) + s_j)]}{\chi_3},\end{aligned} \tag{37}$$

where

$$\begin{aligned}\chi_3 &= \lambda_o m(\theta_j)[\lambda_y^2 + (\lambda_o + m(\theta_s))(m(\theta_j) + m(\theta_s) + \lambda_o) \\ &+ (\lambda_o + m(\theta_s) + \lambda_y)s_j] + m(\theta_j)s_s[\lambda_o^2 + \lambda_y(\lambda_y + m(\theta_j) + s_y) \\ &+ \lambda_o(m(\theta_s) + m(\theta_j) + s_j)].\end{aligned}$$

We now define the steady-state equilibrium for this economy.

Definition 1. A steady-state equilibrium is a set $\{\theta_i^*, p_i^*, p_K^*, w_i^{\kappa *}, Y_i^*, K^*; u^{\kappa *}\}$, where $i = j; s$ and $\kappa = y; o$; such that: (i) The intermediate input markets clear. In particular, Eqs. (3) and (4) are satisfied. (ii) The capital market clears, and Eq. (5) is satisfied. (iii) The free entry Eq. (18) for each sector i is satisfied. (iv) The Nash bargaining optimality Eq. (20) for each sector i and generation κ holds. (v) The flow of employed and unemployed workers as well as that of filled and unfilled vacancies of each type and generation remain constant; that is, Eqs. (29)–(33) are satisfied.

By combining Eqs. (3), (4), (34), and (35), the steady-state equilibrium values of θ_j and θ_s are given by the following reduced system of equations:

$$B_j = \eta\{\eta + (1-\eta)(\frac{Y_s}{Y_j})^{\sigma}[\tau k^{\gamma} + (1-\tau)]^{\frac{\sigma}{\gamma}}\}^{\frac{1-\sigma}{\sigma}} \tag{38}$$

$$B_s = (1-\eta)(1-\tau)[\tau(\frac{K}{Y_s})^{\gamma} + (1-\tau)]^{\frac{1-\gamma}{\gamma}}\{\frac{\eta(\frac{Y_s}{Y_j})^{-\sigma}}{[\tau(\frac{K}{Y_s})^{\gamma} + (1-\tau)]^{\frac{\sigma}{\gamma}}} + (1-\eta)\}^{\frac{1-\sigma}{\sigma}} \tag{39}$$

where $\frac{K}{Y_s} = [\frac{\tau B_s}{(1-\tau)(r+\delta)}]^{\frac{1}{1-\gamma}}$.

Since there are no analytical results for this general model, we will look at three special cases, from which we can learn how different channels work in this model. Then we will quantify the model and estimate the policy effects during the first decade of this century. We also carry out counterfactual experiments to highlight the contribution of each channel.

3. Special cases

3.1. Aging transition only

In this case, we shut down the cross-market matching and capital-skill complementarity channels and focus on the effect of the aging transition channel only. Based on value functions (8)–(17), we let $\phi = 1$ in Eq. (11), remove the term $m(\theta_j)(E_j^o - U^o)$ from Eq. (16), and remove Eqs. (10) and (15). We also set the prices of intermediate goods to be exogenous, and either intermediate good can be used for final goods production with a 1–1 transformation. Then we consider the effects of the tightness of each labor market on wages, employment levels, unemployment rates, and the interaction between the tightness of the two markets.

Lemma 1. *Without cross-market matching assumption and the CES production function, the effects of the tightness of each labor market on both labor markets are (see online Appendix A for proof):*

$$\frac{\partial w_s^o}{\partial \theta_s} > 0, \frac{\partial w_j^y}{\partial \theta_j} > 0, \frac{\partial w_j^y}{\partial \theta_s} < 0, \frac{\partial \theta_j}{\partial \theta_s} < 0,$$

$$\frac{\partial UR^o}{\partial \theta_s} < 0, \frac{\partial e_s^o}{\partial \theta_s} > 0, \frac{\partial UR^y}{\partial \theta_j} < 0, \frac{\partial e_j^y}{\partial \theta_j} > 0,$$

$$\frac{\partial UR^y}{\partial \theta_s} > 0, \frac{\partial e_j^y}{\partial \theta_s} < 0. \frac{\partial UR^o}{\partial \theta_j} < 0, \frac{\partial e_s^o}{\partial \theta_j} > 0,$$

The intuition is the following. An increase in the tightness of the senior sector (junior sector) has a positive effect on w_s^o (w_j^y), because it increases the job-finding rate of unemployed elderly (young) workers and their bargaining power. Higher wages attract more workers. Changes in unemployment rates UR^o and UR^y and employment levels e_s^o and e_j^y follow in both labor markets. For the interaction between these two markets, first, θ_s has a negative effect on w_j^y. This is because w_s^o increases with θ_s which decreases J_s^o. Expecting a lower flow of value with old workers reduces the incentive of firms to post vacancies in the junior sector, because the young workers will get old eventually (that is, θ_j decreases). Therefore, UR^y increases and e_j^y decreases with θ_s.[4] Higher θ_j implies that more young workers become employed when they enter the senior labor market, which has a negative (positive) effect on UR^o (e_s^o).

Next, we consider the impact of the delayed retirement policy and that of an aging population. A delayed retirement policy is achieved by lowering λ_o.

Proposition 1. *Without cross-market matching and the CES production function, the effects of λ_o on the labor market are given:*

$$\frac{\partial \theta_s}{\partial \lambda_o} < 0, \frac{\partial \theta_j}{\partial \lambda_o} \lesseqgtr 0, \frac{\partial w_s^o}{\partial \lambda_o} < 0, \frac{\partial w_j^y}{\partial \lambda_o} \lesseqgtr 0,$$

$$\frac{\partial UR^o}{\partial \lambda_o} \lesseqgtr 0, \frac{\partial UR^y}{\partial \lambda_o} \lesseqgtr 0, \frac{\partial e_j^y}{\partial \lambda_o} \lesseqgtr 0, \frac{\partial e_s^o}{\partial \lambda_o} \lesseqgtr 0.$$

The above proposition shows how λ_o affects both labor markets. A lower λ_o implies a lower probability of retiring. We find that λ_o has negative effects on θ_s and w_s^o. First, a lower λ_o implies that old workers can work for a longer time, which brings firms a higher flow of the value of hiring an old worker (J_s^o). Thus, firms post more vacancies and offer higher wages w_s^o on this market. Although the increase in the number of old workers may lead to more unemployed old workers, the positive effect on θ_s dominates and it increases. According to Lemma 1, θ_j decreases with θ_s. Furthermore, lower λ_o implies that more old workers leave, and hence less young workers enter the economy. Therefore, firms have a greater incentive to hire in the senior market instead of the junior market. The overall effect of λ_o on θ_j is ambiguous. The effects of λ_o on e_s^o, e_j^y, UR^y, and w_j^y are ambiguous, because θ_s and θ_j have opposing effects on these variables.

3.2. Cross-market matching only

In this case, we examine the effect through the cross-market matching channel, that is, allowing old workers to find jobs in the junior sector. At the same time, we shut down the aging transition and capital-skill complementarity channels to highlight our findings. To achieve this, we set $\lambda_y = \lambda_o = 0$, let λ be the mass of old workers and $1 - \lambda$ be the mass of young workers, and normalize the total population to one. We also let the prices of intermediate goods and capital in Eqs. (3)–(5) be exogenous.

We summarize the effects of the tightness of each market on both labor markets in the following lemma.

Lemma 2. *Without the aging transition and the CES production function, the effects of each labor market tightness on both labor markets are (see online Appendix B for proof):*

$$\frac{\partial w_s^o}{\partial \theta_s} \gtreqless 0, \frac{\partial w_j^y}{\partial \theta_j} > 0, \frac{\partial w_j^o}{\partial \theta_s} < 0, \frac{\partial w_s^o}{\partial \theta_j} > 0, \frac{\partial w_j^o}{\partial \theta_j} > 0,$$

$$\frac{\partial UR^o}{\partial \theta_s} < 0 \quad (when\ s_s < s_j), \frac{\partial e_s^o}{\partial \theta_s} > 0, \frac{\partial UR^y}{\partial \theta_j} < 0, \frac{\partial e_j^y}{\partial \theta_j} > 0,$$

$$\frac{\partial UR^o}{\partial \theta_j} < 0, \frac{\partial e_j^o}{\partial \theta_s} < 0, \frac{\partial e_j^o}{\partial \theta_j} > 0.$$

First, θ_s generates opposing effects and has an ambiguous effect on w_s^o. On the one side, firms in the senior sector post more vacancies as θ_s increases, which in turn raises the wage level of old workers in the senior sector. However, on the other side, a higher θ_s causes more old workers in the junior sector to separate from their current jobs, which brings down their wage w_j^o. Moreover w_j^o is the outside option of old workers in the senior sector, so w_s^o decreases as well. Similarly, a higher θ_j pushes w_j^y and w_j^o up. A higher w_j^o, as the outside option of w_s^o, drives up w_s^o.

Second, a higher θ_s decreases the employment level of old workers in the junior sector, because the senior market employment level increases. Here the model requests $s_s < s_j$, under which a negative relationship between θ_s and u_r^o can be derived. By contrast, if the senior sector has a higher separation rate, then older workers in the junior sector will still face a high unemployment rate even though they switch to the senior sector. As a result, the effect will be ambiguous.

Since the number of workers is exogenous for both young and old workers, both the delayed retirement policy and the aging population can be represented by increasing the number of old workers in the economy. We then examine the effects of λ on both labor markets.

[4] Mathematically, $\partial w_s^o / \partial \theta_j = 0$. Intuitively, θ_j has no effect on w_s^o, because it would not change firms' expectations.

Proposition 2. *Without the aging transition assumption and the CES production function, the effects of λ on labor markets are as follows (see online Appendix B for the proof):*

$$\frac{\partial \theta_s}{\partial \lambda} > 0, \frac{\partial \theta_j}{\partial \lambda} < 0, \frac{\partial UR^o}{\partial \lambda} \gtreqless 0, \frac{\partial UR^y}{\partial \lambda} > 0, \frac{\partial w_j^y}{\partial \lambda} < 0, \frac{\partial w_s^o}{\partial \lambda} \gtreqless 0, \frac{\partial w_j^o}{\partial \lambda} < 0.$$

First, a higher λ implies that there are more old workers in the economy. This generates a direct negative effect on tightness of both labor markets. However, there is also a positive effect. Condition $\mu p_j - b^o < p_j - b^y$ implies that it is more profitable for a company to hire an old worker than a young worker. Thus, firms post more vacancies in both markets, since they expect to hire old workers in the junior market. In Appendix B (available online), we show that the positive effect dominates the negative effect in the senior market, while the negative effect dominates in the junior market. Thus, θ_s increases with λ and θ_j decreases with λ.

Second, the effect of λ on the unemployment rate of old workers is ambiguous. On the one side, λ reduces UR^o by raising θ_s. On the other side, a larger supply of old labor has a positive effect on UR^o. For junior workers, a smaller supply of young workers has a negative effect on UR^y. Also, a lower θ_j has a positive effect on UR^y. Under the uniqueness condition, the positive effect dominates, and the unemployment rate of young workers increases with λ.

Finally, λ has a negative relationship with the wage level of the young and that of the old who work in the junior sector because of the decrease in θ_j. Meanwhile, the surge in the supply of old labor also intensifies separation in the junior sector, which further lowers the wage level of the old in the junior sector w_j^o. However, we cannot determine the impact on w_s^o for similar opposing effects on θ_s and u_r^o.

3.3. Capital-skill complementarity only

In this case, we examine the effect through capital-skill complementarity and shut down the cross-market matching and aging transition channels. Eqs. (1)–(5) still hold with $\lambda_y = \lambda_o = 0$, and $\phi = 1$. Eqs. (10) and (15) are removed for no cross-market matching. We summarize the effects of tightness of each market on both labor markets in the following lemma.

Lemma 3. *Without the cross-market matching and the aging transition channels, the effects of each labor market tightness on both labor markets are (see online Appendix C for the proof):*

$$\frac{\partial w_s^o}{\partial \theta_s} \gtreqless 0, \frac{\partial w_j^y}{\partial \theta_j} \gtreqless 0, \frac{\partial w_j^y}{\partial \theta_s} > 0, \frac{\partial w_s^o}{\partial \theta_j} < 0,$$

$$\frac{\partial UR^o}{\partial \theta_s} < 0, \quad \frac{\partial e_s^o}{\partial \theta_s} > 0, \frac{\partial UR^y}{\partial \theta_j} < 0, \frac{\partial e_j^y}{\partial \theta_j} > 0,$$

$$\frac{\partial UR^y}{\partial \theta_s} = 0, \frac{\partial e_j^y}{\partial \theta_s} = 0, \frac{\partial UR^o}{\partial \theta_j} = 0, \frac{\partial e_s^o}{\partial \theta_j} = 0.$$

First, the effect of θ_s on w_s^o is ambiguous. On the one hand, a higher θ_s raises old workers' bargaining power and therefore has a positive effect on their wages. On the other hand, hiring more old workers lowers the marginal product of each old worker, which has a negative effect on their wage. θ_j has similar effects on the wage of young workers.

Second, due to the CES production function, a higher θ_s (θ_j) requires firms to post more vacancies in the junior (senior) market. This, in turn, lowers the unemployment rate of the young (old) and increases the corresponding employment level.

Proposition 3. *Without the cross-market matching and the aging transition channels, if $\sigma = 1$, there is no effect of the retirement delay policy on the labor market; if $\sigma < 1$, the effects of the retirement delay policy on both labor markets are as follows (see online Appendix C for proof):*

$$\frac{\partial \theta_s}{\partial \lambda} < 0, \frac{\partial \theta_j}{\partial \lambda} > 0, \frac{\partial UR^y}{\partial \lambda} < 0, \frac{\partial UR^o}{\partial \lambda} > 0, \frac{\partial w_j^y}{\partial \lambda} \gtreqless 0, \frac{\partial w_s^o}{\partial \lambda} \gtreqless 0$$

If σ equals 1. Y_j and sub-aggregate input Q are perfect substitutes. We can determine that k is constant and so p_s and p_j are also constant. Now, the model is reduced to the standard labor search model in which the tightness of both the junior and senior markets has nothing to do with population fluctuations. Neither do the unemployment rates or other variables.

If σ is less than 1, an increase in the number of old workers pushes the marginal product (p_s) in the senior sector down, while the marginal product in the junior sector (p_j) is driven up. Therefore, the profit in the junior sector is greater than that in the senior sector, such that firms in the former sector will create more vacancies. Apparently, θ_j increases and θ_s decreases. At the same time, the retirement delay policy leads to more old workers searching on the senior market, which further pushes θ_s down. In addition, since the total population is normalized to one, $1 - \lambda$ decreases, which implies that the number of young workers who are searching for jobs decreases. Thus, θ_j increases.

Delayed retirement has opposing effects on each market's tightness, as well as the two unemployment rates. Due to these opposing effects, λ has an ambiguous effect on wages for both young and old workers.

4. Quantitative analysis

4.1. Calibration

We calibrate our model to match monthly U.S. data for 1990–1999. We then simulate the effects of an increase in the retirement age during 2000–2009. We use a Cobb–Douglas matching function, $M = M_0 u_i^{\epsilon} v_i^{1-\epsilon}$, where M_0 measures the efficiency of the matching process and is normalized to one. Our model economy can be characterized by the following 16 parameters: the parameter of the matching function ϵ, the worker's bargaining power β, the interest rate r, the depreciation rate δ, the search costs c_j and c_s, the unemployment benefits b^o and b^y, the average separation rates s_j and s_s, the probability of getting old λ_y, the probability of retirement λ_o and the parameters in the production function η, τ, γ, and σ.

First, we calculate the difference between the average 30-year Treasury bond rate and the average gross domestic product deflator, which implies an annual real interest rate of 4.87% and a monthly interest rate of 0.4% over the sample period, and thus $r = 0.0004$. Second, following the common practice, we set the worker's bargaining power β to 0.5 and let $\epsilon = \beta$ to meet the Hosios condition, which requires that the unemployment elasticity of the matching function and the bargaining power be equal (see Hosios, 1990).

Third, to let the identification of young and old workers be consistent across different variables in the March Current Population Survey (CPS), we define the 16–34-year-old age group as the young and the 35–64-year-old age group as the old.[5] Since the actual retirement date is different for each worker in reality, we calculate the probability of getting old from the data first, and then use the steady-state condition $\lambda_y P_y = \lambda_o P_o$ to calculate the probability of retirement. Using the CPS data, we can obtain the number of 34-year-old workers in the labor force and the total number of workers in the 16–34-year-old age group. We then calculate the probability of getting old to be 0.0682 as the ratio of the number of 34-year-old workers over the total number of workers in the 16–34 age group. The probability of retirement follows at 0.0503.

Fourth, we calculate the unemployment rate of the young to be 8.8% and the unemployment rate of the old to be 3.9% from the CPS. Fifth, using the method proposed by Shimer (2005), we calculate the average monthly separation rate to be 0.031 from the CPS. To get the separation rates for both groups, we use the identity that the average separation rate times the total number of employed workers equals the sum of both groups' unemployed workers: $s(1 - u^y + P_o - u^o) = s_j(1 - u^y) + s_s(P_o - u^o)$. We then further assume that the separation rate of the young is two times larger than that of the old in the baseline model to calculate that $s_s = 0.0220$ and $s_j = 0.0439$, where s is the average separation rate. Sixth, we calculate the capital depreciation rate δ to be 0.0061 from data from the Bureau of Economic Analysis (BEA).[6]

Seventh, we estimate the elasticity of substitution between old workers and capital and the elasticity of substitution between young workers and intermediate goods using the generalized method of moments method proposed by Jaimovich et al. (2013). Due to limited data availability, we use long-run annual data from CPS for 1965–2009 and get average $\gamma = 0.201$, and $\sigma = 0.662$.[7] Eighth, the relative productivity parameter μ is set to 1 in the baseline model, which indicates that once cross-market matching is allowed, old workers have the same productivity as young workers in the junior market. Later, we will conduct sensitivity tests on the assumptions of separation rates and the relative productivity.

For the remaining six parameters, we jointly calibrate them by matching six targets. First, defining old–young wage gap to be (the average wage of the old - the average wage of the young)/the average of the old, we calculate the old–young wage gap to be 0.28 from the CPS data. Second, we calculate the vacancy-to-unemployment ratio to be 0.803. The data on vacancies are from the Conference Board Help-Wanted advertising index (HWI). The unemployment rate from the CPS. Then, since we only have aggregate data, we assume the vacancy to unemployment ratio is the same for both groups. Third, using data from the BEA, we calculate the capital–output ratio to be 1.348.[8]

The last two targets are the unemployment insurance replacement rates (the value of leisure) for both groups. Most papers in the job search literature set the replacement rate to 0.4, as estimated by Shimer (2005). This number captures the unemployment benefit as the value of leisure, which is at the upper end of unemployment insurance replacement rates in the U.S. Instead of unemployment insurance, some recent studies consider the value of non-market activity and estimate the value of leisure to be 0.71 (see Hall and Milgrom, 2008; Pissarides, 2009; and Bruegemann and Moscarini, 2010). We first consider a standard definition of the value of leisure and follow Shimer (2005), to set the replacement rate b_i/p_i to 0.4 for both groups in the baseline calibration. We then discuss how the results change with a broader concept of the value of leisure in the sensitivity tests. We summarize our calibration results in Table 1.

[5] Before 1995, the breakpoint between young and old in the Current Population Survey was age 34 years, which is not consistent with the data thereafter, particularly, for the data on "unemployment less than 5 weeks". To be consistent, we use age 34 years as the break-point for our entire sample period.

[6] Following the work of Chassamboulli and Palivos (2013), we use nonresidential equipment, software, and nonresidential structures to construct the capital stock.

[7] If we restrict the data to our sample period, we only have 10 observations, and the results are not credible. The estimation shows capital-skill substitution, which is not consistent with the findings in the literature. Krusell et al. (2000) and Castro and Coen-Pirani (2008) find capital-skill complementarity. Jaimovich et al. (2013) use age to proxy the labor market experience and find capital-experience complementarity in production.

[8] We define private output as the difference between gross domestic product and gross housing value added plus compensation of government employees.

Table 1
Calibration results.

Parameters	Values	Interpretation
r	0.004	Monthly interest rate. (Federal Reserve Bank of St. Louis)
ϵ	0.5	Unemployment elasticity of the matching function
β	0.5	Worker's bargaining power
δ	0.0061	Depreciation rate (BEA)
		Calculated from the March CPS:
γ	0.201	$(1-\gamma)^{-1}$ is the elasticity of substitution between K and Y_s
σ	0.662	$(1-\sigma)^{-1}$ is the elasticity of substitution between Q and Y_j
λ_y	0.0682	Probability of getting old
λ_o	0.0503	Probability of retirement
s_j	0.0439	Separation rate for junior
s_s	0.0220	Separation rate for senior
		Jointly Calibrated to Match:
b_o	0.2743	The capital–output ratio is 1.348 (BEA)
b_y	0.1783	The senior/junior wage gap is 28% (CPS)
c_s	0.5041	The v/u ratio is 0.803 for both groups (HWI, JOLTS and CPS)
c_j	0.2889	The replacement ratio is 0.4 for both groups (Shimer, 2005)
η	0.2188	
τ	0.0143	

* v/u=vacancy/unemployment.
** BEA = Bureau of Economic Analysis; CPS = Current Population Survey; HWI = Help-Wanted Index; JOLTS = Job Openings and Labor Turnover Survey.

Table 2
Effects of the delayed retirement policy (λ_o) (percentage changes).

Junior workers		Senior workers		Cross-Market Matching	
w_j^y	5.79	w_s^o	−2.08	w_j^o	5.32
θ_j	5.65	θ_s	−7.82	–	–
e_j^y	0.30	e_s^o	31.00	e_j^o	35.91
ER_j^y	0.30	ER_s^o	0.01	ER_j^o	3.75
UR^y	−2.42			UR^o	−3.91
Average wage	1.82			Total UR	−13.49

4.2. Results

Using the CPS data, we find that over 2000–2009, the probability of retirement decreases from 5.03% to 3.84%. To explore the impact of such changes on the labor market, we report the percentage changes of labor market variables by varying the probability of retirement and keeping the other parameters unchanged. We summarize the results in Table 2.

Table 2 shows that with the implementation of the delayed retirement policy, the probability of retirement decreases 23.66%, which causes θ_j to increase by 5.65% and θ_s to decrease by 7.82%; the wages of young workers increase 5.79% and those of old workers decrease 2.08%. For old workers who search for jobs in the junior market, their wages increase 5.32%. We present the expressions for the employment rates, average wage, and total unemployment rate in online Appendix D.

The intuition is as follows. First, as the mass of senior workers becomes bigger, there is a direct negative effect on the tightness of the senior market (θ_s); therefore, the wage in this market decreases. Second, since the senior labor market becomes tighter, more seniors conduct cross-market job search, which has a negative effect on the tightness of the junior market (θ_j). However, the complementarity between seniors and juniors generates a positive effect on vacancies and hence θ_j. The overall effect results in increases in θ_j and w_j. Old workers also earn 5.32% more wages in the junior market.

The unemployment rate of young workers decreases 2.42%, and that of old workers decreases 3.91%, so the positive effect on vacancies dominates in the junior market. Although the number of employed young workers only increases 0.3%, the number of employed old workers increases 31% and 35.91% percent in the senior and junior markets respectively (e_s^o and e_j^o). This is because firms' value of employment increases, which generates a positive effect on vacancy posts. Of course, the employment rate of old workers in the senior market (ER_s^o) increases only 0.01% due to the increase in the base of unemployed old workers. The employment rate of old workers in the junior market (ER_j^o) increases 3.75% due to the increase in vacancies in the junior market. The capital-skill complementarity channel also contributes to the increases in e_j^y and ER_j^y. Finally, the average wage increases 1.82% and the total unemployment rate decreases 13.49%.

4.3. Counterfactual experiments

To highlight the contribution of each channel, we explore three counterfactual experiments by removing the channels one by one. Specifically, in succession, we (i) eliminate the aging transition channel by setting $\lambda_o = \lambda_y = 0$, letting λ be the mass of old

Table 3
Counterfactual experiments: Removing the aging transition channel (percentage changes).

Junior workers		Senior workers		Cross-market matching workers	
w_j^y	0.97	w_s^o	5.07	w_j^o	3.28
θ_j	58.4	θ_s	−55.96	–	–
e_j^y	−11.79	e_s^o	4.37	e_j^o	83.62
ER_j^y	1.85	ER_s^o	−5.00	ER_j^o	67.13
UR^y	−19.08			UR^o	−3.47
Average wage	5.42			Total UR	−17.26

Table 4
Counterfactual experiments: Removing the cross-market matching channel (percentage changes).

Junior workers		Senior workers		Cross-Market Matching Workers	
w_j^y	2.84	w_s^o	−0.84	w_j^o	–
θ_j	46.92	θ_s	−53.43	–	–
e_j^y	1.83	e_s^o	29.78	e_j^o	–
ER_j^y	1.83	ER_s^o	−0.92	ER_j^o	–
UR^y	−15.99			UR^o	34.42
Average wage	1.29			Total UR	−8.68

workers and $1-\lambda$ be the mass of young workers, and remove the corresponding terms from value functions (8)–(17); (ii) eliminate the cross-market matching channel by setting $\phi = 1$, removing value functions (10) and (15) and the corresponding term from Eq. (16); (iii) eliminate the capital-skill complementarity channel by removing the CES production function and setting competitive prices p_s and p_j to the values in the baseline calibration. To make the results comparable to the results in the baseline calibration, we do not recalibrate the model. Despite the changing parameter values, in each experiment, we hold the other parameters constant at their baseline calibration levels and then solve for the endogenous variables.[9]

Table 3 presents the results for eliminating the aging transition channel. We calculate the mass of old workers $\lambda = 0.5757$ for 1990–1999 and $\lambda = 0.6325$ for 2000–2010, from the CPS data. Without the aging transition channel, the most distinguished differences are strong and positive effects on θ_j, and amplified effects on θ_s and e_j^o compared with the baseline model.

When shutting down the aging transition channel, the number of workers in both groups is fixed for a given λ. Since there are no workers getting old or retiring from firms, the values of filled vacancies (J_s^o and J_j^y) increase, which has positive effects on vacancy posts on both markets. When λ increases, more old workers search in both markets. Since the number of young workers decreases, capital-skill complementarity directs firms to post more vacancies on the junior market to employ enough workers for production, including old workers. Thus, θ_j increases as much as 58.4%, e_j^o increases 83.62%, ER_j^o increases 67.13%, and e_s^o increases 4.37% less than in the baseline calibration. Furthermore, w_s^o increases 5.07% instead of decreasing, and w_j^o increases less, to 3.28%. Because of the influx of old workers, θ_s decreases 55.96%, and ER_s^o decreases 5%.

Although θ_j increases a lot, the number of employed young workers (e_j^y) shrinks 11.79%, and the unemployment rate of young workers (UR^y) decreases 19.08%. Since the total number of young workers also decreases, the overall effects on w_j^y and ER_j^y are positive. The total unemployment rate decreases 17.26% and the average wage increases 5.42%. Finally, compared with the baseline results, λ has a negative effect on the average wage and the unemployment rate of the old (UR^o), and it has a positive effect on the unemployment rate of the young (UR^y) and the total UR through the aging transition channel.

In the second counterfactual experiment, we shut down the cross-market matching channel from the baseline calibration. Since there are no old workers in the junior market, we set $\phi = 1$. Then we remove the terms related to the cross-market matching channel from the value functions and labor market equilibrium conditions. The masses of old and young workers are still P_o and 1 as in the baseline calibration. We present the results in Table 4.

The results show that without the cross-market matching channel, UR^y decreases much more, to 15.99%, UR^o turns from negative to 34.42%, and the total UR decreases less, to 8.68% compared with the baseline model. e_j^y increases only 1.83% and e_s^o almost remains the same. The intuition is the following. First, when old workers can only search in the senior market, although firms' value of employment (J_s^o) increases and more vacancies are posted, e_s^o and ER_S^o still decrease slightly compared with the baseline results. UR^o increases as much as 34.42% due to much more unemployed old workers in this market. Second, there are still more young workers employed due to the capital-skill complementarity channel. The changes in ER_j^y and UR_y are bigger. Since fewer workers search in the junior market in this case, firms' value of employment (J_j^y) does not increase as much as before, and w_j^y increases less than in the baseline model. In general, λ_o has a positive effect on UR^y and a negative effect on UR^o and the total UR through the cross-market matching channel.

Third, we eliminate the capital-skill complementarity channel. We remove the production function, and allow the intermediate good to be used for final goods production with a 1–1 transformation technology. We set the values of goods market prices p_s and

[9] The corresponding value functions and key equations to solve each experimental model are listed in online Appendix E

Table 5
Counterfactual Experiments: Removing the capital-skill complementarity channel (percentage changes).

Junior workers		Senior workers		Cross-market matching workers	
w_j^y	−0.05	w_s^o	0.55	w_j^o	−0.10
θ_j	−3.55	θ_s	2.52	–	–
e_j^y	−0.2	e_s^o	31.16	e_j^o	23.98
ER_j^y	−0.2	ER_s^o	0.13	ER_j^o	−5.35
UR^y	−1.62			UR^o	−3.70
Average wage	2.19			Total UR	−10.56

Table 6
Sensitivity with respect to μ: The effects of λ_o.

%Changes	$\mu = 0.8$	$\mu = 1$ (baseline)	$\mu = 1.2$	$\mu = 2$
		Junior workers		
w_j^y	5.82	5.79	5.75	4.66
θ_j	4.96	5.65	6.38	14.34
e_j^y	0.27	0.30	0.34	0.73
ER_j^y	0.27	0.30	0.34	0.73
		Senior workers		
w_s^o	−2.06	−2.08	−2.10	−1.50
θ_s	−5.82	−7.82	−10.55	−50.54
e_s^o	31.04	31.00	30.95	29.75
ER_s^o	0.04	0.01	−0.03	−0.94
		Cross-market matching workers		
w_j^o	4.86	5.32	5.74	9.46
e_j^o	33.72	35.91	38.96	109.14
ER_j^o	2.09	3.75	6.08	59.66
		Aggregates		
UR^y	−2.13	−2.42	−2.71	−5.80
UR^o	−4.22	−3.91	−3.43	7.42
Average wage	1.83	1.82	1.81	2.13
Total UR	−13.34	−13.49	−13.62	−13.98

p_j to those in the baseline calibration. The value functions and labor market equilibrium conditions are the same as in the baseline model.

Table 5 shows that, compared with the baseline calibration, both θ_s and e_s^o increase. Since the intermediate good can be directly transformed to the final good, aging population implies higher J_s^o. Therefore, firms post more vacancies and offer higher wages in the senior market. As more job vacancies are available in the senior market, fewer old workers engage in cross-market searching. As a result, e_j^o increases by a lower percentage to 23.98%, and the overall effect decreases UR^o by 3.7%. Without the CES production function, the demand for labor decreases in the junior market, and firms post fewer vacancies, which lower θ_j, w_j^y, and w_j^o to 3.55%, 0.05%, and 0.1%, respectively. e_j^y and ER_j^y decrease −0.2% and UR^y increases −1.62% compared with the baseline results, which implies that fewer young workers are employed. In short, λ_o has a negative effect on UR^o, UR^y, and hence the total UR through the capital-skill complementarity channel.

4.4. Sensitivity tests

Since the productivity of senior workers could be higher or lower than that of junior workers in various industries, we conduct sensitivity tests by varying μ while holding the other parameters unchanged. Results corresponding to those in Table 2 for different values of μ are listed in Table 6. The table shows that market tightness, unemployment rates, and e_j^o are the most responsive variables. As senior workers become more productive, firms' value of employment in the junior market increases. Since firms cannot determine which type of workers would match with them, they must post more vacancies to match with more old workers. As a result, θ_j increases and e_j^o increases up to 109.14%. Although firms increase the number of vacancies posted in the senior market, the number of employed old workers (e_s^o) barely changes and there is a large decrease in θ_s. This implies that the positive effect in the junior labor market dominates that in the senior market. Eventually, this may lead the changes in UR^o to turn from negative to positive. Finally, UR^y decreases with the difference in productivity.

Next, we conduct sensitivity tests on the separation rates. We test in which (i) the young workers have a relatively low separation rate, and s_j is only one-half of s_s; (ii) the two separation rates are equal; (iii) young workers have a relatively high separation rate, and s_j is four times s_s. Table 7 reports corresponding to those in Table 2. In general, the results are not sensitive to separation rates.

Table 7
Sensitivity with respect to separation ratios: The effects of λ_o (percentage changes).

%Changes	$s_j = 0.5s_s$	$s_j = s_s$	$s_j = 2 * s_s$(baseline)	$s_j = 4s_s$
		Junior workers		
w_j^y	5.74	5.76	5.79	5.84
θ_j	4.58	5.01	5.65	6.45
e_j^y	0.18	0.22	0.30	0.46
ER_j^y	0.18	0.22	0.30	0.46
		Senior workers		
w_s^o	−2.10	−2.10	−2.08	−2.05
θ_s	−7.75	−7.78	−7.82	−7.82
e_s^o	30.95	30.97	31.00	31.06
ER_s^o	−0.03	−0.02	0.01	0.05
		Cross-market matching workers		
w_j^o	5.26	5.29	5.32	5.37
e_j^o	37.28	36.82	35.91	34.22
ER_j^o	4.80	4.45	3.75	2.47
		Aggregates		
UR^y	−2.04	−2.20	−2.42	−2.63
UR^o	−2.59	−3.07	−3.91	−5.26
Average wage	1.87	1.85	1.82	1.75
Total UR	−12.13	−12.69	−13.49	−14.43

Table 8
Sensitivity with respect to the replacement ratio: The effects of λ_o (percentage changes).

%Changes	Replacement ratios = 0.71	Replacement ratios = 0.4 (baseline)
	Junior workers	
w_j^y	5.59	5.79
θ_j	13.53	5.65
e_j^y	0.69	0.30
ER_j^y	0.69	0.30
	Senior workers	
w_s^o	−2.24	−2.08
θ_s	−13.21	−7.82
e_s^o	30.91	31.00
ER_s^o	−0.06	0.01
	Cross-market matching workers	
w_j^o	4.10	5.32
e_j^o	43.55	35.91
ER_j^o	9.59	3.75
	Aggregates	
UR^y	−5.50	−2.42
UR^o	−4.75	−3.91
Average wage	1.63	1.82
Total UR	−15.85	−13.49

Finally, we set the replacement ratio to be 0.4 in the baseline calibration. We then investigate how the results would change if the value of non-market activities is considered as a part of the value of leisure. We set the replacement ratio to be 0.71, following the most recent literature. Table 8 shows that, in general, the results vary little with the change in the replacement ratio, but all three unemployment rates decrease with higher replacement ratios.

5. Conclusion

We employed a labor search model with the CES production function and cross-market matching to study the channels through which the delayed retirement policy affects the unemployment rates of young and old workers. We found that the retirement policy has opposing effects on the unemployment rate of young workers through the cross-market matching and capital-skill complementarity channels. Through the cross-market matching channel, the retirement policy increases unemployment among young workers (the effect is ambiguous for old workers) and has a negative effect on the wages of cross-market matched workers. This effect on the unemployment rate of the young is negative (positive for old workers) through the capital-skill complementarity channel. If we shut down both channels and only model the transition from young to old, the effects on unemployment rates become ambiguous. These results are essentially due to the different effects of the market tightness of the two markets on wages and unemployment rates. Nevertheless, this channel can complicate the analysis of the results. Finally, we calibrated our model to U.S. data and estimated the effects of retirement policy during the first decade of this century. We found that there were significant changes in most wages and the unemployment rates of both markets. The counterfactual experiments provide empirical evidence for our theoretical analysis of each channel.

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Appendix. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jmacro.2021.103387.

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